Abstract: A novel state-space model of a multi-node supply chain is presented, controlled via local proportional inventory-replenishment policies. The model is driven by a stochastic sequence representing customer demand. The model is analyzed under stationarity conditions and a simple recursive scheme is developed for updating the covariance matrix. This allows us to characterize the "bullwhip effect" (demand amplification) in the chain and to solve an optimization problem for a three-node model involving the minimization of inventory subject to a probabilistic constraint on downstream demand. Finally, issues related to estimation schemes based on local historical data are discussed.

Keywords: Bullwhip effect, State-space model, Supply Chain, Covariance matrix Estimation, Information-sharing.

1. INTRODUCTION

The work presented in this paper aims to analyze the effects of certain aspects of proportional (continuous) inventory policies on the stability and performance of serial multi-node supply chains. In contrast to more traditional inventory-replenishment policies commonly used for supply chain control (e.g. \( (S, s) \) policies), continuous policies (e.g., P or PI policies) have only recently been proposed, apparently inspired from the area of classical process control engineering (Lin et al. 2004), (Dejonckheere et al. 2003), (Perea-Lopez et al. 2001). Their main characteristic is that orders take place continuously, rather than being triggered by specific events (e.g., when the inventory falls below a certain target level). Despite possible practical limitations of continuous ordering policies in some cases, these in principle can offer additional flexibility (e.g., by smoothing out flows) which can be beneficial for the stability and performance properties of the supply chain.

In practice, continuous ordering policies are applicable when cost savings due to batch ordering are not significant.

Relative stability is supply chain dynamics is often quantified via the concept of "bullwhip effect". The bullwhip effect is a well known instability phenomenon in supply chains, related to increased volatility in demand profiles in the upstream nodes of the chain (Sterman 1989). This may limit significantly the smooth operation of the chain and result in high costs arising due to its implications on production planning, high levels of inventory costs, poor customer service, etc. The bullwhip effect has been analyzed extensively in recent literature, and many contributing factors for this phenomenon have been identified (Lee et al. 1997a), (Lee et al. 1997b), (Dejonckheere et al. 2003), (Perea-Lopez et al. 2001). These include poor coordination, aggressive stock replenishment/demand forecasting policies and uncertain lead times in the chain. Note that these
factors apply for general ordering policies, not only proportional policies considered in this paper. In this work, we will present explicit methods for analyzing and predicting the bullwhip effect via covariance analysis of proportional control schemes in supply chain models of arbitrary complexity. Moreover, we study issues related to supply chain performance under such schemes, the potential advantages of information-sharing and the applicability of local estimation schemes based on historical data. The state-space supply chain model used in the paper is based on our previous work (Papanagnou and Halikias 2005). The benefits of using a state-space (rather than a transfer-function) approach arise mainly from its suitability for the recursive updating of the covariance matrix of structured multi-node systems of the type used in this work. Also, in this paper we have made the model of the customer-demand stochastic process more concrete, by assuming that it is a collection of independent and identically distributed random variables, drawn from a normal distribution \( N(\mu, \sigma^2) \). Although this assumption is clearly unrealistic for real supply chains (as it ignores, for example, trends, seasonal variations or more complex correlation patterns) it offers the advantage of simplicity and can be easily extended to more complex cases, e.g. ARMA demand-profile models (Zipkin 2000), a topic which we intend to investigate in the future. The assumption is consistent with the main aims of the paper, which not include the analysis of demand forecasting on the stability and performance of the supply chain.

2. THE SUPPLY-CHAIN MODEL

We consider a simple series multi-stage supply chain as shown in Fig.1. There are \( n \) individual stages between generic Customer and Manufacturer and we denote as \( i \) the intermediate supplier index \( (i \geq 1) \). Fig.1 also depicts the flow of goods and information (orders) within the supply chain. Let \( I_i(t) \) denote the inventory level of node \( i \) at time \( t \). We let also \( Y_{i,i-1}(t) \) indicate the amount of goods to be delivered to node \( i-1 \) by the upstream node \( i \) at time instant \( t \). We also introduce a time delay \( \Delta \), which is the lead time needed for the goods to be dispatched to the downstream node (i.e. the goods are delivered at time \( t + \Delta \)). The model is based on (Lin et al. 2004), from where additional details can be obtained.

