5. Probability Multiplication as a New Principle in Psychophysics

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One of Martin Regan’s most interesting, but little tested, ideas has been to generalize the mechanism of the Reichardt detector of motion to spatial vision, in the form of a hypothetical “Coincidence Detector” (Morgan and Regan, 1987; Regan, 2000; Regan and Beverley, 1985). The Reichardt detector (Hassenstein and Reichardt, 1956; Reichardt, 1961; van Santen and Sperling, 1985) works by multiplying the signal from two different spatial detectors, with a delay to one of them. In its full opponent version, the outputs of two such detectors, tuned to opposite directions, are subtracted (figure 5.1). A useful consequence of opponency (discussed at length by Regan in his 2000 book Human Perception of Objects) is contrast independence. Changes in contrast of a stationary stimulus over time (flicker) will not be confounded with movement of the stimulus.

The same idea can be applied to a coincidence detector for spatial hyperacuity (figure 5.2). Changes of only a few arcsec in separation of two lines or dots can be reliably detected by observers making the decision whether the test stimulus is “wider” or “narrower” than a standard stimulus. Observers do not confound changes in separation with changes in contrast of the component lines (Morgan and Regan, 1987). The same is true of vernier acuity. Opponent pairs of coincidence detectors can account for this independence of spatial decisions from target contrast.

In this chapter, we consider psychophysical evidence for opponent Reichardt detectors and coincidence detectors more generally. We begin with motion. Compelling evidence for a multiplication stage in coincidence detection comes from the “amplification effect” (van Santen and Sperling, 1984). The contrast required to detect the direction of movement of a low-contrast, drifting sinusoidal grating is lowered if it is superimposed upon a stationary, flickering grating of the same spatial and temporal frequency. In the case of sampled motion, with a 90° phase shift between frames, such
Figure 5.1: Schematic diagram of the Reichardt Detector.

Figure 5.2: A version of the coincidence detector proposed by Morgan and Regan (1987) to account for the insensitivity of spatial interval acuity to contrast jitter of the component lines (top). The output of spatially localized detectors (circles) are multiplied and subtracted to give a signal proportional to target separation.
a stimulus is formally identical to one in which odd frames have higher contrast than even frames (or vice versa). The contrast thresholds for motion found when varying even-frame contrast decrease as odd-frame contrast is raised, relative to the case where the odd and even frames have the same contrast (which we shall refer to as the “yoked condition”). This is predicted by the multiplication stage in the Reichardt model, since it is the product of even and odd frames that should be constant at threshold (van Santen and Sperling, 1984).

Van Santen and Sperling’s model makes the counterintuitive prediction that contrast for even frames at threshold (for motion detection) should decrease without limit as contrast of the odd frames is raised. This is a consequence of detection limited by late noise after the multiplication stage. Morgan and Chubb (1999) found that thresholds did not decrease in this way, but reached an asymptotically low value when odd frames were greater or equal to three times the yoked threshold. To account for their findings, they explored an “early noise” version of the Reichardt model, in which noise was added to the output of two detectors in quadrature phase and the noisy outputs are then multiplied. This model provided a satisfactory fit to their data for two-frame motion of a 2 cpd sinusoidal grating of temporal frequency 2.5 Hz.

However, the Morgan and Chubb model was defective in not having an opponent stage. It cannot truly be called a Reichardt detector; it is a “half-Reichardt” detector. In fact, Morgan and Chubb point out that their early noise model is difficult to distinguish from one in which motion direction is correctly computed if both frames reach detection threshold in the presence of early noise. Indeed, after the paper was written, we realized that the half-Reichardt detector with early noise is mathematically identical to a high-threshold (HT), early noise model of independent detection of the two frames. “Probability multiplication” in the latter is the equivalent of signal multiplication in the former (for the proof, see appendix 5.A2.3).

