Field Quantization in krein space

By : Farrin Payandeh

Amirkabir U. of Technology

F. Payandeh, M. Mehrafarin, M. V. Takook

6th Internatoinal workshop on Psudo Hermitian Hamiltonians in Quantum Physics. (16-18 July, City University, London)

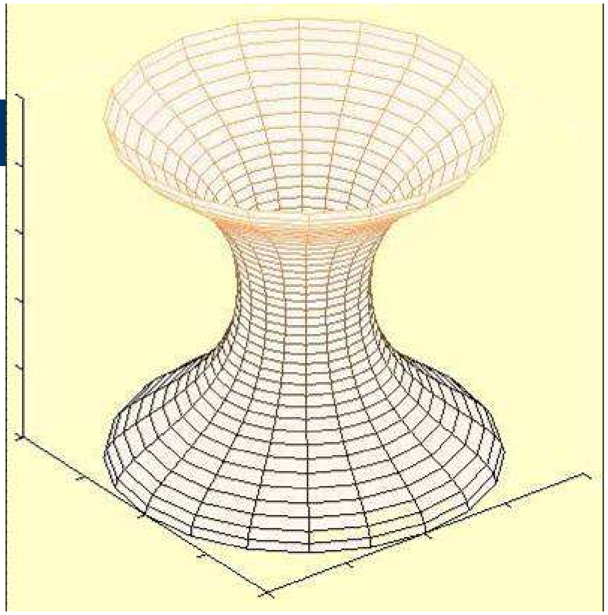
de-Sitter spacetime

$$X_H = \{x \in R^5; x^2 = \eta_{\alpha\beta} x^\alpha x^\beta = -H^{-2}\},\ \ \alpha,\beta = 0,1,2,3,4$$

$$\eta_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1,-1)$$

Einstein equation: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \quad \Rightarrow \begin{cases} \Lambda = \dfrac{R^{(0)}}{4} = 3H^2 \\ R^{(0)}_{\mu\nu} = 3H^2 g^b_{\mu\nu} \end{cases}$$



Close coordinate:

$$X^0 = H^{-1} tg\rho$$

$$X^1 = (H\cos\rho)^{-1} \sin\alpha \sin\theta \cos\varphi$$

$$X^2 = (H\cos\rho)^{-1} \sin\alpha \sin\theta \sin\varphi \qquad \rho = \tan^{-1}(\sinh Ht)$$

$$X^3 = (H\cos\rho)^{-1} \sin\alpha \cos\theta$$

$$X^4 = (H\cos\rho)^{-1} \cos\alpha \qquad 0 \le \varphi \le 2\pi \ \ ,0 \le \theta \le \pi, 0 \le \alpha \le \pi, -\frac{\pi}{2} < \rho < \frac{\pi}{2}$$

de-Sitter spacetime

- **1930-1940** : Electron wave eq. in de Sitter spacetime (Dirac)
- **1940-1960** : Irreducible representations of de Sitter group
  (Newton – Tomas – Dexmier – Takahashi)
- **1960-1970** : Quantum field theory in de Sitter spacetime (Chernikov – Tagirov)
- **1980** : Appearance of de Sitter metric in inflationary model (Linde)
- **1986** : Quantizing the minimally coupled scalar field and calculating the graviton two point function in de Sitter spacetime (Antoniadis - Iliopoulos – Tomaras, Allen – Turyn)

(a) **Breaking of the covariance.**
(b) **Appearance of the infrared divergence.**
- **Allen B.**, Phys. Rev. D, **32** (1985) 3136, "Vacuum states in de Sitter space".

de-Sitter spacetime

- **Covariant Quantization and Negative Frequency States**:

- **Takook M.V**., Phd thesis, Paris (1997)

- **De Bievre S**., **Renaud J**., Phys. Rev. D., **57** (1998) 6230, “Massless Gupta-Bleuler vacuum on the (1+1)-dimensional de Sitter spacetime”.