Balancing the inventory \( I_i(t) \) of node \( i \) at time step \( t \) gives:

\[
I_i(t) = I_i(t-1) + Y_{i+1,i}(t - \Delta) - Y_{i,i-1}(t) \quad (1)
\]

where \( I_i(t-1) \) is the inventory level at node \( i \) at time step \( t-1 \) and \( Y_{i,i+1}(t - \Delta) \) represents the products dispatched by the upstream node \( i + 1 \) to node \( i \), which is assumed to arrive with a delay of \( \Delta \) time steps. Although inventory level is a key variable in supply chain operation, each node \( i \) can better monitor the changes in inventory level at time \( t \) by using inventory position, \( IP_i(t) \), which is given by:

\[
IP_i(t) = I_i(t) + Y_{i+1,i}(t) - Y_{i,i-1}(t) \quad (2)
\]

We denote by \( O_{i,i+1}(t) \) the amount of orders placed by node \( i \) to node \( i + 1 \), given by:

\[
O_{i,i+1}(t) = k_i(SP_i - IP_i(t)) \quad (3)
\]

where \( SP_i \) represents a target set-point (assumed constant) and \( k_i \) is the corresponding inventory replenishment gain factor. Standing orders of node \( i \) at time step \( t \) evolve according to the difference equation:

\[
O^*_i(t) = O_{i-1,i}(t) + O^*_i(t-1) - Y_{i,i-1}(t) \quad (4)
\]

For the purposes of further analysis it is assumed that there is always enough stock at each node to meet the demand, so that \( Y_{i,i-1}(t) = O^*_i(t-1) \). This implies that the amount of goods dispatched to node \( i \) from the downstream stage \( i-1 \) at time \( t \) is the amount of standing orders of node \( i \) at time \( t-1 \). This is essentially a linearisation assumption also made in (Lin et al. 2004) which simplifies the subsequent analysis. In addition, since the covariance analysis which follows does not depend on \( SP_i \), we may set \( SP_i = 0 \) for simplicity.

We now consider the series supply chain model depicted in Fig.1. Each stage \( i \) has two inputs \( w_{id} \) and \( w_{ir} \) and two outputs \( z_{il} \) and \( z_{ir} \) (left and right respectively). It can be inferred from the nature of the figure’s interconnections that \( w_{id} = z_{i-1,r} \) and \( w_{ir} = z_{i+1,l} \). The terminal node \( \Phi \) representing the Manufacturer is assumed to be a simple time delay. Thus the manufacturer always delivers the orders he receives with a delay of one time step. The model equations for each separate node can be expressed in state space form as:

\[
x_i(t+1) = A_i x_i(t) + (B_{i1} B_{ri}) \begin{pmatrix} w_{il} \\ w_{ir} \end{pmatrix}
\]

and

\[
\begin{pmatrix} z_{il} \\ z_{ir} \end{pmatrix} = \begin{pmatrix} C_{il} \\ C_{ir} \end{pmatrix} x_i(t) + \begin{pmatrix} D_{il} & D_{i1} \\ D_{ir} & D_{ir} \end{pmatrix} \begin{pmatrix} w_{il} \\ w_{ir} \end{pmatrix}
\]
The equivalent state-space model of the manufacturer is \( x(t + 1) = A \phi x(t) + B \phi z_{i+1,r} \) and \( w_{i+1,r} = C \phi \), due to our previously assumed \( A_0 = 0 \) and \( B_0 = C \). The state space form of the node \( i \) given above can be written in more concrete form as:

\[
x_i(t + 1) = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} x_i(t) + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} u_i(t)
\]

\[
y_i(t) = \begin{pmatrix} 0 & 1 \\ -k_i & k_i \end{pmatrix} x_i(t) + \begin{pmatrix} 0 \\ 0 \end{pmatrix} u_i(t)
\]

where \( y_i(t) \) and \( u_i(t) \) are the two-dimensional vector outputs and inputs of node \( i \), respectively.

The general \( n \)-node model shown in Fig.1 can be aggregated as \( x(t + 1) = Ax(t) + Be(t) \) where \( e(t) = w_i(t) \) represents customer demand, and \( x(k) = [x_1'(k), x_2'(k), \ldots, x_n'(k), x_{n+1}'(k)]' \).

