# Non-Hermitian von Roos Hamiltonian's $\eta$-weak-pseudo-Hermiticity and exact solvability

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**Abstract**

A complexified von Roos Hamiltonian is considered and a Hermitian first-order intertwining differential operator is used to obtain the related position dependent mass $\eta$-weak-pseudo-Hermitian Hamiltonians. Two "*user -friendly*" *reference-target* maps are introduced to serve for exact-solvability of some non-Hermitian $\eta$-weak-pseudo-Hermitian position dependent mass Hamiltonians. A non-Hermitian $\mathcal{PT}$-symmetric Scarf II and a non-Hermitian periodic-type $\mathcal{PT}$-symmetric Samsonov-Roy potentials are used as *reference* models in a *"user-friendly" reference-target* map and the corresponding isospectral Hamiltonians are obtained. It is observed that for each exactly-solvable *reference* Hamiltonian there is a corresponding set of exactly-solvable *target* Hamiltonians.

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## 1 Introduction

Subjected to von Roos constraint $\alpha+\beta+\gamma=-1$; $\alpha,\beta,\gamma\in\mathbb{R}$, the von Roos position-dependent-mass (PDM) Hamiltonian [1-12] reads

$$H=-\partial_x\left(\frac{1}{M(x)}\right)\partial_x+\tilde{V}(x), \tag{1}$$

with

$$\tilde{V}(x)=\frac{1}{2}(1+\beta)\frac{M''(x)}{M(x)^2}-[\alpha(\alpha+\beta+1)+\beta+1]\frac{M'(x)^2}{M(x)^3}+V(x), \tag{2}$$

and primes denote derivatives. An obvious profile change of the potential $\tilde{V}(x)$ obtains as $\alpha,\beta$, and $\gamma$ change, manifesting in effect an ordering ambiguity conflict in the process of choosing a unique kinetic energy operator

$$T=-\frac{1}{2}\left[M\left(x\right)^{\alpha}\partial_{x}M\left(x\right)^{\beta}\partial_{x}M\left(x\right)^{\gamma}+M\left(x\right)^{\gamma}\partial_{x}M\left(x\right)^{\beta}\partial_{x}M\left(x\right)^{\alpha}\right] \qquad (3)$$

Hence, $\alpha,\beta$, and $\gamma$ are usually called the von Roos ambiguity parameters. Yet, such PDM-quantum-particles (i.e., $M\left(x\right)=m_{\circ}m\left(x\right)$) are used in the energy density many-body problem, in the determination of the electronic properties of semiconductors and quantum dots [1-5].

Regardless of the continuity requirements on the wave function at the boundaries of abrupt herterojunctions between two crystals [6] and/or Dutra's and Almeida's [7] reliability test, there exist several suggestions for the kinetic energy operator in (3). We may recollect the Gora's and Williams' ($\beta=\gamma=0$, $\alpha=-1$) [8], Ben Danial's and Duke's ($\alpha=\gamma=0$, $\beta=-1$) [9], Zhu's and Kroemer's ($\alpha=\gamma=-1/2$, $\beta=0$) [10], Li's and Kuhn's ($\beta=\gamma=-1/2$, $\alpha=0$) [11], and the very recent Mustafa's and Mazharimousavi's ($\alpha=\gamma=-1/4$, $\beta=-1/2$) [3]. Nevertheless, in this work we shall deal with these orderings irrespective to their classifications of being "good-" (i.e., satisfying the continuity requirements on the wave function, mentioned above, and surviving the Dutra's and Almeida's [7] reliability test) or "to-be-discarded-" orderings (i.e., not satisfying the continuity requirements on the wave function and/or failing the Dutra's and Almeida's [7] reliability test). The reader is advised to refer to, e.g., Mustafa and Mazharimousavi [3] for more details.

