

Interbank lending and systemic risk

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Abstract

We simulate interbank lending. Each bank faces fluctuations in deposits and stochastic investment opportunities which mature with delay. This creates the risk of liquidity shortages. An interbank market lets participants pool this risk but also creates the potential for one bank's crisis to propagate through the system. We study banking systems with homogeneous banks, as well as systems in which banks are heterogeneous. With homogeneous banks, an interbank market unambiguously stabilises the system. With heterogeneity, knock-on effects become possible but the stabilising role of interbank lending

words.)

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I. INTRODUCTION

In banking systems, there is a tendency for crises to spread from institution to institution. This tendency is referred to as ‘systemic risk’ and, at a large enough scale, leads to ‘systemic failure’. The East Asian crisis of 1997 displayed systemic failure in at least three respects. First, the initial failure of banks in Thailand set in motion a panic withdrawal of funds from financial institutions in the entire region, leading to further failures. Second, the crisis triggered off adverse changes in asset prices which dramatically undermined the solvency of all financial institution. Third, losses suffered by failing institutions were directly transmitted via a chain of equity and credit relationships to banks and other financial institutions, not just in the region, but also in Russia, Brazil and the West (see, *e.g.*, Radelet and Sachs [1998] or Edison, Luangaram and Miller [1998] for a discussion of these aspects of the 1997 crisis).

The first aspect of systemic failure is essentially a ‘bad’ equilibrium in which depositors lose confidence in the entire system and attempt to simultaneously withdraw illiquid funds. This has attracted a large literature (see Diamond and Dybvig [1985], Donaldson [1992], de Bandt [1999], Jacklin and Bhattacharya [1988], Calomiris and Kahn [1996], Cowen and Kroszner [1989]). The second aspect has been discussed in general by Allen and Gale [1998] and, with particular reference to East Asia, by Edison, Luangaram and Miller [1998]. In general, when those banks which face a liquidity crisis resort to selling off ‘long assets’, this devalues these assets and undermines the portfolios of those banks that did not face liquidity shortages to begin with.

The third aspect of systemic risk arises from inter-locking exposures among financial institutions, whether through equity, debt or participation in a common payments system. The failure of one institution can have ‘knock on’ effects on the balance sheet of other institutions. At a theoretical level, Allen and Gale [1998] have studied this aspect of systemic risk along with the second aspect discussed above. The eventual propagation of a banking crisis rests on a combination of both factors in their model.

The empirical dimensions of the the third type of risk have been considered in Humphrey [1986], Angelini, Maresca and Russo [1996] and Furfine [1999]. The first two papers study knock-on possibilities associated with intra-day netting arrangements between banks; Humphrey for the United States and Angelini *et al.* for the Italian netting system. Furfine studies the bilateral exchange of interbank credit in the federal funds market of the United States. Each of the papers simulates the impact of failure in one bank upon the financial health of the system as a whole. A common theme of their results is that the degree of contagion depends on the relative importance of the failing bank as a debtor within the system.

While the above studies emphasise the *ex post* destabilising implications of one bank’s failure, there is an *ex ante* sense in which interbank credit plays a stabilising role, *i.e.* by allowing participants to pool resources in order to make good their obligations to outside parties (such as depositors or non-bank creditors). By borrowing from banks with surplus liquidity, banks which face a temporary shortfall can survive as a result of interbank credit. This risk sharing, in and of itself, must help keep down the incidence of failures in the system. Of course, in the event of one bank’s failure, an interbank credit market could very well exhibit the symptoms of contagion, but this possibility has to be weighed against the risk sharing role.

In this paper, we develop a simulation model which combines the *ex ante* and *ex post* effects of interbank credit on the stability of the banking system. We compare the dynamics of the system under various scenarios concerning the degree to which banks lend to other banks which face liquidity shortfalls. These strategies are imposed exogenously and the

choice between them is not derived from first principles. The focus is instead on the characteristics of the system and of its individual components; in particular, the heterogeneous nature of banks and the extent of credit linkage between them.¹

The simulation results suggest that interbank credit contributes to a lower incidence of bank failures through the risk sharing role but at the same time does create the tendency for the system to display avalanches, *i.e.* episodes of multiple bank collapse. Avalanches indicate the presence of contagion, *i.e.* the tendency for the failure of one bank to have knock-on effects on other banks. The latter possibility is quite minimal when the interbank market consists of homogeneous banks. But when the market consists of heterogeneous banks, the tendency for contagion becomes manifest as more risky banks can drag down less risky ones. We also find that the holding of large cash reserves, while unambiguously stabilising in the case of isolated banks, can lead to reducing the risk sharing role of the interbank market and can thus make banks more unstable when they are linked through such a market.

Section 2 describes the basic model under the assumption that banks are homogeneous. Section 3 discusses the results of simulating the model and then discusses extensions which introduce heterogeneity into the system. Section 4 concludes.

II. THE BASIC MODEL.

The system operates in discrete time, which is denoted by $t = 0, 1, 2 \dots$. At any time, t , there are a finite number of functioning banks, M_t . Each of these is labelled by k , $k = 1, 2, \dots, M_t$.

Banks receive stochastic deposits from customers and stochastic investment opportunities from entrepreneurs based in the non-financial sector. The primary purpose of banks is to

¹This approach follows recent attempts to use ideas from statistical mechanics to study some aspects of the dynamics of economic and financial systems (see for example Bak et al.[1993], Lux [1999], Iori [2001], Bouchaud and Cont [2000]).

channel funds received from depositors towards productive investment. These opportunities are assumed to be bank-specific in the sense that one bank cannot undertake the opportunity offered to another. It is assumed that resources tied up in investment, become illiquid. Hence, any investment made at time t fully matures only at some time $t + \tau$, even though a return, ρ , is realised at each point, $t + 1, t + 2, \dots, t + \tau$. ρ is exogenous and, in all the cases discussed in this paper, risk-free. Since withdrawals are unpredictable, however, a bank may find it itself unable to repay depositors due to the illiquidity of its assets.

If no interbank market is present, the mere inability to repay depositors triggers off a bank's failure. If interbank lending is possible, an illiquid bank might seek funds not just to pay depositors but also to repay past creditor banks. If despite such efforts, a bank ends up with insufficient funds, we assume for simplicity that it too closes down. Credit linkages between banks are defined by a connectivity matrix, J_{ij} . J_{ij} is either one or zero; a value of one indicates that a credit linkage exists between banks i and j and zero indicates no relationship. J_{ij} are randomly chosen at the beginning of the simulation. c denotes the probability that J_{ij} is one for any two banks. At one extreme, $c = 0$ represents the case of no interbank lending, while $c = 1$ represents a situation in which all banks can potentially borrow and lend from each other.