Further

\[
A = \begin{pmatrix} A_{11} & A_{12} & 0 & \cdots & 0 \\ A_{21} & A_{22} & A_{23} & \ddots & \vdots \\ 0 & A_{32} & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \cdots & A_{nn} & A_{n,n+1} \\ 0 & 0 & \cdots & 0 & A_{n+1,n} \end{pmatrix}
\]

and \( B \) is the zero column vector, except from a one in the \((2, 1)\) position. More specifically, the elements of \( \Phi \) are defined as:

\[
A_{ij} = \begin{cases} A_i + B_i D_{i-1,r} C_{il} & \text{for } i = j; \\
B_{ir} C_{i+1,1} & \text{for } i = j - 1; \\
B_{il} C_{i-1,r} & \text{for } i = j + 1; \\
0 & \text{for } |i - j| > 1.
\end{cases}
\]

Note that this state-space model has been derived under the simplifying assumption that \( D_{i,t} = 0 \) for all \( i = 1, 2, \ldots, n + 1 \) and \( D_{i,r} = D_{i,t} = 0 \) for all \( i = 1, 2, \ldots, n \) (we also define \( D_{0,r} = 0 \)). These relations actually hold in our case.

The state space model given by equations (5) and (6) is obtained by identifying:

\[
x_i(t) = (IP_i(t-1) Y_{i,i-1}(t))'
\]

\[
u_i(t) = (O_{i-1,i}(t) Y_{i,i-1}(t))'
\]

\[
y_i(t) = (Y_{i,i-1}(t) O_{i,i+1}(t))'
\]

Note that the two set-point levels have been set to zero as they do not affect the covariance analysis.

### 3. COMPUTATION OF MODEL’S COVARIANCE MATRIX

In section 2 the state-space model of supply chain was formulated as \( x(t+1) = Ax(t) + Be(t) \) and \( y(t) = Cx(t) \) where \( \{e(t)\} \) denotes customer demand, \( A \) depends linearly on the parameters \( \{k_i\} \) and \( y(t) \) represents an arbitrary output vector. Note further that we have set \( SP_i = 0 \). Assume now that \( e(t) \) is a random sequence of independent and identically distributed random variables of unit variance (plus a constant deterministic component which can be ignored for the purpose of covariance analysis). Then, the (steady-state) covariance of the state-vector \( x(t) \), \( E(z(t)x'(t)) \), is given by the (unique, positive semi-definite) solution of the discrete Lyapunov equation \( P - AP'A - BB' = 0 \) (Davies and Vinter 1985). Further, \( E(yy') = C'PC' \).

The structure of the model allows for a recursive updating of the covariance matrix which is presented in (Papanagnou and Halikias 2005).

The covariance matrices of two and three nodes \( P_3 \) and \( P_5 \), respectively are given in (7) and (10):

\[
P_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
\]

\[
P_5 = \begin{pmatrix} k_1(k_1 - 2) & 1 \\ 1 & k_1 \end{pmatrix}
\]

where \( k = k_1k_2 - k_2 - k_1 \).

### 4. CHARACTERISATION OF BULLWHIP EFFECT

The covariance analysis of the model allows us to analyze the effect of the inventory replenishment policies on the bullwhip effect. Recall that end-customer demand has been modelled as a sequence of independent and identically distributed random variables of unit variance. Hence, the variance of the demand signal at any node of the chain may be calculated easily from the covariance matrix. Consider the three-node model. The orders placed by the second node (on the manufacturer) correspond to signal \( z_{2r} \) and we can write:

\[
z_{2r}(t) = C_{2r} x_2(t) + D_{22r} w_{2r}(t) = C_{2r} x_2(t) - k_2 x_\phi(t)
\]

Thus the demand amplification factor can be obtained from the variance of \( z_{2r} \), \( \sigma^2 \) which is given as:

\[
\sigma^2 = E(z_{2r}^2) = \frac{k_1 k_2 (k_2 + 2) - (k_1 - k_2) k_2}{k_1 - 2}
\]