- **Gazeau J. P**., **Renaud J., Takook M.V.**, Class. Quant. Grav. **17** (2000) 1415, gr-qc / 9904023, “Gupta-Bleuler quantization for minimally coupled scalar field in de Sitter spacetime.

- **Takook M.V**., Mod. Phys. Lett. A, **16** (2001) 1691, gr-qc/0005020, “Covariant two-point function for minimally coupled scalar field in de Sitter spacetime.

Linear quantum gravity in de Sitter spacetime and infrared divergence

- **The appearance of infrared divergence**

- **Antoniadis I**., **Iliopoulos J., Tomaras T. N.**, Phys. Rev. Lett. **56** (1986) 1319, “Quantum instability of de Sitter space”.

- **Allen B**., **Turyn M.** Nucl. Phys. B. **292** (1987) 813, “An evaluation of the graviton propagator in de Sitter space”.

- **The infrared divergence is not physical**

- **Antoniadis I**., **Iliopoulos J., Tomaras T. N.**, Nucl. Phys. B. **462** (1996) 437, “One-loop effective action around de Sitter space”.

- **Takook M.V**., PhD thesis, Paris (1997).

Linear quantum gravity in de Sitter spacetime and infrared divergence

- **Takook M.V**., Proceeding of the Wigsym6, 1999, Istanbul, gr-qc/0001052, "Covariant two-point function for linear gravity in de Sitter space".
- **Garidi T**., **Gazeau J. P., Renaud J., Rohani S., Takook M.V.**, "Linear quantum gravity in de Sitter space".
- **de Vega H. J**., **Ramirez J., Sanchez N.**, Phys. Rev. D. **60** (1999) 044007, astro-ph/9812465, "Generation of gravitational waves by generic sources in de Sitter spacetime".
- **Hawking S. W**., **Hertog T., Turok N.**, hep-th/0003016, "Gravitational waves in open de Sitter space".
- **Higuchi A**., **Kouris S. S.**, gr-qc/0004079, "Large-distance behavior of the graviton two-point function in de Sitter spacetime".

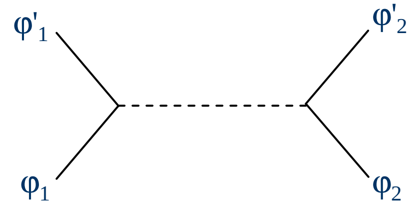
Linear quantum gravity in de Sitter spacetime and infrared divergence

$(x.x') \rightarrow \infty \quad \Rightarrow$ $\boxed{G^{\mu\nu\rho'\sigma'} \rightarrow \text{logarithmic-divergence}}$ $G^{\mu\nu\rho'\sigma'} \propto \{\log(-H^2 x.x')\}$

$G^{\mu\nu\rho'\sigma'}(x,x') = \langle 0 | h(x)^{\mu\nu} h^{\rho'\sigma'}(x') | 0 \rangle$

Tree-level scattering amplitude:

$$M = \int T^{\mu\nu}(x) G^{F}_{\mu\nu\rho'\sigma'}(x,x') T^{\rho'\sigma'}(x') d^4x d^4x'$$



Field eq. in ambient space:

$$h_{\mu\nu}(X) = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x^\beta}{\partial X^\nu} K_{\alpha\beta}(x(X)) \qquad K_{\alpha\beta}(x) = D_{\alpha\beta}(x,\partial)\varphi_{Llm}(x)$$

$\boxed{\varphi = 0}$

Two-point function:

$$W_{\alpha\beta\alpha'\beta'}(x,x) = \langle 0 | \ K_{\alpha\beta}(x) K_{\alpha'\beta'}(x') | 0 \rangle \qquad G^{\mu\nu\rho'\sigma'} = \frac{\partial X^\mu}{\partial x^\alpha}\frac{\partial X^\nu}{\partial x^\beta}\frac{\partial X^{\rho'}}{\partial x'^{\alpha'}}\frac{\partial X^{\sigma'}}{\partial x'^{\beta'}} W^{\alpha\beta\alpha'\beta'}$$

$$\boxed{W_{\alpha\beta\alpha'\beta'}(x,x') = \Delta_{\alpha\beta\alpha'\beta'}(x,x') W_{mc}(x,x')}$$