We now describe the system in detail. We shall first describe the case where the interbank market does not exist at all ($c = 0$) and then discuss the case where such a market exists ($c > 0$).

In the initial period, $t = 0$, the economy starts with a finite number, N_0 , of banks. The sequencing of events within each period is as follows. At the start of each period, each bank inherits an amount, L_{t-1}^k of cash holdings from the previous period, which consists of

$$L_{t-1}^k = A_{t-1}^k + V_{t-1}^k - \sum_{s=1}^{\tau} I_{t-s}^k$$

where A^k denotes deposits held by the general public in bank k , V^k represents its equity, and I^k represents investment by bank k . At $t = 0$, the values of $A_{-1}^k, V_{-1}^k, I_{-1}^k, I_{-2}^k$ etc. are chosen exogenously.

Given A_{t-1}^k , each bank pays an exogenous amount of interest, $r_a \geq 0$, at the start of period t and, at the same time, receives income, $\rho \sum_{s=1}^{\tau-1} I_{t-s}^k +$ and $(1+\rho)I_{t-\tau}^k$ from investments made over the last τ periods. It then faces a stochastic pattern of withdrawals and new deposits from customers, resulting in a new value, A_t^k .

Instead of strictly controlling aggregate deposits, we assume that A_t^k are idiosyncratic random variables whose exact specification will be varied through the paper and will be described as relevant in the following sections.

Since the amount of cash available to a bank can vary within a period, we shall use \hat{L}_t and \tilde{L}_t to represent intra-period cash at various points within each period and L_t to represent end-of-period cash. After paying interest, receiving investment income and facing deposit shocks, intra-period cash is:

$$\hat{L}_t^k = L_{t-1}^k + (A_t^k - A_{t-1}^k) - r_a A_{t-1}^k + \rho \sum_{s=1}^{\tau} I_{t-s}^k + I_{t-\tau}^k. \quad (1)$$

In the absence of an interbank market, any bank with $\hat{L}_t^k < 0$ is unable to make good at least part of its current obligations to depositors. It then shuts down and is removed from the system.

If $\hat{L}_t^k \geq 0$, the bank survives. Such a bank can undertake dividend payments to shareholders along with fresh investment activity. Both dividend payments and investment require cash, and dividends are paid before investment takes place.

Dividends are paid out of “excess” returns. In practice, $\hat{E}_t^k = \hat{V}_t^k/A_t^k$ is calculated for each surviving bank. \hat{V}_t is middle-of-period net worth and is itself defined by:

$$\hat{V}_t^k = \hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - A_t^k.$$

All banks with $\hat{E}_t^k > \chi$, where χ is a target capital:deposit ratio, are chosen as candidates to pay dividends. The actual dividend paid, D_t^k , is given by:

$$D_t^k = \max \left[0, \min \left[\rho \sum_{s=1}^{\tau} I_{t-s}^k - r_a A_{t-1}^k, \hat{L}_t^k - R_t^k, \hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1 + \chi)A_t^k \right] \right],$$

where R_t^k is the minimum reserve kept by bank k and is calculated as $R_t^k = \beta A_t^k$. The above formulation restricts dividend payments to equal investment income net of interest payments (if positive), but not if that violates either reserve or capital:deposit targets. Linking dividends to the capital:deposit ratio also has the advantage that the capital of surviving banks grows in proportion to their average deposits.

After dividends have been paid, the bank is assumed to undertake investment on the basis of its available cash, on the one hand, and its stochastic investment opportunity, on the other. Cash reserves at this point are defined as $\tilde{L}_t^k = \hat{L}_t^k - D_t^k$. The amount available to invest is therefore $\tilde{L}_t^k - \beta A_t^k$.

We assume that at any time, t , each bank receives a random investment opportunity, ω_t^k . This opportunity defines the maximum possible investment it can undertake. The actual investment satisfies:

$$I_t^k = \min \left[\max[0, \tilde{L}_t^k - \beta A_t^k], \omega_t^k \right].$$

The end-of-period cash holding of the bank then becomes:

$$L_t^k = \tilde{L}_t^k - I_t^k.$$

The end-of-period net worth of the bank is defined by:

$$V_t^k = L_t^k + \sum_{s=0}^{\tau-1} I_{t-s}^k - A_t^k.$$

This completes the description of the system without an interbank market. We now discuss the system with lending.

In this case, at the start of each period, the amount, L_{t-1}^k , of cash holdings held by each bank consist of

$$L_{t-1}^k = A_{t-1}^k + B_{t-1}^k + V_{t-1}^k - \sum_{s=1}^{\tau} I_{t-s}^k,$$

where B_{t-1}^k denotes total borrowing by bank k from other banks at time $t - 1$. Note that B^k can be negative or positive and satisfies: $\sum_{k=1}^N B^k = 0$. Borrowing is assumed to consist of one-period loans, which require repayment in full in the period after which they

are undertaken. From here onwards, we shall economise on notation and describe things qualitatively for the most part.

As before, at the very outset of period t , each bank's cash holding changes due to receipt of income from investment, payment of interest to depositors and a stochastic shock to deposits. The difference is that, at this early stage, if cash holdings become negative the bank can issue negotiable debt certificates to cover any excess of payments over its cash reserves. However, the certificates have to be redeemed at the end of the period through borrowing from other banks. If this is not done, the bank fails and its debt certificates become worthless.

Each bank's next priority is to repay creditors, if any, from the previous period. Such payments have to be made in cash. For the purposes of simulations, it is assumed that a bank either immediately pays all it owes, if it can, or pays nothing at this stage. Hence, only banks with cash holdings in excess of their debt obligations (which equal $(1 + r_b)B_{t-1}^j$ for bank j) make payments. Here, r_b is the interest rate on interbank borrowing. Repayments go directly to the creditors from the past period. The intra-period cash holdings of the relevant banks are updated accordingly.

At this point, two types of banks can be distinguished, those with positive cash and those with negative cash. Accordingly, they get classified as potential lenders and potential borrowers. Note that banks which are potential borrowers at this stage could themselves be owed money by debtors from the past period. Borrowing banks make demands for loans equaling their debt obligations (interest plus principal) minus their current cash (which if negative adds to their demand). In this formulation, borrowing on the interbank market is restricted to short-term solvency needs, *i.e.* to repay depositors and creditors, and does not cover 'long-term' investments.

Surplus banks are assumed to give priority to dividend payments and investment. The latter assumption implies that arbitrage between the real sector and the interbank market is imperfect. One justification for this would be the specialised nature of bank investments in the real sector. Banks are more likely to possess privileged (as opposed to shared) infor-

mation about their non-bank borrowers than about other banks within the system; hence, they are more likely to earn economic rents from lending to non-bank entrepreneurs than to other banks. As a result, the returns on ‘real’ assets dominate those on interbank loans.