The growing interest in the non-Hermitian pseudo-Hermitian Hamiltonians with real spectra [13-21], on the other hand, have inspired our resent work on PDM first-order-intertwining operator and $\eta$-weak-pseudo-Hermiticity generators [12]. A Hamiltonian $H$ is pseudo-Hermitian if it obeys the similarity transformation $\eta\,H\,\eta^{-1}=H^{\dagger}$, where $\eta$ is a Hermitian invertible linear operator and $(^{\dagger})$ denotes the adjoint. The existence of real eigenvalues is realized to be associated with a non-Hermitian Hamiltonian provided that it is an $\eta$-pseudo-Hermitian:

$$\eta\,H=H^{\dagger}\,\eta, \qquad (4)$$

with respect to the nontrivial "metric"operator $\eta=O^{\dagger}O$, for some linear invertible operator $O:\mathcal{H}\rightarrow\mathcal{H}$ ($\mathcal{H}$ is the Hilbert space). However, under some rather mild assumptions, we may even relax $H$ to be an $\eta$-weak-pseudo-Hermitian by not restricting $\eta$ to be Hermitian (cf., e.g., Bagchi and Quesne [17]), and linear and/or invertible (cf., e.g., Solombrino [18], Fityo [19], and Mustafa and Mazharimousavi [12,20]).

Whilst in the non-Hermitian pseudo-Hermitian Hamiltonians neighborhood [13-22], the non-Hermitian $\mathcal{PT}$-symmetric Hamiltonians (i.e., a Bender's and Boettcher's [13] initiative on the so called nowadays $\mathcal{PT}$-symmetric quantum mechanics) are unavoidably in point. They form a subclass of the non-Hermitian pseudo-Hermitian Hamiltonians (where $\mathcal{P}$ denotes parity and $\mathcal{T}$ mimics the time reversal). Namely, if $\mathcal{PT}H\mathcal{PT}=H$ and if $\mathcal{PT}\Phi\left(x\right)=\pm\Phi\left(x\right)$ the eigenvalues turn out to be real. However, if the latter condition is not satisfied the eigenvalues appear in complex-conjugate pairs (cf., e.g., Ahmed in [13]).

In this work, we consider (in section 2) a complexified von Roos Hamiltonian (1) (i.e., $\tilde{V}(x) \longrightarrow \tilde{V}(x) + iW(x)$) regardless of the nature of the ordering of the ambiguity parameters as to being "good" or "to-be-discarded" ones. A Hermitian first-order differential PDM-intertwining operator is used to obtain the corresponding non-Hermitian $\eta$-weak-pseudo-Hermitian PDM-Hamiltonian. The related *reference/old-target/new* non-Hermitian $\eta$-weak-pseudo-Hermitian Hamiltonians' map is also given in the same section. Yet, in connection with the resulting effective *reference/old* potential, two feasible *"user-friendly"* forms are suggested to serve for exact-solvability of some non-Hermitian $\eta$-weak-pseudo-Hermitian PDM-Hamiltonians. Such *user-friendly* forms turn out to imply that there is always a set of isospectral *target/new* non-Hermitian $\eta$-weak-pseudo-Hermitian PDM-Hamiltonians associated with *"one"* exactly-solvable *reference/old* non-Hermitian $\eta$-weak-pseudo-Hermitian PDM-Hamiltonian. In section 3, we use two illustrative examples (i.e., a complexified $PT$-symmetric Scarf-II and a periodic-type $PT$-symmetric Samsonov-Roy potentials) as *reference/old* models in one of the two *"user-friendly"* forms and report the corresponding sets of isospectral *target/new* non-Hermitian $\eta$-weak-pseudo-Hermitian PDM-Hamiltonians. Section 4 is devoted for the concluding remarks.

## 2 An $\eta$-intertwiner and $\eta$-weak-pseudo-Hermitian Hamiltonians' *reference-target* map

A complexification of the potential $\tilde{V}(x)$ in (1) may be achieved by the transformation $\tilde{V}(x) \longrightarrow \tilde{V}(x) + iW(x)$, where $\tilde{V}(x), W(x) \in \mathbb{R}$ and $\mathbb{R} \ni x \in (-\infty, \infty)$. Hence, Hamiltonian (1) becomes non-Hermitian and reads

$$H = -\mu(x)^2 \partial_x^2 - 2\mu(x)\mu'(x)\partial_x + \tilde{V}(x) + iW(x), \tag{5}$$

with $\mu(x) = \pm 1/\sqrt{M(x)}$. A Hermitian first-order intertwining PDM-differential operator (cf., e.g., Mustafa and Mazharimousavi [12] on the detailed origin of this PDM-operator) of the form

$$\eta = -i\left[\mu(x)\,\partial_x + \mu'(x)/2\right] + F(x); \qquad F(x), \mu(x) \in \mathbb{R} \tag{6}$$

would result, when used in (4),

$$W(x) = -\mu(x) F'(x), \tag{7}$$

$$\tilde{V}(x) = -F(x)^2 - \frac{1}{2}\mu(x)\mu''(x) - \frac{1}{4}\mu'(x)^2 + \alpha_\circ, \tag{8}$$

where $\alpha_\circ \in \mathbb{R}$ is an integration constant. One may then recast $V(x)$ as

$$\begin{aligned} V(x) &= \alpha_\circ - F(x)^2 + \left(\frac{1}{2} + \beta\right)\mu(x)\mu''(x) \\ &\quad + \left[4\alpha(\alpha + \beta + 1) + \beta + \frac{3}{4}\right]\mu'(x)^2. \end{aligned} \tag{9}$$