We seem to have a paradox here. The model that satisfactorily fits Morgan and Chubb’s data is not a motion energy model after all. It is compatible with independent detection of the component frames, and thus with a model based on “local sign” (Morgan, 1990) more akin to a long-range motion mechanism (Braddick, 1980). Does the same model fit van Santen and Sperling’s data? We fit both probability multiplication and late noise models to their full psychometric functions, with the results shown in figure 5.3. Neither model fits all the data. However, there is some evidence that the late-noise model is a better fit at the higher temporal frequency of 12.5 Hz while the probability multiplication model is at least as good a fit at the lower frequency of 1.8 Hz, closer to the frequency (2.5 Hz) used by Morgan and Chubb. Note particularly the unacceptable failure of the late noise model at 1.8 Hz in observer NB in the highest amplification condition.

We considered two further models. The opponent or contrast discrimination model is based on a formal similarity between amplification and facilitation in contrast discrimination. We consider direction discrimination in an opponent mechanism, one half-detector of which receives an input corresponding to one frame alone (the one of higher contrast) and the other half-detector receives an input that is the sum of the contrasts in the two frames. In each case the input is transduced by a power function, to give an accelerating nonlinearity, required for facilitation. Finally, we considered a convoy model, so called because a convoy moves at the speed of its slowest mem-
Figure 5.3: The figure shows a reanalysis of the data in van Santen and Sperling (1984). Each row shows the data from a single observer at a particular temporal frequency of the motion. The top two rows are for 1.8 Hz, and the bottom two for 12.5 Hz. Each panel shows how probability correct (vertical axis) changes as a function of the contrast of the odd-numbered frames in the movie sequence (horizontal axis). The contrast of the odd-numbered frames is fixed within each panel, and is indicated by the figure in the bottom right hand corner of each panel. The contrast of even-numbered frames is shown on the x-axis. The open circles are data, and the error bars represent 95% confidence intervals derived from the binomial distribution. The curves represent fits to the data by the various models described in the text: Solid line, the opponent model; Dotted line, the probability multiplication model; Dashed line: the late-noise Reichardt model. Note that the probability multiplication model is the best fit to the data at the lower temporal frequency, while at high temporal frequencies the data are better fit by the late-noise and opponent models.

ber. In this case, the detector receives an input from the lower contrast frame only. It is assumed that the higher-contrast frame is always detected, and that the observer is infallibly correct if both frames are detected.

To document the failure of the late noise model further, we repeated Morgan and Chubb’s conditions with a new observer (AJ). Once again (figure 5.4), the probability multiplication model was a better fit than the late-noise Reichardt model.

To confirm that we were able to get late noise behavior as well as early noise, we tried various combinations of temporal and spatial frequency, and number of motion frames, with various spatial envelopes for the gratings. We concentrated on thresholds obtained with an amplifying frame of $3 \times yoked$ threshold, which is optimal for distinguishing the models. Results were frequently mixed, with neither model fitting well. However, we found at least one case where the late noise model was better than probability multiplication: a 12.5 Hz grating windowed with a stationary Gaussian window (figure 5.5).
Figure 5.4: Data from one observer (AJ) in a first-order motion detection task, with two quadrature frames of a Gaussian windowed 2 cpd sinusoidal carrier, replicating the procedure of (Morgan and Chubb, 1999). The frame duration was 100 msec with no ISI; thus temporal frequency was 2.5 Hz. Conventions are as in Figure 5.2.

Figure 5.5: Data for two observers performing a first-order motion detection task in which the stimulus was a 2 cpd sinusoidal carrier moving within a stationary Gaussian envelope. Frame duration was 20 msec, thus temporal frequency 12.5 Hz. Other conventions are as in figure 5.3.
We conclude that observers do have access to a Reichardt detector, in which the predominant source of noise enters after the multiplication stage. What mechanism are they using for probability multiplication? Morgan and Chubb considered and rejected the idea that the phase of the stimulus is known to the observer as soon as it is detected (the local sign model). Their evidence contrary to this idea was that contrast thresholds for detection were lower than for phase discrimination (sine vs. cosine). This leaves the possibility that observers use a specialized coincidence mechanism such as the Reichardt detector, but one in which there is early noise with a high threshold on the output of the detectors. Observers may have access to both of these mechanisms (early/HT and late noise Reichardt) and use different mechanisms on different trials, ensuring that no single model will fit all the data, as we observed.