After investment, any excess left over is made available to other banks. There is no mechanism in the model to guarantee that the total demand for loans equals the total supply. Hence, as far as the simulation is concerned, it is assumed that each borrowing bank contacts relevant lending bank in a random order. Each borrowing bank can only contact lending banks with which it is linked in an interbank relationship. The extent of linkage is parametrised by c , $c \leq 1$, which represents the proportion of other banks that each bank shares an interbank relationship with. $c = 1$ implies that all banks are connected to each other in a single market.

Once a borrowing bank contacts a lending bank, an agreement is reached between the two about how much credit will be exchanged. This is the minimum of the two banks’ respective demand and supply. The bank that is left with an unfulfilled trade stays on and contacts another bank on the opposite side of the market and tries reaching further agreements. A borrowing bank does not receive actual funds until it has lined up enough credit to ensure that it will not fail during the current period. Once a bank has obtained sufficient credit, funds are transferred and the cash positions of all banks involved is updated. This continues until either all loanable funds are exhausted or all demands for credit are satisfied.

The process now reiterates itself through the following steps: banks which had not repaid creditors in the first round but now have borrowed enough cash to pay off past debt entirely, do so; these payments go to their creditors and money holdings get updated accordingly; potential borrowers and lenders are determined for the next round; lenders undertake further investment, assuming they had unfulfilled investment opportunities from the previous round. Finally, a fresh round of borrowing and lending takes place. In principle, the process repeats itself until reiteration produces no further exchange of credit.

All banks which are left with negative cash holdings or cash holdings which fall short of their remaining debt obligations are deemed to be in default. These banks are removed

from the system. Their remaining assets (upon liquidation) are distributed to depositors; if, afterwards, there is still some value left, this goes to creditors from the previous period. All claims in excess of the bank's liquidation value are subtracted from the system. In the case of deposits, the aggregate is reduced by the amount owed to depositors minus that recovered from liquidation. In the case of interbank debt, each creditor's net worth is directly reduced by the amount owed to it by the failing bank minus the amount recovered.

III. SIMULATIONS AND RESULTS.

We started each simulation with 400 banks. In all the simulations presented here, the return on investment, ρ , was set at one percent, the interest rate on interbank borrowing, r_b , was 0.5 percent and the interest rate paid to depositors was $r_d = 0$. χ , the capital:deposit ratio was 30 percent. Parameters which vary across the simulations are identified separately.

Each bank, labeled by an index $k = 1, 2, \dots, M$, is characterized by its size s_k . This size represents the initial amount of customer deposits. In the homogeneous case it is assumed that all banks have the same size and $s_k = \bar{A}$. In the heterogenous case banks' sizes are sampled from the positive side of a Normal distribution $|N(\bar{s}, \sigma_s^2)|$.

A. Homogeneous case

In the initial set of simulations, we assumed that the banks were identical in the stochastic sense, *i.e.* that $s^k = \bar{A}, \forall k$ and $\bar{\omega}^k = \bar{\omega}, \forall k$. Each bank holds deposits, \bar{A} , worth 1000 units and initial equity, V_0 , equal to 0.3 times the initial deposit. At the initial period, $t = 1$, each bank is assumed to already have made investments. The maturity period is set at $t = 3$.

Within this framework, we consider two alternative models of how bank deposits fluctuate over time. In the first one, Model A, we take the traditional view that fluctuations are caused by random but mutually uncorrelated withdrawals of depositors. Under the assumption of independence, fluctuations in deposits are proportional to the square root of their size

$$A_t^k = \bar{A} + \sigma_A \sqrt{\bar{A}} \epsilon_t, \quad (2)$$

$$\omega_t^k = \bar{\omega} + \sigma_\omega \eta_t. \quad (3)$$

An alternative, Model B, is to assume that local correlations exist in the behaviour of each bank's depositors. These may arise from the fact that depositors of a given bank can observe and copy each other's withdrawal pattern. In this case, banks will experience fluctuation in their deposit proportional to their mean size

$$A_t^k = \bar{A} + \bar{A} \sigma_A \epsilon_t, \quad (4)$$

$$\omega_t^k = \bar{\omega} + \sigma_\omega \eta_t. \quad (5)$$

In both cases $\epsilon_t \sim N(0, 1)$, $\eta_t \sim N(0, 1)$ and $\bar{\omega} = \delta \bar{A}$, with $0 < \delta < 1$. Note that this herding behaviour only acts locally so that fluctuation of deposits are still uncorrelated across different banks.

Since the purpose of the exercise was to study the dynamics of a self-contained system with a given initial number of banks, we excluded the possibility that failing banks would be replaced by new entrants. Along with the homogeneity of banks this led to a system that, depending on the average size of investment opportunities, was either highly stable with practically no banks failing or a system in which all banks failed in the long run. For this reason, we use as our measure of relative stability, not the total long-run incidence of bank failures, but the number of failures over any given period of time.

Both model A and B generate qualitatively similar results, so we report simulations only for model B.

Figure 1 compares bank failures with different degrees of linkage in the interbank market. Linkage is described by a percentage, c . A linkage of c percent implies that each bank can exchange credit with c percent of other banks in the system. The vertical axis shows the number of surviving banks and the horizontal axis measures time (up to $t = 1000$). Other parameters were $\beta = 0.2$, $\sigma_\omega = \sigma_A = 0.5$. Increasing linkage adds stability, in the sense that

in any period, there are more surviving banks the greater the degree of linkage. Indeed, with 100 percent linkage, the system becomes entirely stable. This is consistent with Allen and Gale's [1998] result on the risk sharing role of interbank lending. This pattern was very robust to changing the parameters of the simulation.

FIGURE 1 HERE

A second question concerned the role of reserve requirements. **Figure 2** shows how different reserve ratios affect the incidence of surviving banks for the case of no interbank credit market. Of parameters other than β , only σ_A was changed to 0.25 while the others were set as in Figure 1. As the reserve ratio, β , increases from 10 percent to 70 percent in increments of 20 percent, the incidence of bank failures clearly falls. This conforms with conventional wisdom regarding the role of reserves.

FIGURE 2 HERE

Figure 3 depicts the same experiment as Figure 2, except that interbank linkages are set at 1 percent. With an interbank market, the role of reserve requirements is less clear cut. In both cases, increasing β initially leads to an *increase* in the incidence of bank failures, but if β crosses a critical level, increasing it further results in fewer bank failures.