One should, nevertheless, be reminded that an anti-Hermitian first -order operator of the form $\eta = \mu(x)\,\partial_x + \mu'(x)/2 + F(x)$ will exactly do the same job (cf., e.g., Mustafa and Mazharimousavi [12]). Moreover, as a result of this intertwining process, a non-Hermitian $\eta$-weak-pseudo-Hermitian Hamiltonian is obtained.

We may now consider our non-Hermitian $\eta$-weak-pseudo-Hermitian Hamiltonian in (5), along with (7) and (8), in the one-dimensional Schrödinger equation

$$H\,\psi(x) = E\,\psi(x) \tag{10}$$

and construct the so-called *reference/old-target/new* non-Hermitian $\eta$-weak-pseudo-Hermitian Hamiltonians' map (equation (10) is the so-called *target/new* Schrödinger equation). A task that would be achieved by the substitution

$$\psi(x) = \varphi(q(x))/\sqrt{\mu(x)}, \tag{11}$$

to imply, with the requirement

$$q'(x) = 1/\mu(x) \tag{12}$$

that removes the first-order derivative $\partial_q\varphi(q)$, a so-called *reference/old* Schrödinger equation

$$-\partial_q^2\varphi(q(x)) + \left[\tilde{V}_{eff}(q(x)) - E\right]\varphi(q(x)) = 0, \tag{13}$$

where

$$\begin{aligned}\tilde{V}_{eff}(q(x)) \;=\;& (\beta+1)\,\mu(x)\,\mu''(x) + [4\alpha(\alpha+\beta+1)+\beta+1]\,\mu'(x)^2 \\ & -F(x)^2 + \alpha_\circ - i\mu(x)\,F'(x)\,.\end{aligned} \tag{14}$$

This effective *reference/old* potential suggests two "*user-friendly*" forms. The first of which can be achieved through the choice

$$(\beta+1)\,\mu(x)\,\mu''(x) + [4\alpha(\alpha+\beta+1)+\beta+1]\,\mu'(x)^2 = 0, \tag{15}$$

to imply

$$\tilde{V}_{eff,1}(q) = \alpha_\circ - F(q)^2 - iF'(q)\,. \tag{16}$$

where

$$\frac{dF(x)}{dx} = \frac{dF(q(x))}{dx} = \frac{dq(x)}{dx}\frac{dF(q)}{dq} = \frac{1}{\mu(x)}\frac{dF(q)}{dq},$$

is used. Hence $\mu'(x)\,\mu(x)^{\delta} = const.$ and

$$\mu(x) = [C_1x + C_2]^{1/(\delta+1)}\;;\;\delta = \left[4\alpha + 1 + \frac{4\alpha^2}{\beta+1}\right], \tag{17}$$

where $C_1$ and $C_2$ are two constants and $C_1, C_2 \in \mathbb{R}$ . Nevertheless, one should notice that the Ben Danial's and Duke's ($\alpha = \gamma = 0, \beta = -1$) ordering (although $\beta = -1$ is not allowed by (17) but satisfies (15)) has already been discussed by

Mustafa and Mazharimousavi [12]. Hence, the Ben Danial's and Duke's ordering shall not be considered in the forthcoming studies. Moreover, under such mass settings, we may report that; for Gora's and Williams' ($\beta = \gamma = 0$, $\alpha = -1$) and Li's and Kuhn's ($\beta = \gamma = -1/2$, $\alpha = 0$) orderings $\delta_{GW} = \delta_{LK} = 1$, for Zhu's and Kroemer's ($\alpha = \gamma = -1/2$, $\beta = 0$) ordering $\delta_{ZK} = 0$, and for Mustafa's and Mazharimousavi's ($\alpha = \gamma = -1/4$, $\beta = -1/2$) ordering $\delta_{MM} = 1/2$.

