DAE 2. Example 1: shaft selection

$P = 100\,kW$

$N = 4$

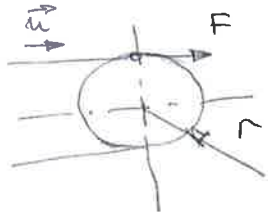
$n = 6000$ rpm

$u = 50$ m/s

ANSI 1080

$a = 0{,}16\,m$

$b = 0{,}04\,m$



$d = ?$

$P = F \cdot u = T \cdot \omega \Rightarrow F = \frac{P}{u} = \frac{100\,000}{50} = 2000\,N$ $\quad F = 2000\,[N]$

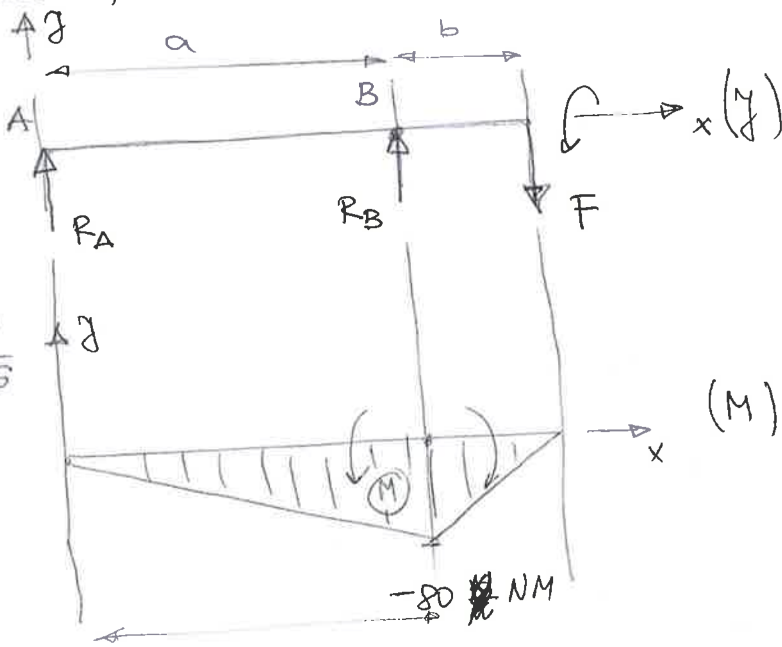
$\left(\omega = \frac{2n\pi}{60} = \frac{n\pi}{30} = \frac{6000 \cdot 3{,}14}{30} = 628\,\frac{rad}{s} \right)$

$\left(u = r \cdot \omega \Rightarrow r = \frac{u}{\omega} = \frac{50}{628} = 0{,}0796\,m \right)$

$r = 79{,}58\,[mm]$ – radius

$T = F \cdot r = 2000 \cdot 0{,}07958 \Rightarrow T = 159{,}15\,[Nm]$

Figure (b) – Free body diagram of the shaft



$\Sigma y = 0: \; R_A + R_B - F = 0 \qquad (1)$

$\Sigma M_B = 0: -R_A \cdot a - F \cdot b = 0 \qquad (2)$

$(2) \rightarrow \quad R_A = -F \cdot \frac{b}{a} = -2000 \cdot \frac{0{,}04}{0{,}16}$

$R_A = -500\,[N]$

$(1) \rightarrow \quad R_B = F - R_A = 2000 + 500$

$R_B = 2500\,[N]$

$M_B = -F \cdot b = -2000 \cdot 0{,}04$

$M_B = -80\,[Nm]$ – maximum bending moment

CRITICAL SECTION
point B
at $x = a = 160$ mm

Torsional stress on the shaft:

$$\tau_{\cancel{x}y} = \frac{c \cdot T}{J} = \frac{\frac{d}{2} \cdot \cancel{4}159{,}15}{\frac{\pi \cdot d^4}{32}} \quad c = \frac{d}{2} \quad J = \frac{\pi d^4}{32} \text{ - second polar moment of area}$$