FIGURE 3 HERE

We explain these results as follows. Holding reserves keeps resources liquid and enables banks to better meet unpredictable shifts in demand by depositors. Obviously, increasing

reserves adds to the stability of individual banks. However, with interbank linkages, higher reserves also reduce the insurance provided by banks to each other. If reserves are set high enough relative to the degree of risk, the individual effect completely dominates and overall stability increases with reserves. Indeed, in this regime our simulations detected no activity on the interbank market, since the need to keep high reserves mopped up all the excess liquidity at the disposal of surplus banks. At lower values of β , reserves do not kill off interbank activity and the individual stabilising effect is relatively small compared to the insurance effect. It is in this regime that increasing reserves leads to more frequent failures.

Changing the other parameters of the system had predictable effects. For example, increasing the volatility on deposits increased the rate of bank failures, while increasing the recovery rate from failed banks reduced the rate at which banks failed. However, the qualitative nature of the effects reported in Figure 1-3 were not affected by varying these other parameters. These comparative static experiments will therefore not be discussed in further detail.

We now turn to the issue of contagious failures. Contagion would suggest that ‘avalanches’, *i.e.* periods in which many banks collapse together, occur. There appears to be no evidence of this in Figure 1 where the time path of surviving banks declines more or less smoothly over time.

We also simulated a variant of the above model. Instead of letting the *aggregate* levels of deposits and investment opportunities average out on the basis of the law of large numbers, we controlled them deterministically. Aggregate investment opportunities were assumed to be a fixed constant over time. Aggregate deposits were fixed initially and subsequently reduced appropriately whenever a failing bank left behind obligations which exceeded the liquidation value of its remaining assets. Individual banks received stochastic shares of the aggregate deposits and investment opportunities.

In the variant, the basic results described by Figures 1-3 were also observed. But in the counterparts to Figures 1 and 3, whenever the interbank market was present, instead of a relatively uniform decline in the number of banks over time, there was an initial period of

slow decline followed by a subsequent sharp decline. On a casual glance, this appeared to be a sign of contagious failure. But it turned out that the sharp decline was driven by the cessation of all activity on the interbank market rather than via knock-on effects.² When we tried a third variant in which aggregate quantities remained deterministic but both the aggregate deposit and the investment opportunity shrank proportionately as banks failed, activity on the interbank market did not dry up and the path of decline remained roughly uniform, as in the present version.

We report these results in order to emphasise that despite having explored several formulations of the stochastic risk facing individual banks, two properties were consistently observed: (i) increasing inter-bank linkages slows down the rate at which banks fail, (ii) there are no significant ‘knock-on’ effects from the failure of individual banks.

One feature of the results shown in Figures 1 and 3 was that no single bank became a significant debtor and hence, the knock-on effects of any bank’s failure did little damage to other banks. The homogeneity of banks appears to be a cause of this. The implication is that with homogeneous banks, the risk sharing role of the interbank market is dominant while the absence of significant amounts of borrowing or lending on an individual basis minimises the potential for knock-on effects within the system.

We therefore made a departure from the homogeneous case by introducing different types of banks.

²Essentially, as the number of surviving banks fell, surviving banks were able to get larger and larger *per capita* opportunities to invest, which eventually absorbed all their available deposits. This feature has been ruled out in the present model where *per capita* values of both deposit and investment opportunity are constant by construction.

B. Heterogeneous case

The first manner in which we pursued heterogeneity was by letting banks vary in size. Each bank's average deposit was chosen from a Gaussian distribution. Other characteristics, such as the average investment opportunity, were scaled in proportion to this parameter. We again consider two models of each bank's deposit fluctuations over time.

In model A,

$$A_t^k = s^k + \sigma_A \sqrt{s^k} \epsilon_t \quad (6)$$

$$\omega_t^k = \bar{\omega} + \sigma_\omega \eta_t. \quad (7)$$

Note that, with different sizes, in model A, the deposits of bigger banks will have a lower volatility-mean ratio than those of smaller banks. In model B,

$$A_t^k = s^k + \sigma_A s^k \epsilon_t, \quad (8)$$

$$\omega_t^k = \bar{\omega} + \sigma_\omega \eta_t. \quad (9)$$

In model B, all bank deposits have the same volatility-mean ratio. Hence, banks differ not in terms of the idiosyncratic risk arising from depositors' demands, but in terms of the size of their borrowing and lending requirements. Big banks are capable of generating both larger demands for and larger supplies of funds. In both cases $\bar{\omega} = \delta s^k$ and $s^k \sim |N(\bar{s}, \sigma_s^2)|$.

The second way in which we introduced heterogeneity was in the specification of investment opportunities. We call this variant, model C. In this case, banks face identical distributions of deposits but differ by a scale factor in the distribution of investment opportunities,

$$A_t^k = \bar{A} + \bar{A} \sigma_A \epsilon_t \quad (10)$$

$$\omega_t^k = i^k + \sigma_\omega \eta_t \quad i_k \sim |N(\bar{i}, \sigma_i)|. \quad (11)$$

Before showing the results of the simulation we derive some qualitative results on the behaviour of the aggregate system, under the assumption that banks are all connected by lending and borrowing to each other, i.e. $c = 1$. We need to compare quantities averaged over different formulations of heterogeneity (distribution of bank sizes and investment opportunities). In model A, the aggregate reserves, $A^T = \sum_{k=1}^M A^k$, have a variance

$$\sigma_T^2 \sim \sum_{i=1}^M s_k.$$

This is a random variable which depends on the realization of the banks size. Averaging over the realization of the s_k we obtain

$$E[\sigma_T^2] = \sum_{i=1}^M \bar{s} = M\bar{s}.$$

Also, the mean size of the total deposit

$$E[A^T] = M\bar{s}.$$

Thus the volatility/mean ratio of the overall deposit, and hence the probability of default, on average decreases with the square root of the number of banks linked together but does not depend on the heterogeneity parameter σ_s . In model B, the aggregate variance is

$$\sigma_T^2 \sim \sum_{i=1}^M s_k^2$$

and averaging over the realization of the s_k we obtain

$$E[\sigma_T^2] = \sum_{i=1}^M E[(s_k)^2] = \sum_{i=1}^M (\bar{s}^2 + \sigma_s^2) = M(\bar{s}^2 + \sigma_s^2).$$

Also in this case the the volatility/mean ratio of the overall deposit decreases with the square root of M and the probability of default is reduced, on average, when the interbank market is in place. Nonetheless, in this case, defaults become more likely to appear as the heterogeneity of the system increases.