The second choice

$$F(x) = \mu'(x) \Longrightarrow \mu(x) = \int^{x} F(y)\, dy, \tag{18}$$

on the other hand, would lead to

$$\tilde{V}_{eff,2}(q) = -iF'(q) + (\beta + 1) F'(q) + [4\alpha(\alpha + \beta + 1) + \beta] F(q)^2 + \alpha_\circ, \tag{19}$$

Obviously, a $\beta = -1$ (consequently, $\alpha = \gamma = 0$ by the von Roos constraint $\alpha + \beta + \gamma = -1$) would lead to (16) (Ben Danial's and Duke's ordering is to be discarded in the current study for the reasons mentioned above).

## 3 Isospectral PDMs with $\mu'(x)\,\mu(x)^{\delta} = const.$

It is evident that the position-dependent-mass $M(x)$ under the current settings is strictly determined through (15) and consequently through (17) to read

$$M(x) = \mu(x)^{-2} = [C_1 x + C_2]^{-2/(\delta+1)}. \tag{20}$$

This form identifies a class of isospectral position-dependent-mass functions satisfying the effective *reference/old* potential $\tilde{V}_{eff,1}(q)$ of (16), regardless of the form of the $\eta$-weak-pseudo-Hermiticity generator $F(q)$, and implies

$$q(x) = \int^{x} \mu(y)^{-1} dy = \begin{cases} \frac{(\delta+1)}{\delta C_1} [C_1 x + C_2]^{\delta/(\delta+1)} & ;\text{ for } \delta \neq 0 \\ \frac{1}{C_1} \ln(C_1 x + C_2) & ;\text{ for } \delta = 0 \end{cases}. \tag{21}$$

Unlike the case we have very recently considered in [12], where Ben Danial's and Duke's ordering (i.e., $\alpha = \gamma = 0$, $\beta = -1$) was used and the position-dependent-mass was left arbitrary instead (but, of course, a positive-valued function).

Nevertheless, one should notice that the form of our $\tilde{V}_{eff,1}(q)$ in (16) depends only on the choice of our $\eta$-weak-pseudo-Hermiticity generator $F(q)$ (the choice of which should be oriented in such a way that an exactly-solvable $\eta$-weak-pseudo-Hermitian *reference/old* Hamiltonian is obtained). Therefore, a set of exactly-solvable *target/new* potentials of (14) would obtain and depends only on the class of the strictly determined position-dependent-mass functions in (20). Two illustrative examples are in order.

## 3.1 A complexified $PT$-symmetric Scarf-II model

Let us recollect (cf., e.g., Mustafa and Mazharimousavi [12]) that an $\eta$-weak-pseudo-Hermiticity generator of the form

$$F(q) = -V_2 \operatorname{sech} q \Longrightarrow F'(q) = V_2 \operatorname{sech} q \tanh q \tag{22}$$

would yield (with $\alpha_\circ = 0$) a *reference/old* effective complexified $PT$-symmetric Scarf-II potential of the form

$$\tilde{V}_{eff,1}(q) = -V_2^2 \operatorname{sech}^2 q - iV_2 \operatorname{sech} q \tanh q \ ; \ \ \mathbb{R} \ni V_2 \neq 0. \tag{23}$$

Which, in turn, would imply a *target/new* effective potential of the form

$$\tilde{V}_{eff,1}(x) = -4V_2^2 \frac{f(x)^2}{\left(f(x)^2 + 1\right)^2} \mp 2iV_2 \frac{f(x)\left(f(x)^2 - 1\right)}{\left(f(x)^2 + 1\right)^2}, \tag{24}$$

where $f(x) = \pm \exp[q(x)]$, with $q(x)$ given in (21). In this case, the *target/new* effective potentials in (24) form a set of isospectral potentials the eigenvalues of which are readily reported in [12,17] as

$$E_n = -\left[|V_2| - n - \frac{1}{2}\right]^2 \ ; \ \ n = 0, 1, 2, \cdots, n_{\max} < (|V_2| - 1/2). \tag{25}$$

## 3.2 A periodic-type $PT$-symmetric Samsonov-Roy model

We may also recycle our $\eta$-weak-pseudo-Hermiticity generator

$$F(q) = -\frac{4}{3\cos^2 q - 4} - \frac{5}{4}, \tag{26}$$

that implies (with $\alpha_\circ = 0$) an effective periodic-type $PT$-symmetric Samsonov's and Roy's [12,14] *reference/old* potential

$$\tilde{V}_{eff,1}(q) = -\frac{6}{[\cos q + 2i \sin q]^2} - \frac{25}{16} \ ; \ \ \mathbb{R} \ni q \in (-\pi, \pi). \tag{27}$$