$$\boxed{\tau_{xy} = \frac{2546{,}4}{\pi \cdot d^3}}$$

$$I = \frac{\pi d^4}{64} \text{ - second moment of area}$$

Bending stress

$$\sigma_x = \frac{c \cdot M}{I} = \frac{\frac{d}{2} \cdot 80}{\pi \cdot \frac{d^4}{64}} = \boxed{\frac{2560}{\pi \cdot d^3}}$$

$$d \geq \sqrt[3]{\frac{32 \cdot N}{\pi \cdot S_y} \cdot \sqrt{M^2 + \frac{3}{4} T^2}}$$

for AISI 1080 $S_y = 380$ MPa

$$d \geq \sqrt[3]{\underbrace{\frac{32 \cdot 4}{\pi \cdot 380 \cdot 10^6}}_{1{,}0722 \cdot 10^{-7}} \cdot \underbrace{\sqrt{80^2 + \frac{3}{4} \cdot 159{,}15^2}}_{159{,}36}} = \sqrt[3]{1{,}70866 \cdot 10^{-5}}$$

$$\boxed{d \geq 0{,}02576 \text{ m} \Rightarrow d = 26 \text{ mm}}$$

$$\tau_{xy} = 47{,}43 \text{ MPa} = 47{,}43 \cdot 10^6 \text{ N/m}^2$$

$$\sigma_x = 47{,}67 \text{ MPa} = 47{,}67 \cdot 10^6 \text{ N/m}^2$$

Example 2: — Shaf dynamic loading

$15 \leq d_2 \leq 50 \text{ mm} \qquad \Delta d_2 = 5 \text{ mm}$

$d_2 = 0{,}75\, d_3$

$d_1 = 0{,}65\, d_3$

$M = \pm 70 \text{ Nm} \qquad f_s = N = 2{,}5$

$T = 45 \text{ Nm}$

$d_2 = ?$

Iterative calculation of ~~To calculate~~ min dia...
Assume d; calculate S_e & K_f, calculate d min

From table (properties of steels)

$S_u = 615 \text{ MPa} \qquad S_y = 380 \text{ Nm}$

S-N diagram or Wöhler diagram gives the STRESS ENURANCE LIMITS of steel for different loading (approximated):

$S_e' = 0{,}5\, S_u$ for bending

$S_e' = 0{,}45\, S_u$ for axial loading

$S_e' = 0{,}29\, S_u$ for torsional loading

$S_e' = 0{,}5 \cdot S_u = 0{,}5 \cdot 615 \qquad \boxed{S_e' = 307{,}5 \text{ MPa}}$ Stress endurance limit

$\cancel{k_s = 1{,}189 \cdot d^{-0{,}112} =}$ Size factor

$S_e = k_f \cdot k_s \cdot S_e'$ — modified endurance limit

assume d_2 ⇒ calculate d_1 ⇒ calculate S_e, K_f ⇒ calculate d

$$d = \sqrt[3]{\frac{32 f_s}{\pi S_y} \cdot \sqrt{\left(\underbrace{\cancel{M_m}}_{0} + \frac{S_y}{S_e} K_f \cdot M_a\right)^2 + \left(T_m + \frac{S_y}{S_e} K_{fs} \cdot \underbrace{\cancel{T_a}}_{0}\right)^2}}$$

1) Assume $d_2 = 25$ mm

$$d_3 = \frac{d_2}{0{,}75} = \frac{d_1}{0{,}65} \Rightarrow \frac{d_2}{d_1} = \frac{0{,}75}{0{,}65} = 1{,}154 \Rightarrow \boxed{d_1 = \frac{d_2}{1{,}154}}$$