We split the simulation analysis into two parts. First we investigate the role of connectivity and heterogeneity in the system before contagion effects could possibly take place, i.e.

up to the time the first event of default occurs (note that at this time, several defaults may occur simultaneously). To this end, we calculate the first time of default τ_1 , averaged over 1000 different realizations of A^k and ω^k (sampled from the same distribution) as a function of c , σ_s and σ_i . In all the following simulations, we set γ , the recovery rate, to zero.³ Increasing connectivity improves the stability of the system and increasing heterogeneity makes the system more stable in model A (**Figure 4a**) and more unstable in model B (**Figure 4b**). In model A, bigger banks are inherently more stable, so heterogeneity seems to play a stabilising role because the introduction of larger banks contributes to the overall stability of the system. This is not true of model B in which all banks have the same mean:variance ratio.

FIGURE 4 HERE

To identify contagion effects more precisely, we analyzed a model with only two types of banks. Both faced identical distributions of deposits but differed in the distribution of investment opportunities: ‘low’ and ‘high’. Due to low and relatively constant investment opportunities, the former set of banks tended to possess excess liquidity. In the absence of interbank lending, they never failed (see **Figure 5a**: ‘liquid’ banks are type 0 and ‘illiquid’ ones are type 1).

FIGURE 5 HERE

However, with lending, they became exposed to the risk of default because of non-

³The connection between low recovery rates and the knock-on effects of bank failures has been considered in Furfine (1999), whose simulation suggests that for contagion to be possible, recovery rates should not be too large.

repayment by borrowing banks. Since non-repayment implies failure, the collapse of a borrowing bank was a direct trigger of the collapse of the lending one. **Figure 5b** and **Figure 5c** show that as the degree of connectivity was increased across banks, the ‘liquid’ banks defaulted in increasing numbers. Nonetheless, the *overall* incidence of bank failures remained lower with interbank lending than without. This illustrates the trade-off between the insurance role of interbank credit and the possibility that it will result in ‘knock-on’ effects.

Having obtained these insights into the role of heterogeneous investment opportunities in the binary case, we went into the study contagion for our model proper. We believe that contagion effects may play a more important role in model B and C. We present results for model C in the following.

In **Figure 6**, the variance of types set at 10, we show that increasing connectivity from 5 to 50 percent leads to less failures during the interval studied, but increasing connectivity to 80 percent leads to more failures than at 50 percent. Hence, in contrast to the case of homogeneous banks, increasing the extent of the interbank market need not increase overall stability. Also, the time path now shows ‘avalanches’, *i.e.* many banks collapse over brief periods of time. Since the aggregate environment is on average constant, the non monotonic response of the system to increasing connectivity together with the presence of avalanches suggest that knock-on effects may be now taking place.

FIGURE 6 HERE

In **Figure 7**, we compare the size of avalanches when the level of connectivity increases (in the case where investment opportunities are normally distributed with a variance which is set at 10). As connectivity increases, episodes of failure become more sparse, but when they happen, the number of banks involved gets larger and larger. An interesting effect which can be observed is that as linkages increase, the system moves from a dynamics where

only small avalanches are observed to one where avalanches of many different sizes become possible.

FIGURE 7 HERE

To detect contagion in a statistical sense, we measured the probability of a bank defaulting conditional on it being a creditor of a previously defaulted bank. We indicate a bank in this category as B_c . For this purpose we measured the ratio, ξ , averaged over 500 realizations of the initial distribution of investment opportunities, of B_c banks defaulting at times $\tau_1 < t < \tau_2$ over the total number of banks defaulting over the same interval of time, where τ_1 is, as before, the first time one (or more) bank defaults and τ_2 is the time needed to observe 50 defaults. In **Figure 8** we plot ξ as a function of σ_i and c . Heterogeneity does not seem to play a major role to generate contagion in this experiment and in fact the curves collapse on each other for different values of σ_i . The monotonic decrease of ξ with the system's connectivity also suggests that, even if contagion effects may happen, widening the scope of the interbank does not make creditor banks become more prone to default on average. Our interpretation of this is that while heterogeneity may be a factor behind individual episodes of contagion and avalanches (as in Figure 5 and 7), it does not do so in a statistically frequent manner (at least not over time interval considered in this experiment); most of the time, the stabilising role of connectedness dominates.

FIGURE 8 HERE

IV. CONCLUSIONS:

When banks are homogeneous and face idiosyncratic deposit shocks, the insurance role of interbank lending prevails. In this situation, higher reserve requirements can lead to a higher incidence of bank failures. When banks are heterogeneous in average liquidity or

average size, contagion effect may arise. Our simulation results indicate that when banks differ in size but not in average liquidity, heterogeneity alone can contribute to instability, depending on the nature of the idiosyncratic deposit shocks, but interbank lending stabilises the system. But when banks differ in terms of average liquidity, interbank lending can create knock-on effects which may work against the insurance function of the market.

One solution which suggests itself is that interbank lending relationships be confined to banks which share similar liquidity characteristics. This is not necessarily an argument for government intervention. Before advocating that, one would need to explain why banks do not endogenously sort themselves in the first place. Rochet and Tirole (1996) argue that a willingness by regulatory authorities to bail out failing institutions might reduce the incentive for banks to monitor and screen each other. The possibility of government bailouts might hinder endogenous separation.

Finally, the model of this paper can lend itself to some extensions regarding the design and stability of payments systems in general. For example, Carletti, Hartmann and Spagnolo [2002] have argued that bank mergers can reduce liquidity in the interbank market. Similarly, while in the present paper, in order to focus on the autonomous behaviour of the banking system, we have neglected any activity by the central bank, in practise, the latter often intervenes especially when ‘large’ banks face crises. The implications of such developments on systemic stability could be studied by appropriate modifications of the present model.

The implications of this for systemic stability could be studied using a variant of our model.