Massless minimally coupled scalar field

$$L=\frac{1}{2}[-g(x)]^{1/2}\{g^{\mu\nu}(x)\nabla_\mu\varphi(x)\nabla_\nu\varphi(x)-[m^2+\zeta R(x)]\varphi^2(x)\}$$

$$\delta S=0\Rightarrow \quad \boxed{[\ \partial^2_H+(m^2+\zeta R)]\varphi(x)=0} \qquad \partial^2_H=\frac{1}{\sqrt{-g}}\partial_\nu\sqrt{-g}\,g^{\nu\mu}\partial_\mu \qquad g=\det g_{\mu\nu}$$

field solution: $\boxed{\varphi(x)=\chi(\rho)D(\Omega)}$ $\quad D(\Omega)=Y_{Llm}(\Omega)$

$$\chi(\rho)=A_L(\cos\rho)^{\frac{3}{2}}[P^\lambda_{L+\frac{1}{2}}(\sin\rho)-\frac{2i}{\pi}Q^\lambda_{L+\frac{1}{2}}(\sin\rho)] \qquad A_L=H\frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(L-\lambda+\frac{3}{2})}{\Gamma(L+\lambda+\frac{3}{2})}\right)^{\frac{1}{2}}$$

$$\lambda=\sqrt{\frac{9}{4}-k} \qquad \frac{9}{4}\geq k\geq 0$$

$$\lambda=i\sqrt{k-\frac{9}{4}} \qquad \frac{9}{4}\leq k$$

$$k=\frac{m^2}{H^2}+12\zeta.$$

Massless minimally coupled scalar field

$$<\phi,\psi>=\frac{i}{H^2}\int_{\rho=0}\varphi^*(\rho,\Omega)\overleftrightarrow{\partial}_\rho\psi(\rho,\Omega)d\Omega$$

$$<\varphi^\lambda_{L'l'm'},(\varphi^\lambda_{Llm})^*>=0$$

$$<\varphi^\lambda_{L'l'm'},\varphi^\lambda_{Llm}>=\delta_{LL'}\delta_{ll'}\delta_{mm'}$$

Massless minimally coupled: $m=\zeta=0 \Rightarrow$ $\varphi=0$

$$M_{\mu\nu}=-i(X_\alpha\partial_\beta-X_\beta\partial_\alpha) \quad , \quad (M_{03}+iM_{04})\varphi_{1,0,0}=-i\frac{4}{\sqrt{6}}\varphi_{2,1,0}+\varphi_{2,0,0}+\frac{3H}{4\pi\sqrt{6}}$$

Zero mode problem: $A_0=H\frac{\sqrt{\pi}}{2}\frac{\Gamma(0)}{\Gamma(3)}=\infty$

Correct zero mode: $\varphi_{000}=\psi_g+\frac{\psi_s}{2}$

$$\psi_g=\frac{H}{2\pi} \qquad \psi_s=-i\frac{H}{2\pi}[\rho+\frac{1}{2}\sin 2\rho] \qquad \rho=tg^{-1}(\sinh Ht)$$

Massless minimally coupled scalar field

$$(M_{03}+iM_{04})\varphi_0=(M_{03}+iM_{04})\psi_s=-i\frac{\sqrt{6}}{4}\varphi_{1,0,0}-i\frac{\sqrt{6}}{4}\varphi^*_{1,0,0}-\frac{\sqrt{6}}{4}\varphi_{1,1,0}-\frac{\sqrt{6}}{4}\varphi^*_{1,0,0}$$