From figure

$$\frac{r}{d_1} = \frac{(d_2-d_1)/2}{d_1} = \frac{1}{2}\left(\frac{d_2}{d_1}-1\right) = 0{,}0769 \Rightarrow r = 0{,}0769\, d_1$$

$$r = 0{,}0769 \cdot \frac{d_2}{1{,}154} \Rightarrow r = 0{,}06672\, d_2$$

$$\boxed{d_1 = 21{,}63}$$

$$\boxed{r = 1{,}668 \text{ mm}} \Rightarrow \frac{r}{d} = 0{,}0769 \Rightarrow \boxed{K_c = 1{,}9}$$

$$K_f = 1 + (K_c - 1)\cdot q_n$$ ← Fatigue stress concentration factor

$$K_f = 1 + (1{,}9-1)\cdot 0{,}75 = \boxed{1{,}675}$$

$$q_n = 0{,}75$$

$$S_e = k_f \cdot k_s \cdot S_e'$$ ← Modified endurance limits

From diagram $k_f = 0{,}75$ ← surface finish factor

$k_s = 1{,}189 \cdot d_1^{-0{,}112}$ ← size factor, d. in [mm]

$$k_s = 1{,}189 \cdot 21{,}63^{-0{,}112} = 0{,}8425$$

$$S_e = 0{,}75 \cdot \cancel{1{,}189}\; 0{,}8425 \cdot 307{,}5 = 194{,}3 \text{ MPa}$$

midrange
$$M_m = \frac{M_{max} + M_{Min}}{2} = \frac{70-70}{2} = 0\ [Nm] \qquad T_m = \frac{T_{max} + T_{min}}{2} = \frac{45+45}{2} = 45\ [Nm]$$

alternating
$$M_a = \frac{M_{max} - M_{min}}{2} = \frac{70+70}{2} = 70\ [Nm] \qquad T_a = \frac{T_{max} - T_{min}}{2} = \frac{45-45}{2} = 0\ [Nm]$$

$$d_1 = \sqrt{\underbrace{\frac{32\cdot 2{,}5}{\pi \cdot 380\cdot 10^6}}_{6{,}701\cdot 10^{-8}} \cdot \sqrt{\underbrace{\left(\frac{380}{194{,}3}\right)^2 \cdot 1{,}675^2 \cdot 70^2}_{52583} + \underbrace{45^2}_{2025}}} = \boxed{0{,}02509 \text{ mm}}$$

Example 2 – continue

Calculated $d_1 = 0{,}02509 \not= 25{,}1$ mm

$$d_2 = 1{,}154 \cdot d_1 = 1{,}154 \cdot 25 = 28{,}96 \text{ mm}.$$

Initial guess of 25 mm is not correct.
The next size is 30 mm and we need to recalculate with this:

Assume $d_2 = 30$ mm $\Rightarrow$ $d_1 = 25{,}99 = 26$ mm

$$d_3 = \frac{d_2}{0{,}75} = \frac{30}{0{,}75} = 40 \text{ mm}$$

$$\left.\begin{array}{l} r = \dfrac{d_2 - d_1}{2} = \dfrac{30-26}{2} = 2 \text{ mm} \\ \dfrac{r}{d} = \dfrac{2}{26} = 0{,}076923 \approx 0{,}077 \end{array}\right\} \Rightarrow \begin{array}{l} q_n = 0{,}78 \\ K_c = 1{,}9 \end{array}$$

$$k_s = 1{,}189 \cdot d_1^{-0{,}112} = 1{,}189 \cdot 25{,}1^{-0{,}112} \Rightarrow k_s = 0{,}82874$$ ← Size factor

$$K_f = 1 + (k_c - 1) \cdot q_n = 1 + (1{,}9 - 1) \cdot 0{,}78 \quad K_f = 1{,}702$$ ← Fatigue stress

$$S_e = 0{,}75 \cdot 0{,}82876 \cdot 307{,}5 \cdot 10^6 \quad S_e = 191.13 \text{ MPa}$$ ← Endurance limit