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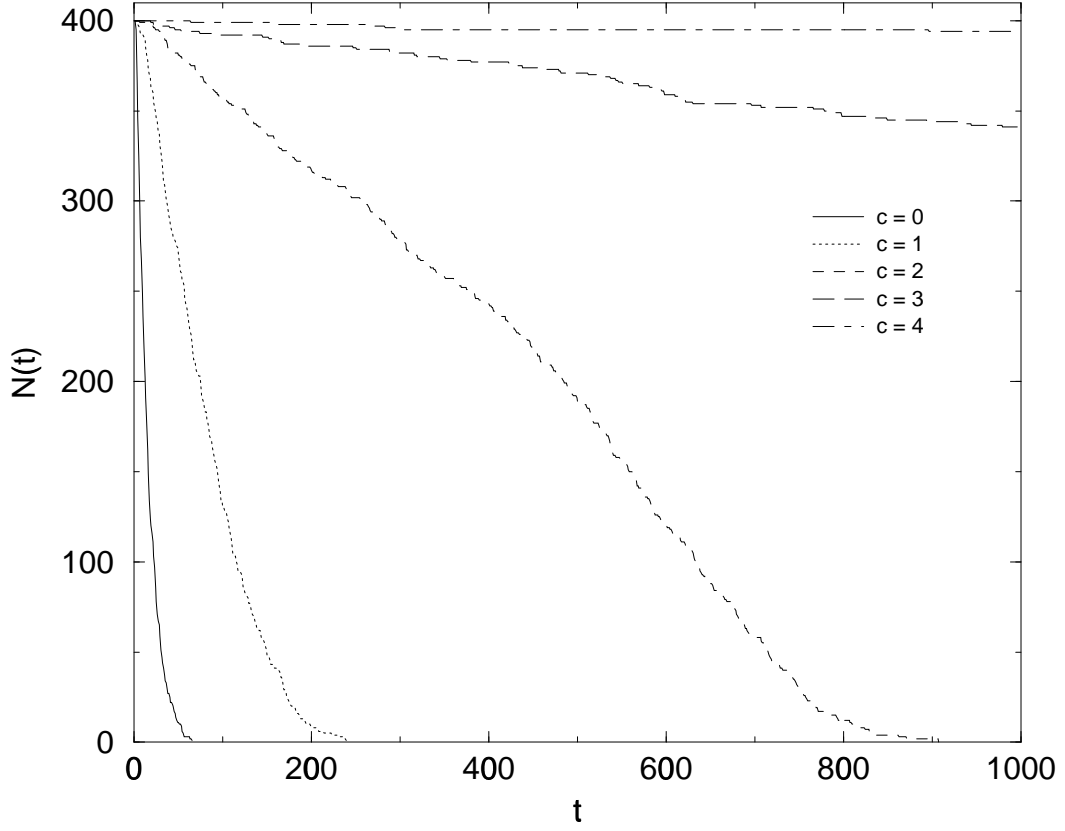


FIG. 1. Surviving banks with different interbank linkages.

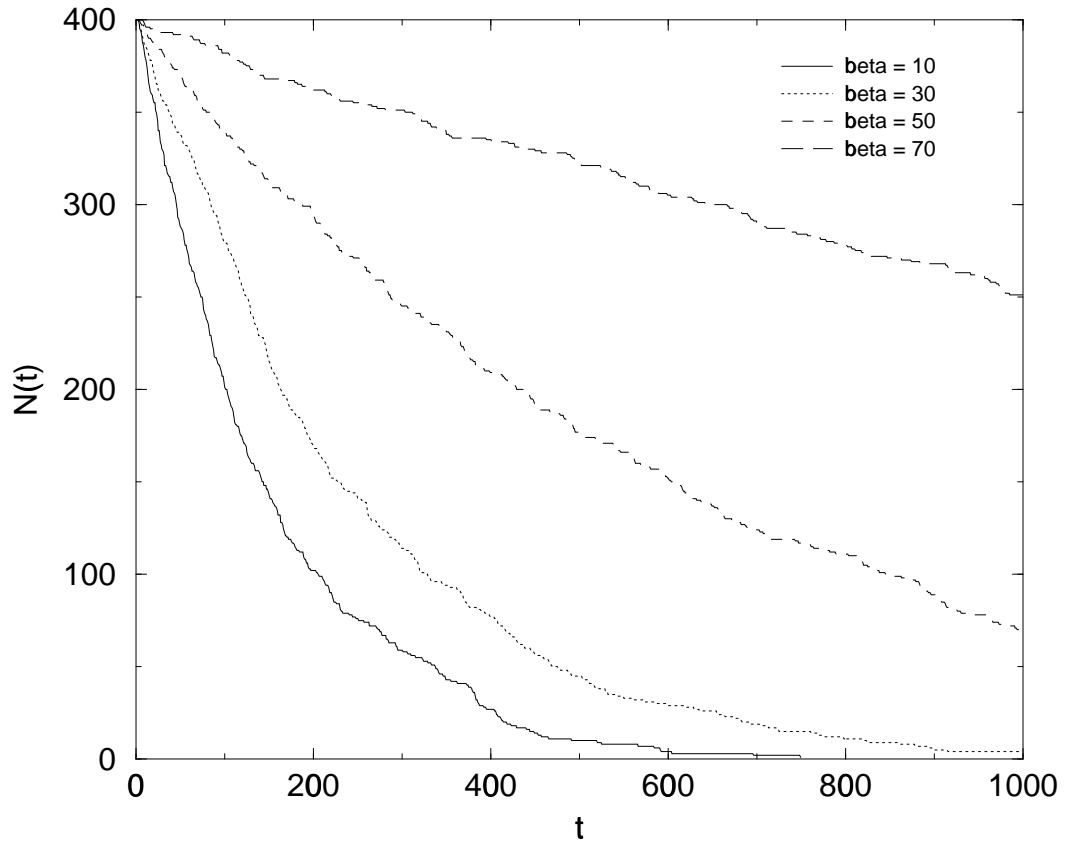


FIG. 2. Surviving banks with different reserves and no interbank linkage.