To find the regions in the \((k_1, k_2)\) plane where demand amplification and demand attenuation occurs, this expression was set to one, and the resulting equation was solved to give \( k_2 \) as a function of \( k_1 \). This gives two solutions:

\[
k_2 = f(k_1) = \frac{2 - 5k_1 + 2k_1^2}{2(k_1 - 1)} \pm \sqrt{4 - 12k_1 + 13k_1^2 - 4k_1^3}
\]
which are valid for \( k_1 \neq 1 \). It can be easily seen that the positive square root should be chosen, as with this choice, \( k_1 \) values in the interval \( 0 \leq k_1 \leq 2 \) are mapped to \( k_2 \) values inside the same interval. The resulting curve is plotted in Fig. 2, and indicates the boundary between the demand-amplification and demand-attenuation regions. As expected, aggressive replenishment policies (i.e., large values of \( k_1 \) and \( k_2 \)) reinforce the bullwhip effect. A similar analysis was performed in (Papanagnou and Halikias 2005) for the four-node model to obtain the boundary surface between the demand-amplification and attenuation regions in \((k_1,k_2,k_3)\) parameter space.

5. ANALYSIS OF OPTIMAL POLICIES, CO-OPERATION AND ESTIMATION ISSUES

In this section we specialize our system to a three node model. We assume linear dynamics (i.e., that all inventories are sufficiently high to meet downstream demand with no back-ordering). Assume further that customer demand is normally distributed as \( e(t) = O_{i1}(t) \sim N(\mu, \sigma^2) \). Provided that the system is stable \((0 < k_1 < 2 \text{ and } 0 < k_2 < 2)\) all signals in the limit are stationary. The expected values of the state variables can be found by the steady state model, which is of the form:

\[
x(t+1) = Ax(t) + Be(t)t + F(SP)
\]

where \( SP \) is the (deterministic) vector of set-points \( SP = (SP_1,SP_2)' \) (assumed constant). Thus under stationary conditions,

\[
x(t) = (I - A)^{-1}Be(t) + (I - A)^{-1}F(SP)
\]

and hence

\[
E[x(t)] = \mu(I - A)^{-1}B + (I - A)^{-1}F(SP)
\]

Note that the indicated matrix inverse exists as the spectral radius of \( A \) is less than one as long as \( 0 < k_1 < 2 \text{ and } 0 < k_2 < 2 \). Thus, the five state variables are distributed as:

\[
\begin{align*}
IP_1(t) & \sim N(SP_1 - \frac{\mu}{k_1}, \frac{\sigma^2 k_1}{2 - k_1}), \\
Y_{1,0}(t) & \sim N(\mu, \sigma^2), \\
IP_2(t) & \sim N(SP_2 - \frac{\mu}{k_2}, \frac{\sigma^2 k_2 (k_1 k_2 - k_1 - k_2 + 2)}{k_2 (2 - k_1) (2 - k_2) (k_1 + k_2 - k_1 k_2)}), \\
Y_{2,1}(t) & \sim N(\mu, \frac{\sigma^2 k_1}{2 - k_1})
\end{align*}
\]

\( O_{2,1}(t) \sim N\left(\mu, \frac{\sigma^2 k_1 (k_1 k_2 - k_1 - k_2 + 2)}{(2 - k_1) (2 - k_2) (k_1 + k_2 - k_1 k_2)}\right) \)

Note that the mean of \( IP_i \) \((i = 1,2)\) does not track the set-point \( SP_i \) exactly, but with a steady-state error \( \mu/k_1 \) characteristic of type zero feedback systems. As expected, the information pattern is asymmetric, i.e. node 2 (distributor) is affected by the inventory policy of node 1 (retailer) but not vice versa. Suppose now that the retailer makes his policy gain-factor \( k_1 \) known. In this case the distributor can make use of this information to minimize his own costs, typically related to excessive inventory levels. Although this objective is situation-specific (e.g. due to possible existence of capacity constraints, depreciation effects, etc) it is reasonable to assume that the objective of the distributor is to minimize both his average inventory and his inventory fluctuations. Note that in our model the distributor is always capable of controlling his average inventory-level through his choice of \( SP_2 \), which can be used to shift \( E(IP_2) \) to any required level. In fact, a more general cost function could be formulated, involving the integral of an appropriate inventory cost-function, weighted by the distribution of \( IP_2 \).