Field operator in Krein space: $\varphi(x)=\frac{1}{\sqrt{2}}(\varphi_p(x)+\varphi_n(x))$

$$\varphi_p(x)=\sum_k a_k\varphi_k(x)+H.C\quad,\quad \varphi_n(x)=\sum_k b_k\varphi^*_k(x)+H.C$$

$$k\in K'=K+\{0\}$$

$$a^+_k|0\rangle=|\text{physical state}\rangle,\quad b^+_k|0\rangle=|\text{unphysical state}\rangle\quad,\quad a_k|0\rangle=b_k|0\rangle=0$$

$$W^p_{mc}=\frac{H^2}{8\pi^2}\left[\frac{1}{1-z}-\ln(1-z)+f_{\alpha\beta}(\eta,\eta')+\text{const}\right]$$

$$W_{mc}(x,x')=\frac{H^2 i}{8\pi}\varepsilon(x^0-x'^0)[\delta(1-z)+\theta(z-1)]$$

$$\varepsilon(x^0-x'^0)=\begin{cases}1 & x^0\succ x'^0\\ 0 & x^0=x'^0\\ -1 & x^0\prec x'^0\end{cases}$$

$$z(x,x')=-H^2x.x'=-H^2\eta_{\rho\sigma}x^\rho x^\sigma\qquad \rho,\sigma=0,1,2,3,4$$

Covariant graviton field operator

$$K_{\alpha\beta}(x) = \sum_{\lambda,L} a^{\lambda}_{Llm} D^{\lambda}_{\alpha\beta}(x,\partial)\varphi_{Llm}(\rho,\Omega) + H.C$$

Vacuum definition: $\mathbf{a}^{\lambda}_{\mathbf{Llm}} |\mathbf{0}\rangle = \mathbf{0} \qquad \forall\ \mathbf{0} \le \mathbf{l} \le \mathbf{L} \qquad -\mathbf{l} \le \mathbf{m} \le \mathbf{l}$

Covariant field operator in Krein space:

$$K_{\alpha\beta}(x) = \sum_{\lambda,L} a^{\lambda}_{Llm} D^{\lambda}_{\alpha\beta}(x,\partial)\varphi_{Llm}(\rho,\Omega) + H.C + \sum_{\lambda,L} b^{\lambda}_{Llm} D^{\lambda}_{\alpha\beta}(x,\partial)\varphi^{*}_{Llm}(\rho,\Omega) + H.C$$
$$\forall\ 0 \le l \le L \qquad -l \le m \le l$$

Gupta-bleuler vacuum: $a^{\lambda}_{Llm}|0\rangle = 0 \qquad b^{\lambda}_{Llm}|0\rangle = 0$

Generalization to QFT in Minkowski spacetime

The principles of QFT:

(1) Covariance
(2) Causality
(3) Existence of the Vacuum
(4) Positivity

The principles of QFT with the negative frequency states (N.F.S) :

(1) Covariance
(2) Causality
(3) Existence of the Vacuum

Krein QFT calculation

(a) Free scalar field: $(\Box + m^2)\phi(x) = 0$

The two sets of solutions:

$$u_p = u_k(x) = \frac{e^{-ik.x}}{\sqrt{(2\pi)^3 2\omega_k}}$$ **Positive energy**

$$u_n = u_k^*(x) = \frac{e^{ik.x}}{\sqrt{(2\pi)^3 2\omega_k}}$$ **Negative energy**

$$k^0 = \omega_k = \omega_{-k} = \sqrt{\vec{k}^2 + m^2} \geq 0$$

$$u_p(k,x) = u_n^*(k,x) \quad , \quad u_n(k,x) = u_p^*(k,x)$$

$$\phi_K(x) = \frac{1}{\sqrt{2}}\left[\phi_p(x) + \phi_n(x)\right]$$

$$\phi_p(x) = \int d^3\mathbf{k}\left[a(\mathbf{k})u_p(k,x) + a^+(\mathbf{k})u_p^*(k,x)\right]$$

$$\phi_n(x) = \int d^3\mathbf{k}\left[b(\mathbf{k})u_n(k,x) + b^+(\mathbf{k})u_n^*(k,x)\right]$$

The nonzero commutation relations:

$$[a_{\vec{k}}, a_{\vec{k}'}{}^\dagger] = +\delta^3(\vec{k} - \vec{k}'), \quad [b_{\vec{k}}, b_{\vec{k}'}{}^\dagger] = -\delta^3(\vec{k} - \vec{k}')$$