$$d_1 = \sqrt[3]{\underbrace{\frac{32 \cdot 2{,}5}{\pi \cdot 380 \cdot 10^6}}_{6{,}701 \cdot 10^{-8}} \cdot \underbrace{\sqrt{\underbrace{\left(\frac{380}{191{,}13}\right)^2 \cdot 1{,}702^2 \cdot 70^2}_{56107{,}98} + \underbrace{45^2}_{2025}}}_{241{,}1}} =$$

$$\boxed{d_1 = 0.0253 \text{ mm}}$$

Since assumed $d_{1_a} = 26 \text{ mm} > d_{1_c} = 25{,}3$ mm → satisfies

$$d_2 = 30 \text{ mm}$$

$$d_3 = 40 \text{ mm}$$

$$\sigma'_{max} = \sqrt{\left(\frac{32\,K_f\,(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16\,K_{fs}\cdot(T_m + T_a)}{\pi d^3}\right)^2} =$$

$$= \sqrt{\left(\frac{32 \cdot 1{,}702 \cdot (70+0)}{\pi \cdot 0{,}025^3}\right)^2 + 3\left(\frac{16 \cdot 1{,}702 \cdot 45}{\pi \cdot 0{,}025^3}\right)^2} = 88{,}89$$

$\cancel{n_y = 380}$

$$\sigma'_{max} = \frac{S_y}{\sigma'_{max}} = \frac{380}{88{,}89} = \boxed{4{,}79}$$

Example 3 – Critical shaft speed

low-carbon solid shaft

$x_1 = 762$ mm $\qquad P_A = 356$ N

$x_2 = 1016$ mm $\qquad P_B = 534$ N

$x_3 = 508$ mm

$d = 50$ mm $= 0{,}05$ m

$l = x_1 + x_2 + x_3 = 2286$ mm $= 2{,}286$ m

a) $\omega_{1n} = ?$

b) $\omega_{1R} = ?$

c) $\omega_{1D} = ?$

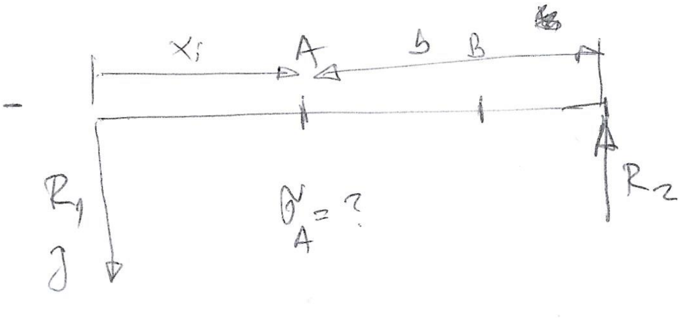
For low-carbon steel Young modulus:

$E = 207$ GPa

Density: $\rho = 7860$ kg/m^3

Second moment of area (area moment of inertia):

$$I = \frac{\pi d^4}{64} = \frac{\pi \cdot (0{,}05)^4}{64} = 3{,}068 \cdot 10^{-7}\ m^4$$



$$\delta_{ij} = \frac{b_j \cdot x_i}{6EI \cdot l}\left(l^2 - b_j^2 - x_i^2\right)$$
for $x_i \le a$

$$\delta_{ij} = \frac{a_j(l-x_i)}{6EIl}\cdot\left(2lx_i - a_j^2 - x_i^2\right)$$

$6EIl = 6 \cdot 207 \cdot 10^9 \cdot 3{,}068 \cdot 10^{-7} \cdot 2{,}286$

$6EIl = 871{,}07 \cdot 10^3$

– Deflection at point A due to load P_A:

$l = 2{,}286$; $b_j = x_2 + x_3 = 1{,}524$ m ; $x_i = x_1 = 0{,}762$ m

$x_i \le a$

$$y_{AA} = P_A \cdot \delta_{AA} = \frac{0{,}762 \cdot 1{,}524 \cdot 356}{871 \cdot 10^3}\left(2{,}286^2 - 1{,}524^2 - 0{,}762^2\right)$$

$$\boxed{y_{AA} = 0{,}0011023\ m \approx 1{,}1\ mm}$$ in the positive direction of y axis