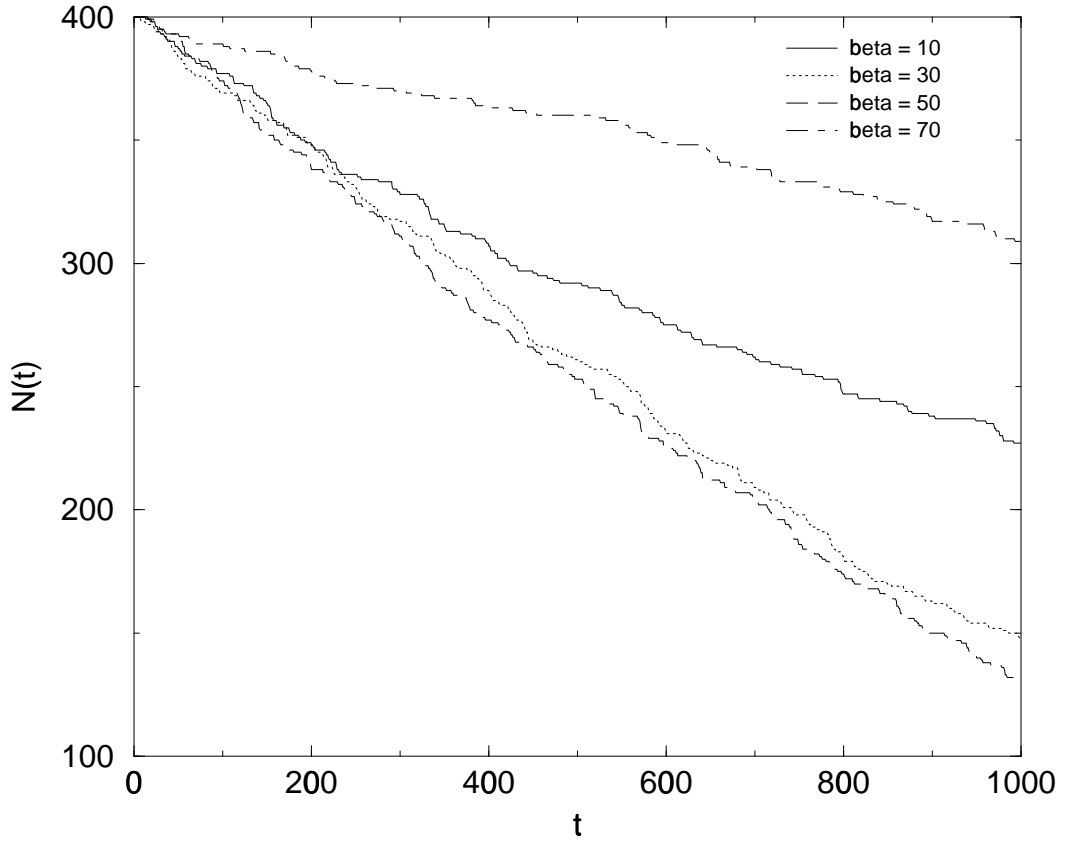


FIG. 3. Surviving banks with different reserves and 1 percent linkage.

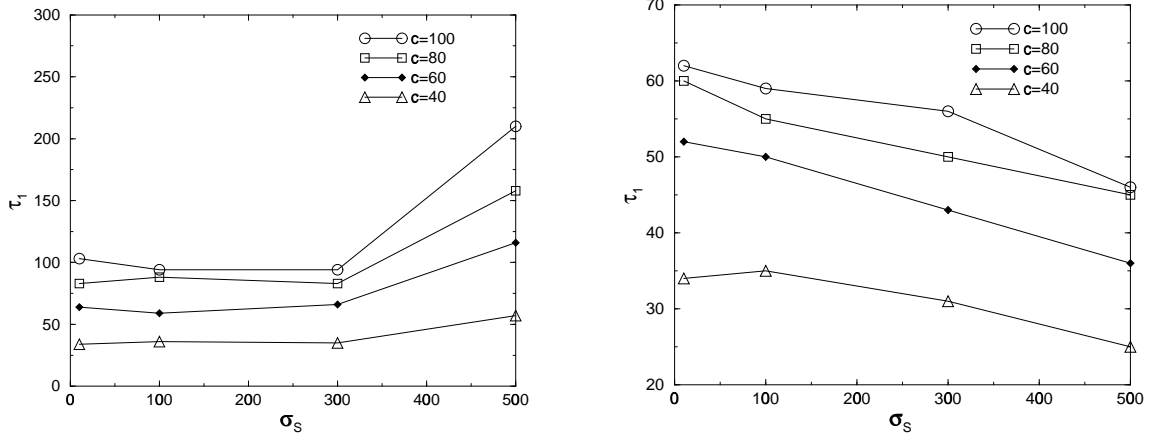


FIG. 4. First failing time τ_1 , versus σ_s for model A (left), and for model B (right).

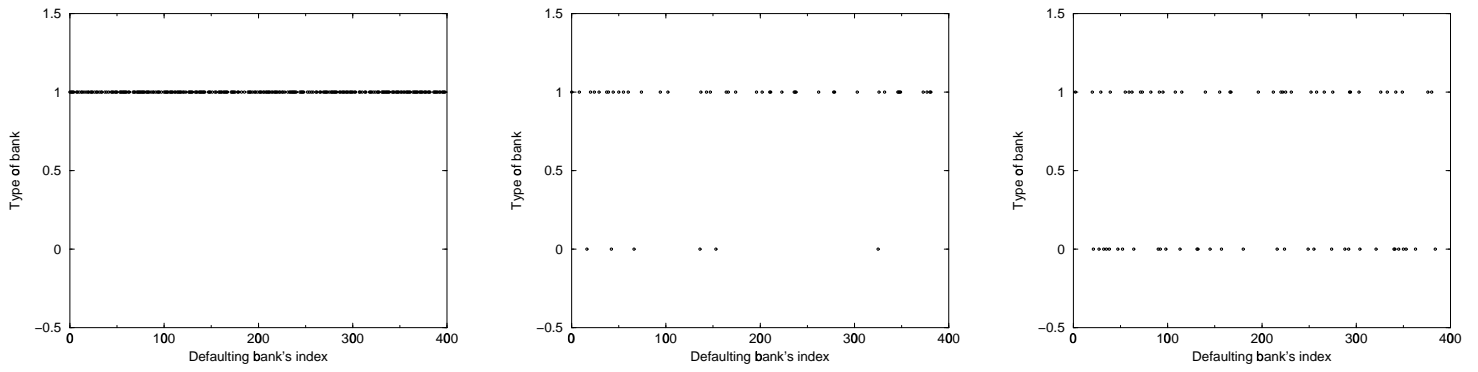


FIG. 5. Incidence of bank failures by bank type with (a) no interbank linkage, (b) 1% linkage, (c) 5% linkage.