Krein QFT calculation

The vacuum state :

$$a^{+}(\mathbf{k})\,|\,0>=|1_k> \quad = |Physical-state\rangle \;; \qquad a(\mathbf{k})\,|\,0>=0 \quad \forall\, \mathbf{k}$$

$$b^{+}(\mathbf{k})\,|\,0>=|\bar{1}_k> \quad = |Unphysical-state\rangle \;; \qquad b(\mathbf{k})\,|\,0>=0 \quad \forall\, \mathbf{k}$$

$$a(\mathbf{k})\,|\,unphysical-state>=0 \;, \quad b(\mathbf{k})\,|\,physical-state>=0 \qquad \forall$$

The norm in Krein space: $<0\,|\,0>=1$

$$<1_{\vec{k}'}|1_{\vec{k}}>=+\delta^3(\vec{k}-\vec{k}'), \quad <\bar{1}_{\vec{k}'}|\bar{1}_{\vec{k}}>=-\delta^3(\vec{k}-\vec{k}')$$

The vacuum energy:

$$H=\int d^3\mathbf{k}\left[a^{+}(\mathbf{k})a(\mathbf{k})+b^{+}(\mathbf{k})b(\mathbf{k})+a^{+}(\mathbf{k})b^{+}(\mathbf{k})+a(\mathbf{k})b(\mathbf{k})\right]k^0$$

$$<0|\,a^{+}a\,|\,0>=<0\,|\,b^{+}b\,|\,0>=<0\,|\,a^{+}b^{+}\,|\,0>=<0\,|\,ab\,|\,0>=0$$

$$<0\,|\,H\,|\,0>=0$$

Krein QFT calculation

The two point function: $iG_T(x,x') = <0|T\phi(x)\phi(x')|0>$

$$G_T(x,x') = \mathrm{Re}\, G_F^P(x,x') = -\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}$$

$$2\sigma_0 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu) \geq 0$$

The Feynman propagator:

$$G_F^P(x,x') = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik.(x-x')}}{k^2-m^2+i\varepsilon} = -\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\left[\frac{J_1(\sqrt{2m^2\sigma_0}) - iN_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}\right] - \frac{im^2}{4\pi^2}\theta(-\sigma_0)\frac{K_1(\sqrt{2m^2(-\sigma_0)})}{\sqrt{2m^2(-\sigma_0)}}$$

The divergence appears in the imaginary part of this eq. and the real part is convergent :

$$\lim_{\sigma_0\to 0}\mathrm{Re}\, G_F^P\ (x,x') = \frac{m^2}{16\pi} - \frac{1}{8\pi}\delta(\sigma_0) \quad , \quad \lim_{\sigma_0\to\infty}\mathrm{Re}\, G_F^P\ (x,x') = 0$$

$$\lim_{z\to 0}\frac{J_1(z)}{z} = \frac{1}{2} \quad , \quad \lim_{z\to 0}\frac{N_1(z)}{z} = -\frac{2}{\pi}\frac{1}{z^2} \quad , \quad \lim_{z\to 0}\frac{K_1(z)}{Z} = \frac{1}{z^2}$$

$$J_1(z) = \frac{z}{2}\sum_{s=0}^{\infty}\frac{(-1)^s}{s!(s+1)!}\left[\frac{z}{2}\right]^{2s}$$

$$N_1(z) = 2J_1(z)\log\frac{z}{2} - \frac{2}{z}$$

$$K_1(z) = -\frac{\pi}{2}[J_1(iz) + iN_1(iz)]$$

Krein QFT calculation

Considering quantum metric fluctuations:

$$< G_T(x,x') >= \frac{-1}{8\pi}\sqrt{\frac{\pi}{2\langle \sigma_1^{\ 2}\rangle}}\exp(-\frac{\sigma_0^{\ 2}}{2\langle \sigma_1^{\ 2}\rangle}) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}$$

(Divergence – free Propagator)