– Deflection at point A due to load P_B

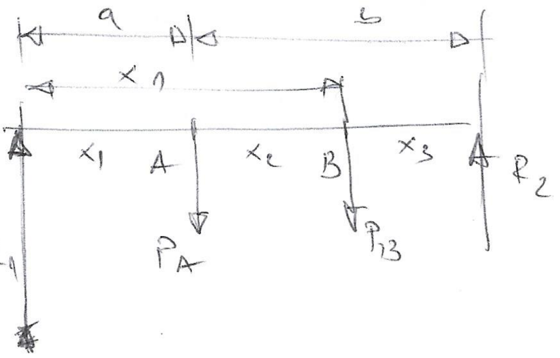
$x = x_1 = x_i = 0{,}762$; $a = x_1 + x_2 = 1{,}524$; $b = x_3 = 0{,}507$

$x_i \le a$

$$y_{AB} = P_B \cdot \delta_{AB} = \frac{534 \cdot 0{,}507 \cdot 0{,}762}{871 \cdot 10^3}\cdot\left(2{,}286^2 - 0{,}762^2 - 0{,}508^2\right)$$

$y_{AB} = 1{,}045$ mm

– Deflection at loc. B due to load A



$x_i > a$

$x_i = x_1 + x_2 = 1{,}778\ m$

$a = x_1 = 0{,}762$

$b = x_2 + x_3 = 1{,}016 + 0{,}508 = 1{,}524$

$$y_{BA} = P_B \cdot \delta_{BA} = \frac{P_B \cdot a (l - x_i)}{6EIl} \cdot \left(2lx_i - a_i^2 - x_i^2\right) =$$

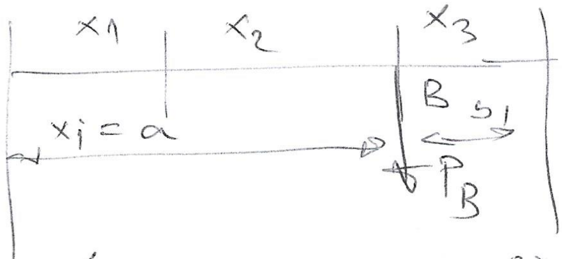
$$= \frac{534 \cdot 0{,}762 \cdot (2{,}286 - 1{,}778)}{871 \cdot 10^3} \cdot \left(2 \cdot 2{,}286 \cdot 1{,}778 - 0{,}762^2 - 1{,}778^2\right)$$

$$\boxed{y_{BA} = 1{,}041\ mm}$$

– Deflection at location B due to the load P_B

$a = x_i = x_1 + x_2 = 1{,}778\ m$

$b_i = x_3 = 0{,}508$



$$y_{BB} = P_B \cdot \delta_{BB} = \frac{534 \cdot 0{,}508 \cdot 1{,}778}{871{,}07 \cdot 10^3} \cdot \left(2{,}286^2 - 0{,}508^2 - 1{,}778^2\right) =$$

$$\boxed{y_{BB} = 1\ mm}$$

Total deflection at point A ⇒ $y_A = y_{AA} + y_{AB} = 1{,}1 + 1{,}045$

$$\boxed{y_A = 2{,}245\ m}$$

–"– deflection at point B ⇒ $y_B = y_{BA} + y_{BB} = 1{,}041 + 1$

$$\boxed{y_B = 2{,}041\ mm}$$

Example 3

a) critical speed without load:

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \cdot \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \cdot \sqrt{\frac{gEI}{A\gamma}} = \left(\frac{\pi}{l}\right)^2 \cdot \sqrt{\frac{4\cdot \cancel{g} E I}{D^2 \cdot \pi \cdot \rho \cdot \cancel{g}}}$$

$$\omega_1 = \left(\frac{\pi}{2,286}\right)^2 \cdot \sqrt{\frac{4\cdot 207\cdot 10^9 \cdot 3,068\cdot 10^{-7}}{0,05^2 \cdot \pi \cdot 7860}} = 121,003 \ \text{rad/s}$$

$$\boxed{n_1 = \frac{30\cdot\omega_1}{\pi} = \frac{30\cdot 121}{\pi} = 1155,5 \ \text{rpm}}$$

b) Critical speed with load using Reyliegh eq:

$$\omega_{1R} = \sqrt{\frac{g\left(P_A\cdot y_A + P_B\, y_B\right)}{P_A\, y_A^2 + P_B\cdot y_B^2}} = \sqrt{\frac{9,81\cdot\left(356\cdot 2,275 + 534\cdot 2,071\right)\cdot 10^{-3}}{356\cdot 0,002275^2 + 534\cdot 0,002071^2}}$$

$$\omega_{1R} = 67.91 \ \frac{\text{rad}}{\text{s}} \Rightarrow \boxed{n_{1_R} = 648.5 \ \text{rpm}}$$

c) Critical speed using Dunkerley eq:

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_{crA}^2} + \frac{1}{\omega_{crB}^2}$$

$$\omega_{cr_A} = \sqrt{\frac{g}{y_{AA}}} = \sqrt{\frac{9.81}{1,1\cdot 10^{-3}}} = 94,4 \ \frac{\text{rad}}{\text{s}} \Rightarrow n_{crA} = 901 \ \text{rpm}$$

$$\omega_{crB} = \sqrt{\frac{g}{y_{BB}}} = \sqrt{\frac{9.81}{1\cdot 10^{-3}}} = 99.05 \ \frac{\text{rad}}{\text{s}} \Rightarrow n_{crB} = 945 \ \text{rpm}$$

$$\omega_1 = \sqrt{\frac{\omega_{crA}^2\cdot\omega_{crB}^2}{\omega_{crA}^2 + \omega_{crB}^2}} = \sqrt{\frac{94.4^2\cdot 99.05^2}{94,4^2 + 99,05^2}}$$

$$\omega_1 = 68,33 \ \frac{\text{rad}}{\text{s}}$$

$$\boxed{n_{1_D} = 68,23}$$

$$\left.\begin{array}{l}\text{Critical speed Rayleigh} \quad n_{1R} = 648 \text{ rpm} \\ \text{Critical speed Dunkerly} \quad n_{1D} = 651.48 \text{ rpm}\end{array}\right\} \underline{\underline{\text{Range}}}$$

Example 4 – key selection

$d = 100\ mm = 0{,}1\ m$

$w = 25\ mm = 0{,}025\ m$

$h = 25\ mm = 0{,}025\ m$

high-carbon

$f_s = 2$

low-carbon

shear

compression

$\tau_{all} = 0{,}4 \cdot S_y$

$\sigma_{al} = 0{,}9 \cdot S_y$

$l = ?$

From table: $S_{y_S} = 380$ MPa for shaft ; $S_{y_k} = 295$ MPa (key)

~~shaft~~ Design stress:

$$\tau_{design} = \frac{\tau_{all}}{f_s} = \frac{0{,}4 \cdot S_{y_S}}{2} = \frac{0{,}4 \cdot 380}{2} = 76\ MPa$$

$$\sigma_{design} = \frac{\sigma_{al}}{f_s} = \frac{0{,}9 \cdot S_{y_S}}{2} = \frac{0{,}9 \cdot 380}{2} = 171\ MPa$$

Second polar moment of area:

$$J = \frac{\pi d^4}{32} = \frac{\pi \cdot 0{,}1^4}{32} = 9{,}81 \cdot 10^{-6}\ m^4$$