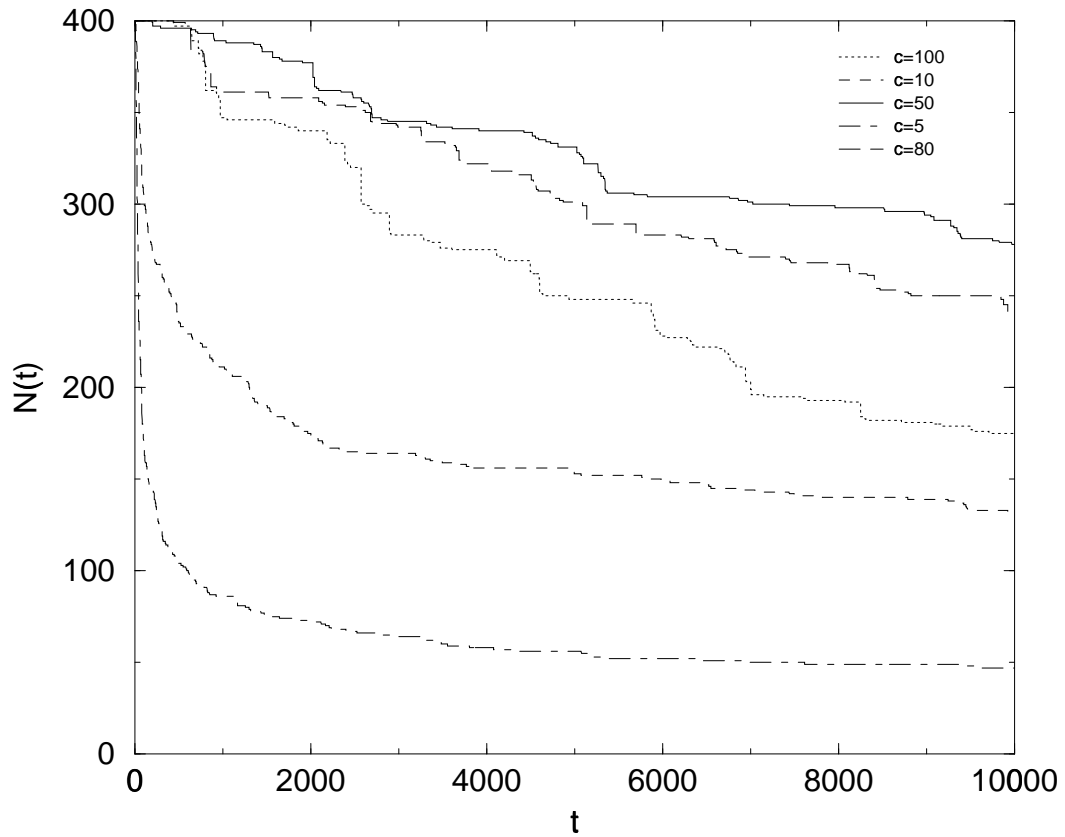


FIG. 6. Effect of increasing linkages on failure; heterogeneous investment opportunities.

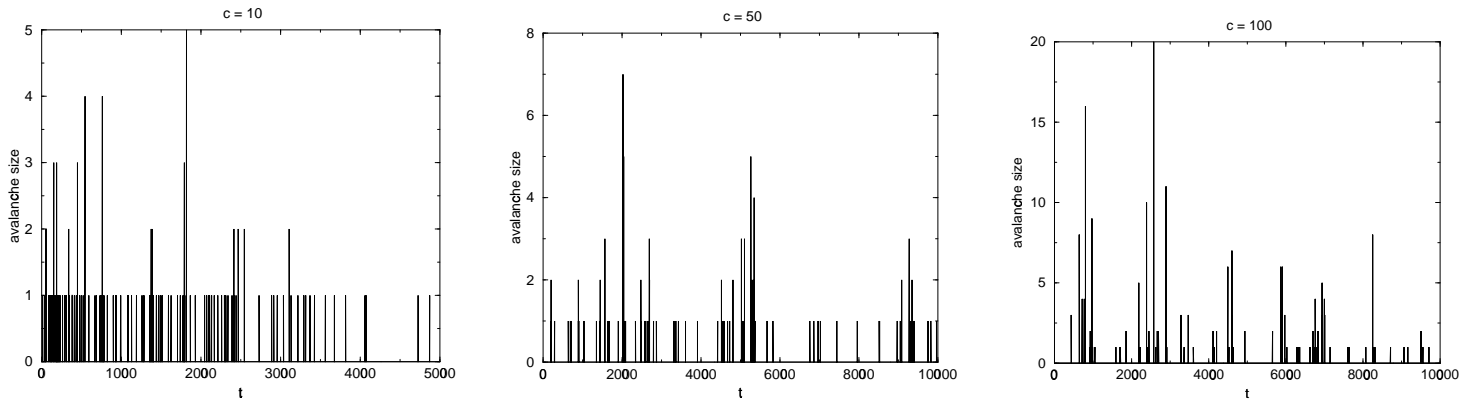


FIG. 7. Effects of increasing linkages on avalanches: variance of investment opportunity at 10.

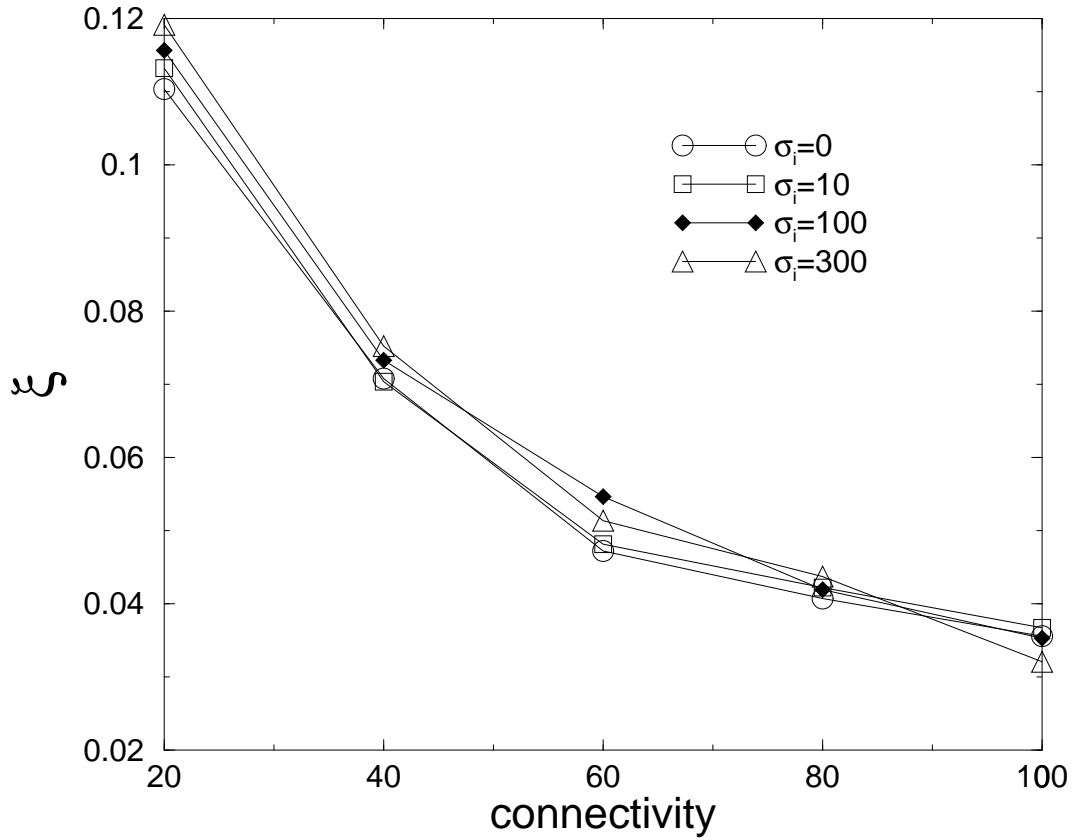


FIG. 8. ξ as a function of connectivity for different values of σ_i in model C.