This results, in effect, a *target/new* effective potential of the form

$$\tilde{V}_{eff,1}(x) = -\frac{6}{[g(x) - 2i\mu(x) g'(x)]^2} - \frac{25}{16}, \tag{28}$$

where $g(x) = \cos(q(x))$, $\mu(x)$ and $q(x)$ are as given in (17) and (21), respectively. Hence, the set of *target/new* effective potentials in (28) are isospectral and the corresponding eigenvalues [12,14] are given by

$$E_n = \frac{n^2}{4} - \frac{25}{16} \ ; \ \ n = 1, 3, 4, 5, \cdots, \tag{29}$$

with a missing $n = 2$ state (the details of which can be found in Samsonov and Roy [14]).

## 4 Concluding remarks

As long as $\eta$-weak-pseudo-Hermitian Hamiltonians are in point, their solvability-nature/type (i.e., e.g., exact-, quasi-exact-, conditionally-exact-, etc.) is still fresh and not yet adequately explored. Amongst is the $\eta$-weak-pseudo-Hermitian von Roos PDM-Hamiltonian. In this work, we tried to (at least) partially fill this gap and add a flavour into such solvability territories of the $\eta$-weak-pseudo-Hermitian Hamiltonians associated with position-dependent-mass settings.

We have suggested two *"user-friendly"* forms for the *reference/old* $\eta$-weak-pseudo-Hermitian PDM-Hamiltonians' map. Only one of which (i.e., $\tilde{V}_{eff,1}(q)$ of (16)) is exemplified through a non-Hermitian $\mathcal{PT}$ -symmetric Scarf II and a non-Hermitian $\mathcal{PT}$ -symmetric Samsonov-Roy periodic-type models. It is observed that for each of these models there is a set of exactly-solvable isospectral *target/new* $\eta$-weak-pseudo-Hermitian PDM-Hamiltonians (documented in (24) for Scarf II and in (28) for Samsonov-Roy). However, we were unlucky to find any illustrative example that can be classified as "successful" for the *"user-friendly"* form $\tilde{V}_{eff,2}(q)$ in (19). Nonetheless, the corresponding *target/new* isospectral set of $\eta$-weak-pseudo-Hermitian PDM-Hamiltonians is anticipated to be feasibly large (as documented by (18)) and not restricted to the position-dependent-mass form (unlike the case of $\tilde{V}_{eff,1}(q)$ in (16), which is restricted to the position-dependent-mass function $M(x)$ in (20)).

Moreover, we may report that a generating function $F(q) = a\exp(-q)$ would lead to (with $\alpha_\circ = 0$) to

$$\tilde{V}_{eff,1}(q) = -a^2 \exp(-2q) + ia\exp(-q) \tag{30}$$

of (16), and

$$\tilde{V}_{eff,2}(q) = a^2 \left[4\alpha(\alpha+\beta+1)+\beta\right]\exp(-2q) - a(\beta+1-i)\exp(-q) \tag{31}$$

of (19). The bound-states of the former (30) (a non-Hermitian Morse model) are reported to form an empty set of eigenvalues and, hence, labeled as "unfortunate" for it leads to an empty set of "unfortunate" isospectral $\eta$-weak-pseudo-Hermitian *target* PDM-Hamiltonians (cf., e.g., Mustafa and Mazharimousavi [12], Bagchi and Quesne [23], and Ahmed [24]). The latter (31), on the other hand, does not fit into any of the "so-far-known" exactly-solvable non-Hermitian Morse-type models, to the best of our knowledge.

Finally, one may add that the current strictly-determined set of *target/new* effective potentials $\tilde{V}_{eff,1}(x)$ in (24) forms a subset of the *target/new* effective potentials reported in equations (25) and (26) by Mustafa and Mazharimousavi [12]. Similar trend is also observed for $\tilde{V}_{eff,1}(x)$ in (28) as it forms a subset of the effective potentials in equations (34) and (35) of [12]. Hence, the scenario of the *energy-levels crossing* and the feasible manifestation of the *flown away states* discussed in [12] remains effective, as long as the our two illustrative examples are concerned.

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