For $2\sigma_0 = 0$: $h_{\mu\nu}$, $\langle \sigma_1^{\ 2}\rangle \neq 0$

$$< G_T(0) >= \frac{-1}{8\pi}\sqrt{\frac{\pi}{2\langle \sigma_1^{\ 2}\rangle}} + \frac{m^2}{8\pi}$$

Krein QFT calculation

(b) Free massless vector field:

$$\partial^2 A_\mu = 0 \qquad , \qquad A_\mu^K(x) = \frac{1}{\sqrt{2}}[A_\mu^p(x) + A_\mu^n(x)]$$

$$A_\mu^K(x) = \frac{1}{\sqrt{2}} \int d^3\vec{k} \sum_{\lambda=0}^{3} \epsilon_\mu^\lambda(\vec{k})[(a_{\vec{k}}^\lambda + b_{\vec{k}}^{\lambda\dagger})\, u_p(k,x) + (a_{\vec{k}}^{\lambda\dagger} + b_{\vec{k}}^\lambda) u_n(k,x)]$$

$$[a_{\vec{k}}^\lambda, a_{\vec{k}'}^{\lambda'\dagger}] = -\eta^{\lambda\lambda'}\delta^3(\vec{k}-\vec{k}'), \quad [b_{\vec{k}}^\lambda, b_{\vec{k}'}^{\lambda'\dagger}] = +\eta^{\lambda\lambda'}\delta^3(\vec{k}-\vec{k}')$$

$$|one\ \ physical\ \ state >= |1_{\vec{k},a}^\lambda > \propto a_{\vec{k}}^{\lambda\dagger}|0> \qquad \lambda = 1,2$$

$$|one\ \ unphysical\ \ state >= |\bar{1}_{\vec{k},a}^\lambda > \propto a_{\vec{k}}^{\lambda\dagger}|0> \qquad \lambda = 0,3$$

$$|one\ \ unphysical\ \ state >= |\bar{1}_{\vec{k},b}^\lambda > \propto b_{\vec{k}}^{\lambda\dagger}|0> \qquad \lambda = 0,1,2,3$$

Four of the above states have negative norms, and two of them are unphysical positive norm states.

Krein QFT calculation

The energy operator in Krein space:

$$H = \int d^3\vec{k}\, \omega_{\vec{k}} \sum_{\lambda=0}^{3} (-\eta_{\lambda\lambda})(a_{\vec{k}}^{\lambda\dagger} a_{\vec{k}}^{\lambda} + b_{\vec{k}}^{\lambda\dagger} b_{\vec{k}}^{\lambda} + a_{\vec{k}}^{\lambda\dagger} b_{\vec{k}}^{\lambda\dagger} + a_{\vec{k}}^{\lambda} b_{\vec{k}}^{\lambda})$$

$$D_{\mu\nu}^{T}(x,x') = -\eta_{\mu\nu} G_T(x,x')$$

The vacuum energy and momentum : $< 0 | \mathrm{H} | \; 0 >= 0$

$$< 0 | P | \; 0 >= 0$$

The vacuum energy is automatically zero.

Krein QFT calculation

(c) Free Spinor field ($s=\frac{1}{2}$):

$$(i\not\partial - m)\psi(x) = 0 \quad , \quad \psi(x) = \frac{1}{\sqrt{2}}\,[\psi_p(x) + \psi_n(x)]$$

$$\psi_K(x) = \frac{1}{\sqrt{2}}\int d^3\vec{k}\sum_{s=1,2}[(b_{\vec{k}s} + c^{\dagger}_{\vec{k}s})\mathcal{U}^s(k,x) + (d^{\dagger}_{\vec{k}s} + a_{\vec{k}s})\mathcal{V}^s(k,x)]$$

$$\mathcal{U}^s(k,x) = \sqrt{\frac{m}{(2\pi)^3\omega_{\vec{k}}}}\; u^s(\vec{k})e^{-ik.x} \qquad \text{(positive energy)}$$

