Dissipative Scattering Systems

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Conversely: Each operator function $S(\cdot)$ with unitary values $S(\lambda)$ is the scattering matrix of some scattering system $\{A_1, A_0\}$.

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Assume $A \subset A^*$ of defect n and A_0 selfadjoint extension, A_D maximal dissipative extension (pseudo-Hamiltonian).

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Summary Simple model for open quantum system, scattering theory works, but Hamiltonians $\widetilde{K}, K_0$ in the closed system are NOT semibounded.

PART III: "Real World"

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Example $A(\lambda) = -\frac{d^2}{dx^2} + V$ regular Sturm-Liouville operators in $L^2(a,b)$

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Open quantum system described by a family $\{A(\lambda)\}$, $\lambda \in \mathbb{C}_+$, of maximal dissipative operators in $\mathcal{H}$.

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Quantum-transmitting Schrödinger Poisson system: Model for carrier transport in semiconductors

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