This discussion of motion mechanisms sets the stage for other kinds of coincidence detector. Some of the tasks we have examined, to measure amplification, are as follows:

1. Vernier acuity with abutting gratings.
2. Alignment acuity for vertically separated Gabor patches, in which the observer must decide whether the imaginary line joining the two patches is tilted clockwise or anticlockwise of the vertical. Both the envelope and carrier are displaced.
3. Stereo-defined motion. A grating defined by stereo disparity in otherwise random noise moves between frames.
4. Second-order alignment. A Gabor patch appears randomly at either the top-left or bottom-right hand corner of a notional square. A second patch is either horizontally or vertically aligned with the first. The patches have random carrier phase, making the task second order. Logically, both patches have to be detected to perform the task; otherwise the observer must guess. Therefore, this is a task in which we would expect probability multiplication.
5. The same as (4) except that the stimuli were Gaussian blobs defined by disparity in low-pass-filtered random noise, not luminance.

The results are easily summarized. In no case did we find an amount of amplification greater than predicted by probability multiplication. Figures 5.6 and 5.7 give examples from abutting vernier and second-order alignment tasks.

Conclusions

The principle of probability multiplication says that contrast thresholds in tasks like vernier acuity can be predicted completely from the independent probability of detecting the component stimuli. No extra mechanisms are required in the tasks we have examined, with the sole exception of motion at highish temporal frequencies (12.5 Hz). In the latter case, we observe amplification greater than that predicted by probability multiplication, consistent with a dominant source of late noise (after multiplication). Physiological identification of this and the late noise mechanisms awaits investigation. We might consider simple versus complex cells, or parvocellular versus magnocellular pathways, with equally little evidence at present.
Figure 5.6: Performance of two observers in a vernier alignment task, using a large field 2 cpd sinusoidal grating with a horizontally oriented $90^\circ$ phase boundary in the middle of the screen. The observer’s task was to decide whether the bottom half-grating was shifted left or right. Exposure was 100 msec. Other conventions are as in figure 5.3.

Figure 5.7: Performance of one observer in a second-order alignment task where the stimuli were Gaussian blobs defined by disparity. Thresholds now refer to disparity, not to contrast. Other conventions as in figure 5.3.
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Appendix

5.A1 Methods

The following describes the methods for first-order motion task. Differences in the other tasks such as vernier are described in the figure legends. To maximize the rate of data collection, two parallel experimental systems were used to collect data in different experiments. The difference between the systems is not thought to be relevant to the interpretation of the data. Both used a Cambridge Research Systems Graphics VSG card in a PC platform to generate the stimuli at a frame rate of 100 Hz. One system used 12-bit luminance resolution from the card and a Barco high-resolution RGB monitor; the other used 15-bit resolution and a Mitsubishi Diamond Pro monitor. The two systems were used to generate two kinds of stimuli. The first system (Barco) was used for experiments with Gabor patches, which consisted of a horizontal 2 cycle/deg sine wave gratings multiplied by a circular Gaussian window of standard deviation 0.25°. Mean luminance was 5 cd/m². The second system (Mitsubishi) was used for rectangularly windowed horizontal 2 cycle/deg sine wave gratings of dimensions 3.3 deg². Mean luminance was 19 cd/m². The stimuli were viewed from a distance of 2 m in a darkened room. Each trial began with the disappearance of the central fixation point followed by a brief motion sequence in which the sine wave carrier moved either up or down inside the stationary window. Each motion step consisted of a single 90° phase change in the carrier. There was a either a single step (two-frame sequence) or three steps (four-frame sequence). A new luminance lookup table was loaded between each frame to control contrast. In the yoked condition all the frames had the same contrast, which was varied between trials in order to determine the psychometric function relating contrast to the probability of the observer making a correct identification of the motion direction (up or down). Five different contrast levels were pseudo-randomly interleaved. The yoked condition was randomly interleaved with the fixed contrast condition, in which the contrast of even numbered frames was fixed within a block of trials. Odd-numbered frames had varying contrast, as in the yoked condition, to determine the psychometric function. Each block of trials consisted of 10 repeats of each contrast level in both yoked and fixed conditions, giving a total of $2 \times 5 \times 10$ trials. At the end of each block the observer rested before beginning another block of trials. We aimed to collect at least 100 trials for each point on the psychometric function. The observers were three of the authors (MJM, JAS, and AJ) who are experienced motion observers, and FF, who was previously inexperienced at motion observing. Further checks on the generality of the findings were performed with a naive student observer (TA), whose results were similar to those of the other observers but are not presented here.
5.A2 Models and Theory

