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Quantization Of Massless Conformally Vector Field In de Sitter Space-Time

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An open problem of theoretical physics :

Unifying theory that include all fundamental forces.

The major difficulty :

Unifiying gauge interactions and massless fields.

A solution that maybe work :

Indefinite metric.

Why de Sitter space time?

Astrophysical data indicate that the universe is in a de Sitter phase.

Einstein's equation:

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = -K T_{\mu\nu}$$

$$T_{\mu\nu} = 0$$

$$\Lambda = 12 H^2 > 0$$

How can we quantize a field?

Streater and wightman's method (axiomatic field theory):

$$w(x,x') = \langle \Omega | k(x), k(x') | \Omega \rangle$$

$w(x,x')$ *the two point function,*

$|\Omega\rangle$ *the vacuum state,*

$k(x)$ *the massless vector field.*

Ref.[1] R.F.Streater and A.S.Wightman,Benjamin,NewYork (1964)"PCT,Spin And Statistics "

What equation does k(x) satisfy?

So(1,4) (de Sitter group) leaves this form invariant:

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2 = constant, \quad (3)$$

So(2,4) (conformal group) leaves the following form invariant:

$$u_0^2 - u_1^2 - u_2^2 - u_3^2 - u_4^2 + u_5^2 = constant \quad (4)$$

$\underline{de\ Sitter\ \ group}$		$\underline{conformal\ \ group}$
$representation$	$\rightarrow$	$rep.(positive\ energy)$ $\oplus$ $rep.(negative\ energy)$

[2] A.O. Barut and A. Bohm, J.Math.Phys.11, 2938, (1970)
E. Angelopoulos and M. Laoues, Rev.Math.Phys. 10, 271,(1998)

Conformally invariant field equation

Dirac's method finds conformally invariant equation.

$\underline{Dirac's\ cone}$		$\underline{physical\ space}$
conformally invariant equations	$\xrightarrow{projection}$	*conformally equations invariant*

[3] P.A.M. Dirac, Ann.Math. 36 657 (1935)

Dirac's cone:

A 5- dimensional supersurface in R^6

$$u^2 = \eta_{ab} u^a u^b \qquad (5)$$

$$\eta_{ab} = diag(1,-1,-1,-1,-1,1)$$

Conformally invariant system on the cone:

$$\begin{cases} (\partial_a \partial^a)^p \Psi = 0 \\ N_5 \Psi = (p-2)\Psi \end{cases} \qquad (6)$$

$$N_5 \equiv u^a \partial_a$$

[4] S.Behroozi, S.Rouhani, M.V.Takook, M.R.Tanhayi, Phys.Rev.D. 74 124014 (2006)

The projection on de Sitter space:

$$\begin{cases} (\partial_a \partial^a)\ \Psi_a = 0 \\ N_5\ \Psi_a = -\Psi_a \end{cases} \qquad (7)$$

$$\begin{cases} x^{\alpha} = (u^5)^{-1}\ u^a \\ x^5 = u^5 \end{cases} \qquad (8)$$

[4] S.Behroozi, S.Rouhani, M.V.Takook, M.R.Tanhayi, Phys.Rev.D. 74 124014 (2006)

Massless conformally invariant vector field in de Sitter space-time:

$$Q^1 k + D_{1\alpha}\,\bar{\partial}\cdot k = 0 \qquad D_{1\alpha} = \bar{\partial}_\alpha$$

$$(Q^0 - 2)\,\bar{\partial}\cdot k = 0, \qquad (9)$$

$$k_\alpha = \varepsilon_\alpha(x,\zeta,z,\sigma)\,(x.\xi)^\sigma$$

$$\varepsilon_\alpha(x,\zeta,z,\sigma) = (\bar{z}_\alpha + a\frac{z\cdot x}{x\cdot\xi}\,\bar{\xi}_\alpha) \qquad (10)$$

$$\sigma = 0,-1,-2,-3$$

$a =$ *arbitrary constan parameter.*

Ref.[4]