$$\mathcal{V}^s(k,x) = \sqrt{\frac{m}{(2\pi)^3\omega_{\vec{k}}}}\; v^s(\vec{k})e^{ik.x} \qquad \text{(negative energy)}$$

$$\{b_{\vec{k}s}, b^{\dagger}_{\vec{k}'s'}\} = \{d_{\vec{k}s}, d^{\dagger}_{\vec{k}'s'}\} = \delta_{ss'}\;\; \delta^3(\vec{k} - \vec{k}')$$

$$\{a_{\vec{k}s}, a^{\dagger}_{\vec{k}'s'}\} = \{c_{\vec{k}s}, c^{\dagger}_{\vec{k}'s'}\} = \delta_{ss'}\;\; \delta^3(\vec{k} - \vec{k}')$$

Krein QFT calculation

$$< 1^b_{\vec{k},s} \mid 1^d_{\vec{k}',s} >= \delta_{bd}\ \delta^3(\vec{k}-\vec{k}') \quad , \quad < \bar{1}^a_{\vec{k},s} \mid \bar{1}^c_{\vec{k}',s} >= \delta_{ac}\ \delta^3(\vec{k}-\vec{k}')$$

$$H = \int d^3\vec{k}\ \omega_{\vec{k}} \sum_{s=1,2} [b^\dagger_{\vec{k}s} b_{\vec{k},s} + b^\dagger_{\vec{k}s} c^\dagger_{\vec{k}s} + c_{\vec{k},s} b_{\vec{k},s} + c_{\vec{k},s} c^\dagger_{\vec{k}s} - d_{\vec{k},s} d^\dagger_{\vec{k}s} - d_{\vec{k},s} a_{\vec{k}s} - a^\dagger_{\vec{k}s} d^\dagger_{\vec{k}s} - a^\dagger_{\vec{k}s} a_{\vec{k}s}]$$

$$< 0 \mid H \mid 0 >= 0$$

$$\begin{aligned} S_T(x,x') &= (i\ \not\partial + m)\frac{1}{2}[G^p_F(x,x') + G^{p*}_F(x,x')] = (i\ \not\partial + m)G_T(x,x') \\ &= \frac{1}{2}\left(S^p_F(x,x') + \gamma^5\gamma^0[S^p_F(x,x')]^\dagger\gamma^0\gamma^5\right). \end{aligned}$$

$$\begin{aligned} S_T(x,x') = \frac{1}{8\pi}\ i\gamma^\mu(x_\mu - x'_\mu)\{&\sqrt{\frac{\pi}{2<\sigma_1^2>}}\ e^{-\frac{\sigma_0^2}{2<\sigma_1^2>}}[\frac{\sigma_0}{<\sigma_1^2>} + m^2\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}] \\ &+\frac{m}{2\sqrt{2}}\ \theta(\sigma_0)[\sqrt{2m^2\sigma_0}\ J_0(\sqrt{2m^2\sigma_0}) - 2J_1(\sqrt{2m^2\sigma_0})]\} \\ &+\frac{m}{8\pi}\ [-\sqrt{\frac{\pi}{2<\sigma_1^2>}}e^{-\frac{\sigma_0^2}{2<\sigma_1^2>}} + m^2\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}] \end{aligned}$$

Krein QFT calculation

$b^{\dagger}_{\vec{k}s}$, $d^{\dagger}_{\vec{k}s}$ are respectively the creation operators of physical one-particle and one-antiparticle states with positive energy, and $c^{\dagger}_{\vec{k}s}$, $a^{\dagger}_{\vec{k}s}$ are called the creation operators of unphysical one-particle and one-antiparticle states respectively with negative energy.

Presence of the "unphysical" states plays the role of an automatic renormalization tool for the theory. The physical interpretation of the unphysical negative energy states is not yet clear and any further progress calls for more investigations [28–31]. However, in the case of spinor field one can interpret as the unphysical particle and antiparticle running in the inverse of the associated time direction to physical ones!