5.A2.1 The Late-Noise Reichardt Model

An elaborated Reichardt detector (van Santen and Sperling, 1985) generates two visual signals before and after a stimulus is displaced, as illustrated in figure 5.1. Before displacement, it generates visual signals $A$ and $B$. After displacement, the signals are $A'$ and $B'$. If these signals are not perturbed by noise, then for sinusoidal stimuli, displaced by 90°, $A$ and $B'$ may be considered equal to zero. Thus an elaborated Reichardt detector without early noise is equivalent to a simple Reichardt detector, in which direction discrimination depends on the product of two signals elicited by the stimulus, one before displacement, and the other after.

In all of our models, visual signals are allowed to vary as any arbitrary power of target intensity. Thus, if $t_1$ and $t_2$ represent the pre- and post-displacement target-intensities, then the late-noise Reichardt model’s two non-zero visual signals are given by $t_1^p$ and $t_2^p$, where $p$ is a free parameter. In all of our models, internal noise is assumed to have a Gaussian distribution. This noise perturbs the product of visual signals in the late-noise Reichardt model. Thus, accuracy is given by

$$\Psi_{LNR}(t_1, t_2) = \Phi\left(\frac{t_1^p t_2^p}{\sigma}\right)$$

where $\sigma$, like $p$, is a free parameter and $\Phi$ is the standard normal CDF.

5.A2.2 The Opponent (Contrast Discrimination) Model

Like the probability multiplication model (below), the opponent model asserts that one random variable $Y$ must exceed another $X$ for a correct response. Thus accuracy is given by

$$\Psi_0(t_1, t_2) = \int_{-\infty}^{+\infty} F_X(z) f_Y(z) dz.$$

In this expression, $Y \sim N((t_1 + t_2)^p, \sigma^2)$ is the noisy signal elicited by the two targets’ combined energies and $X \sim N(\text{Max}(t_1^p, t_2^p), \sigma^2)$ is the noisy signal elicited by the target with the maximum intensity. $F_X(z)$ is the cumulative density function of $X$ and $F_Y(z)$ is the probability density function of $Y$.

5.A2.3 The Probability-Multiplication Model

The probability multiplication model assumes that the pre- and post-displacement targets are detected with probabilities $p_1 = 2\Phi(t_1^p/\sigma) - 1$ and $p_2 = 2\Phi(t_2^p/\sigma) - 1$, respectively. When both targets are detected the observer will respond correctly. When at least one target is not detected, the observer will have a 50% chance of responding correctly. Thus accuracy is given by

$$\Psi_{PM}(t_1, t_2) = p_1 p_2 + (1 - p_1 p_2)/2 = (1 + p_1 p_2)/2$$

Morgan and Chubb previously (1999) proposed a model in which accuracy was given by

$$\Psi_{MC}(t_1, t_2) = \Phi(t_1/\sigma)\Phi(t_2/\sigma) + (1 - \Phi(t_1/\sigma))(1 - \Phi(t_2/\sigma))$$
(their equation A3). With a little algebra, we note that this is equivalent to the probability-multiplication model without a nonlinear transducer:

\[
\Phi_{MC}(t_1, t_2) = 2\Phi(t_1/\sigma)\Phi(t_2/\sigma) - \Phi(t_1/\sigma) - \Phi(t_2/\sigma) + 1
\]

\[
= (1 + (2\Phi(t_1/\sigma) - 1)(2\Phi(t_2/\sigma) - 1))/2
\]

\[
= (1 + p_1 p_2)/2
\]

provided the exponent \( p = 1 \).

5.A2.4 The Convoy Model (not considered here and a poor fit to all the data)

Accuracy is given by:

\[
\Phi_{CM}(t_1) = \Phi(t_1/\sigma)
\]

where \( t_1 \) is the strength of the weaker of the two signals.

Data were fit to the various models described in the text using the MATLAB version of Nelder-Mead simplex (direct search) method. Where a figure contains more than one psychometric function, the data from all the functions were fitted simultaneously.

References