Space of solutions and the indefinite metric

$$k_\alpha = \varepsilon_\alpha(x,\zeta,z,\sigma)\,(x.\xi)^\sigma, \qquad (10)$$

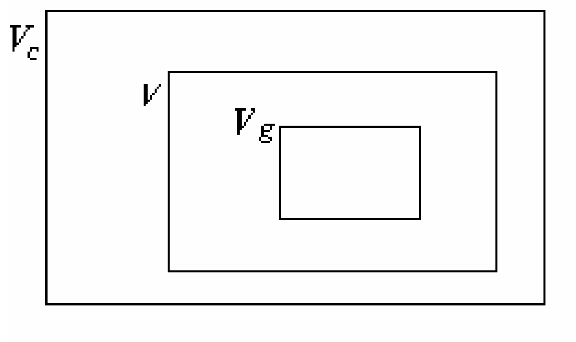
de Sitter invariant inner product on the space of solutions:

$$(k_1,k_2) = \frac{i}{H^2}\int_\rho [k_1^* \cdot \partial_\rho k_2 - c((\partial_\rho x)\cdot k_1^*)(\partial\cdot k_2) - (1^* \overrightarrow{\leftarrow} 2)]d\Omega$$

(11)

[5] J.P.Gazeau, M.Hans, R.Murenzi, Class.Quan.Grav.6 329 (1989)

$$in\,V_c \quad : \quad (k_1, k_2) > 0 \qquad\qquad indefinite$$



$$< 0$$

$$= 0,$$

$$in\,V: \quad \partial \cdot k = 0 \;\; \Rightarrow \quad (k_1, k_2) = \frac{i}{H^2} \int_\rho [k_1^* \cdot \partial_\rho k_2 - c((\partial_\rho x) \cdot k_1^*)(\partial \cdot k_2)] d\Omega$$

$$(k_1, k_2) > 0 \qquad semi\!-\!definite \qquad\qquad (12)$$

$$= 0$$

$$in\,V_g: \qquad (k_1, k_2) = 0.$$

$$physical\,space \;\; V/V_g \quad , \quad (k_1, k_2) \;\; is \;\; positive\!-\!definite$$

It has been proven that the use of an indefinite metric is unavoidable if one insists on the preservation of causality and covariance in gauge quantum field theories.

[6] F.Strochi, Phys.Rev.D. 17 2010 (1978)

The two-point function

$$w(z,z') = \langle \Omega | k(z) k(z') | \Omega \rangle, \qquad (13)$$

$$w_{\alpha\alpha'}(z,z') = c_s \int \sum_{\lambda} \varepsilon_{\alpha}^{\lambda}(z,\xi,Z,\sigma_1) \varepsilon_{\alpha'}^{\lambda}(z',\xi,Z,\sigma_2)$$

$$(z\cdot\zeta)^{\sigma_1}(z\cdot\zeta)^{\sigma_2}\, d\mu(\zeta), \qquad (14)$$

$$w_{\alpha\alpha'}(x,x') = bv\, w_{\alpha\alpha'}(z,z'), \qquad (15)$$

[7] J.Bros, J.P.Gazeau and U. Moschella,
Phy.Rev.Lett 73 1746 (1994)
ibid. Rev.Math.Phys. 8 327 (1996)

The causality condition as in Ref. [1]

$$w_{\alpha\alpha'}(x,x') = w_{\alpha'\alpha}(x',x), \tag{16}$$

$$[k_\alpha(x), k_{\alpha'}(x')] = \frac{H^2}{24} D_{\alpha\alpha'}\, \varepsilon(x^0, x'^0)(Z(x,x') - 1), \tag{17}$$

$$Z(x,x') = -H^2\, x \cdot x'$$

$$\varepsilon(x^0, x'^0) = \begin{cases} 1 & x^0 > x'^0 \\ 0 & x^0 = x'^0 \\ -1 & x^0 < x'^0 \end{cases}$$

Conclusion

It was pointed out that Einstein’s theory of gravitation can be interpreted as a theory of metric field. In the background field method, $g_{\mu\nu} = g^{B.G}_{\mu\nu} + h_{\mu\nu}$, it can consider as a *massless symmetric tensor field* on fixed background.

We have shown to obtain conformally invariant wave equation for graviton, mixed-symmetry rank-3 tensor is needed.

[8] S.Rouhani, M.V.Takook, M.R.Tanhayi “Conformally invariant wave equation for massless spin-2 field in de Sitter space” appears in Phy.Rev.D

So it seems that the use of an indefinite metric is unavoidable for quantization of gravition.

We would like to thank Dr. Takook for his very useful discussions.