Digital Signal Processing
Matlab Assignement

Introduction: Notch filters are often used in DSP applications to remove spurious signals and their harmonics; these are often present due to power-line noise. Such filters have zeros on the unit circle (corresponding to the fundamental and harmonic frequencies to be rejected) and poles near the zeros (but inside the unit circle - why?). This pole-zero pattern ensures that the magnitude frequency response is almost flat at all frequencies, except at the fundamental/harmonic frequencies which must be removed. The purpose of the exercise is to design and simulate the response of such a comb filter using Matlab.

Problem: A DSP system operating at a sampling rate of 500 Hz, is plagued by 50 Hz power-frequency interference noise and its harmonics. Design a notch filter that removes the 50 kHz signal and its harmonics, but remains reasonably flat over all other frequencies.

Procedure: The transfer function of an appropriate comb filter is of the form:

$$H(z) = \frac{N(z)}{N(\rho^{-1}z)}$$

where $N(z)$ is a polynomial of the form $N(z) = 1 - z^{-m}$, in which m is an integer. The zeros of the filter are found by setting:

$$z^m N(z) = z^m - 1 = 0$$

i.e. they lie on the unit circle and correspond to the m roots of unity. These zeros should be selected to correspond to the fundamental and harmonic frequencies that must be eliminated. Parameter ρ controls the location of the poles; If ρ is chosen near 1, the poles will be placed near the unit circle.

Exercises: Using hand calculations and/or Matlab programming/simulations, answer the following questions. Your report should include answers to these questions, together with appropriate plots and your Matlab code:

- Determine m by finding the number of harmonics inside the Nyquist interval $[0, f_s/2]$. (Assume that the notch filter should also reject the DC and the Nyquist frequency components). Select a reasonable value of ρ and using Matlab, display the location of the poles and zeros in the complex plane. How does the magnitude of the poles depend on ρ? (You may use Matlab's function *roots.m* to find the poles and zeros and Matlab's plotting functions to display them in the complex plane).
- Using Matlab, display the magnitude frequency response of the filter and explain how this depends on the parameter ρ. You may need to normalize the magnitude frequency response of the filter to get a clearer picture of this dependence - for two different values of ρ, say. (Use Matlab's function *freqz.m* to calculate the frequency-response of the filter).

- Modify the design of your filter so that the DC and Nyquist frequencies are *no longer* rejected. (Polynomial multiplication and division can be performed by functions *conv.m* and *deconv.m* respectively).

- Illustrate the time-domain response of your filter to an appropriate input using Matlab's function *filter.m*. Your input signal should contain sinusoidal components at 50, 100, 150 and 200 Hz.