Conclusions

-The problem of singularity in the QFT (the regularization and renormalization procedure)

-The problem of renormalizability in Quantum gravity

-The anomally may be in the QFT, not in the general relativity (which is not renormalized in the quantum level)

-The problem may be solved in the :

Quantum Field Theory with the N.F.S.

The new method of quantization using unphysical negative frequency states results in an automatically renormalized theory.

References

[1] De Bievre S., Renaud J., Phys. Rev. D **57**, 6230 (1998).

[2] Gazeau J. P., Renaud J., Takook M. V., Class. Quant. Grav. **17**, 141 (2000).

[3] Dirac P. A. M., Proc. Roy. Soc. A **180**, 1 (1942).

[4] Gupta S. N., Proc. Phys. Soc. A **63**, 681 (1950).

[5] Bleuler K., Helv. Phys. Acta **23**, 567 (1950).

[6] Hawking S. W., Hertog T., Phys. Rev. D **65**, 103515 (2002).

[7] Tomas L. H., Annals of Math., **42**, 113 (1941).

[8] Newton T. H., Annals of Math., **51**, 730 (1950).

[9] Dexmier J., Bull. Soc. Math. France, **89**, 9 (1961).

[10] Takahashi B., Bull. Soc. Math. France, **91**, 289 (1963).

[11] Chernikov N. A., Tagirov E. A., Ann. Inst. Henri Poincare, **IX**, 109 (198).

[12] Allen B., Phys. Rev. D, **32**, 3136 (1985).

[13] Antoniadis I., Iliopoulos J., Tomaras T. N., Phys. Rev. Lett. **56**, 1319 (1986).

[14] Allen B., Turyn M., Nucl. Phys. B, **292**, 813 (1987).

[15] Antoniadis I., Iliopoulos J., Tomaras T. N., Nucl. Phys. B, **462**, 437 (1996).

References

[16] Takook M.V., *Théorie quantique des champs pour des systèmes élémentaires "massifs" et de "masse nulle" sur l'espace-temps de de Sitter* (Thèse de l'université Paris VI, 1997).

[17] Takook M. V., Proceedings of the 6th International Wigner Symposium (WIGSYM6), Istanbul, Turkey, 16-22 Aug. 1999.

[18] de Vega H. J., Ramirez J., Sanches N., Phys. Rev. D, 60, 044007 (1999).

[19] Hawking S. W., Hertog T., Turok N., hep-th/0003016.

[20] Higuchi A., Kouris S. S., gr-qc/ooo4079.

[21] Garidi T., Gazeau J. P., Takook M. V., J. Math. Phys. 44, 3838 (2003).

[22] Takook M. V., Int. J. Mod. Phys. E 14, 219 (2005).

[23] Takook M. V., Mod. Phys. Lett. A 16, 1691 (2001).

[24] Payandeh F., Khani S., Takook M. V., Mehrafarin M., "Quantization of free boson fields in Krein space", submitted for publication.

[25] Fatahi N., Payandeh F., Rouhani S., Takook M.V., "Quantization of spinor fields in generalized Krein space", submitted for publication.

References

[26] Ford H. L., gr-qc/9707062 (1997).

[27] Rouhani S., Takook M. V., gr-qc/0607027 (2006).

[28] Ramirez A., Mielinik B., Rev. Mex. Fis. 49S2, 130 (2003).

[29] Helfer A. D., hep-th/9811081 (1998).

[30] Visser M., Barcelo C., gr-qc/0001099 (2000).

[31] Blinnikov S., Surv. High Energy. Phys. 15, 37 (2000).

[32] Rouhani S., Takook M. V., Europhys. Lett. 68, 15 (2004).

[33] Takook M. V., Int. J. Mod. Phys. E 11, 509 (2002).

[34] Khosravi H., Naseri M., Rouhani S., Takook M. V., Phys. Lett. B, 640, 48 (2006).

[35] Naseri M., Rouhani S., Takook M. V., gr-qc/0704.2302 (2007).

[36] Payandeh F., Mehrafarin M., Takook M. V., "One-loop approximation of Møller scattering in generalized Krein space quantization", in preparation.

THE END