An additional requirement is that the distributor should have enough inventory to meet (fluctuating) downstream demand, at least for most orders placed on him. This is in order to ensure the smooth operation of the chain, to which he has an interest as a participant. One way of modelling this requirement is to include explicit penalty-terms in the distributor’s “objective function”, reflecting real or virtual costs (e.g. penalty terms for not fulfilling a contract, loss of sales due to customer dissatisfaction, etc). Here we impose a probabilistic constraint for fulfilling orders, i.e. we require that \( Proz[IP_2 \geq O_{1,2}] \leq \delta \) for some (small) parameter \( \delta \).

Let \( \Phi(z) \) denote the probability density function of the normal distribution \( N(0,1) \). Then, using the distribution of \( IP_2 - O_{1,2} \) above, the “order-fulfilling” constraint takes the form given in (12).

Thus, the optimization problem faced by the distributor is to choose his inventory replenishment policy parameters, \( k_2 \) and \( IP_2 \), to minimize his inventory costs subject to the constraint of equa-
resulting in the constrained minimum once the optimal policy takes positive, zero and negative values in this assumed to take values in the interval 0 \leq k_1 < 2 we seek the optimal choice of k_2, k_2^* = f^*(k_1) say, which minimizes the variance of IP_2; subsequently we minimize the mean of IP_2 subject to constraint (12). Note that once the optimal policy k_2^* = f^*(k_1) has been determined, we need to set SP_2 given in (14), resulting in the constrained minimum E(IP_2) = SP_2^2 - \frac{\mu}{k_2^*}. It has been assumed that the customer demand parameters \mu and \sigma are known or can be estimated accurately from the data. Due to the high complexity of the expressions involved, we rely (in part) of Matlab’s symbolic toolbox to obtain an analytic expression for the optimal policy k_2^* = f^*(k_1). The following procedure was followed: First, the variance of IP_2 was differentiated with respect to k_2 and the resulting expression was set to zero. This was then solved to express k_1 as a function of k_2, resulting in three (apparently complex) solutions:

\[
\begin{align*}
k_2 &= \frac{3\sqrt{7} + 16/\sqrt{7} + 6k_1 - 8}{k_1 - 1}, \\
k_2 &= \frac{-3\sqrt{7} + 16/\sqrt{7} - 12k_1 - 16 + i\sqrt{3}(3\sqrt{7} - 16)/\sqrt{7}}{k_1 - 1},
\end{align*}
\]

where

\[
l = 108k_1^2 - 216k_1 + 64 + 12\sqrt{3k_1(k_1 - 2)(27k_1^2 - 54k_1 + 32)}.
\]

We each investigate each of the following three solutions in turn. Consider first the term m(k_1) = 27k_1^2 - 54k_1 + 32 appearing inside the square root defining l. It is easy to see that this term is always positive for every value of k_1 (attaining a minimum value m(1) = 5). Thus, the term inside the square root defining l is always negative in the interval 0 < k_1 < 2. Hence l is complex and can be written in terms of its real and imaginary parts as:

\[
l = 108k_1^2 - 216k_1 + 64 + 12\sqrt{3k_1(2 - k_1)(27k_1^2 - 54k_1 + 32)}.
\]

Thus parameter l can be written in polar form as

\[
l = r \exp(i\phi) \quad \text{where}\ r = 64, \quad \phi = \tan^{-1}\left(\frac{3\sqrt{3k_1(2 - k_1)(27k_1^2 - 54k_1 + 32)}}{27k_1^2 - 54k_1 + 16}\right)
\]

To avoid any confusion arising from the fact that \tan^{-1}(\cdot) is a multi-function, we stress that \phi is assumed to take values in the interval 0 \leq \phi < \pi (since the imaginary part of l is positive on the interval 0 < k_1 < 2, while the imaginary part of l takes positive, zero and negative values in this interval). On noting that:

\[
l^{1/2} = \{4 \exp(i\phi/3), 4 \exp(i(\phi + 2\pi)/3), 4 \exp(i(\phi - 2\pi)/3)\}
\]

and substituting into the three expressions for k_2, it can be easily seen after some algebra, that each expression reduces to the same set of solutions, given by the three (real-valued) functions given in equation (15). It may now be easily verified that the minimizing solution corresponds to:

\[
k_2^* = f^*(k_1) = \frac{4 \sin(\phi/3 - 2\pi/3) + 3k_1 - 4}{3(k_1 - 1)}.
\]

where \phi = \phi(k_1) is defined in equation (13). Again f^*(1) is not formally defined by this equation, so we set f^*(1) = 1 to make the function continuous and continuously differentiable at k_1 = 1. A plot of k_2^* = f^*(k_1) (along with the boundary between the attenuation and amplification regions) is shown in Fig. 2. An important observation is that the optimal curve lies entirely in the attenuation region. Thus, under co-operation (disclosure of policy parameter k_1 to the distributor), a “selfish” policy by the distributor (resulting from his attempt to minimize his own inventory costs) can not give rise to the bullwhip effect. Of course this conclusion should be qualified by the assumptions of the model (e.g. linearity, normally-distributed demand profile, no demand forecasting, etc).

We continue our analysis of the three node model by removing the assumption that policy parameter k_1 is communicated to the distributor. A natural question arising in this case is whether k_1 can be estimated by the distributor (node 2). Naturally, the data on which the estimation should be based are restricted only to the input/output and state variables local to node 2. 

![Fig. 2. Optimal policy k_2^* = f^*(k_1) and boundary between amplification and attenuation regions](image-url)
First note that since $E(Y_{2,1}) = \mu$, the mean customer demand ($\mu$) can be estimated from $Y_{2,1}(t)$, which is an input signal to node 2 (e.g. an unbiased estimate $\hat{\mu}$ of $\mu$ can be obtained asymptotically). Consider next the part of the covariance matrix $P_t$ corresponding to the state variables of node 2; this is the diagonal block of $P_t$ corresponding to the third and fourth rows and columns, $a = \frac{P_{12}}{P_{22}} \Rightarrow k_1 = \frac{1 + \alpha k_2}{1 + \alpha k_2 - a}$ and note that:

$$\sigma^2 = \frac{P_{22}(2 - k_1)}{k_1}.$$ 

Now, using the data $\{I_P(t), Y_{2,1}(t)\}$ and noting that parameter $k_2$ is known, we can obtain estimates for $P_{11} = \text{Var}(I_P)$, $P_{22} = \text{Var}(Y_{2,1})$ and $P_{12} = E([I_P - E(I_P)](Y_{2,1} - E(Y_{2,1})))$, say $\hat{P}_{11}$, $\hat{P}_{22}$ and $\hat{P}_{12}$ respectively, and use them to estimate $k_1$ and $\sigma$ via equations:

$$\hat{a} = \frac{\hat{P}_{12}}{\hat{P}_{22}}, \hat{k}_1 = \frac{1 + \alpha \hat{k}_2}{1 + \alpha (\hat{k}_2 - 1)}, \hat{\sigma}^2 = \frac{\hat{P}_{22}(2 - \hat{k}_1)}{\hat{k}_1}.$$ 

This estimation scheme can be implemented recursively.

A limitation of the above method is that it does not take full advantage of the available information structure (e.g. the information contained in $\text{Var}(I_P)$ is ignored). A superior approach is to formulate the estimation problem as a structured-covariance approximation problem, e.g.,

$$\min_{k_1 \in (0, 2), \sigma > 0} \left\| W c \left( \hat{P} - \text{Cov}(I_P, Y_{2,1}) \right) \right\|_F^2$$

in which $\hat{P}$ denotes the estimated covariance matrix (constructed from the data). The choice of Frobenious-norm makes the problem easily transformable into a scalar sum-of-squares type nonlinear optimization, while $W$ is a weighting matrix which can be used to emphasize/de-emphasize different matrix elements in the approximation (here \(\circ\) denotes the Hadamard product, i.e., element by element product, of two matrices).

6. CONCLUSION

A novel state-space model has been presented for analyzing the effect of proportional policies in multi-node supply chains. The covariance matrix of the model was obtained in closed form, as a function of two inventory-replenishment parameters. This allows for a full characterization of the bullwhip effect, and leads to the formulation and solution of a constrained inventory minimization problem under information sharing. It has been shown that under information-sharing, selfish policies cannot lead to demand amplification. Finally, local estimation schemes have been investigated in the absence of information sharing. Extensive simulation results related to the optimal policy and the estimation algorithms will be presented at the Conference.

ACKNOWLEDGEMENTS

C. Papanagnou would like to thank Greek State Scholarships Foundation (I.K.Y.) for the financial support of his research.

REFERENCES