Max. torque

$$\tau_{\cancel{z}} = \frac{T \cdot c}{J} = \frac{T \cdot r}{J} \Rightarrow T_{max} = \frac{\tau_{des} \cdot J}{r}$$

$$T_{max} = \frac{76 \cdot \cancel{10^6} \cdot 9{,}81 \cdot \cancel{10^{-6}}}{0{,}05} = 14\,911\ Nm$$

Max Force = $\boxed{F_{max}} = \frac{T}{r} = \frac{14911}{0{,}05} = \boxed{298\,224\ N = 298{,}224\ kN}$

(12)

Key:

$$\tau_{des_k} = \frac{0{,}4 \cdot S_{yk}}{f_s} = \frac{0{,}4 \cdot 295}{2} \Rightarrow \boxed{\tau_{des} = 59\ MPa}$$

$$\sigma_{des_k} = \frac{0{,}9\, S_{yk}}{f_s} = \frac{0{,}9 \cdot 295}{2} \Rightarrow \boxed{\sigma_{des} = 132{,}75\ MPa}$$

From $\tau = \frac{F}{A} = \frac{2T}{DWL}$ $\Rightarrow$ $L = \frac{2T}{DW\tau}$

From $\sigma = \frac{F}{A} = \frac{4T}{DhL}$ $\Rightarrow$ $L = \frac{4T}{Dh\sigma}$

~~Stress~~ Consider required length to avoid shear failure

$$L_{cr\tau} = \frac{2\, T_{max\,s}}{D \cdot W \cdot \tau_{des_k}} = \frac{2 \cdot 14911}{0{,}1 \cdot 0{,}025 \cdot 59 \cdot 10^6}$$

$$\boxed{L_{cr\tau} = 0{,}2022\ m = 202{,}2\ mm}$$

$$L_{cr\sigma} = \frac{4\, T_{max\,s}}{D \cdot h \cdot \sigma_{des_k}} = \frac{4 \cdot 14911}{0{,}1 \cdot 0{,}025 \cdot 132{,}75 \cdot 10^6}$$

$$\boxed{L_{cr\sigma} = 0{,}17972\ m = 179{,}7\ mm}$$

Example 5 - Bolt-joint stiffness

(13)

$l = 55\ mm$ $\quad \alpha = 30^\circ$

$l_{m_1} = 30\ mm$ — Al $\quad$ M14 $\begin{pmatrix} d_t = 12\ mm \\ d_d = 14\ mm \end{pmatrix}$

$l_t = 20\ mm$ $\quad l_{m_2} = 25\ mm$ — Fe

$k_j = ?\ ;\ k_b = ?$ $\qquad$ From table $E = 207\ GPa$

$$k_b = \frac{A_d \cdot A_t \cdot E}{A_d \cdot l_t + A_t \cdot l_d}$$ — stiffness of the bolt

$l_t = 0{,}02\ m\ ;\ l_d = 0.035\ m$

$$A_d = \frac{d_d^2 \pi}{4} = \frac{0{,}014^2 \cdot \pi}{4} = 1{,}54 \cdot 10^{-4}\ m^2$$

$$A_t = \frac{d_t^2 \pi}{4} = \frac{0{,}012^2 \pi}{4} = 1.131 \cdot 10^{-4}\ m^2$$

$$\boxed{k_b = \frac{1{,}131 \cdot 10^{-4} \cdot 1{,}54 \cdot 10^{-4} \cdot 207 \cdot 10^9}{1{,}54 \cdot 10^{-4} \cdot 0{,}02 + 1{,}131 \cdot 10^{-4} \cdot 0{,}035}} = \frac{3605{,}4}{7{,}66 \cdot 10^{-6}} = \boxed{4{,}703 \cdot 10^8\ N/m}$$

Stiffness of the bolt

Stiffness of the members: $\rightarrow$

$$\boxed{k = \frac{4 \pi \cdot \tan 30 \cdot E \cdot d}{\ln \dfrac{(1.155 \cdot t + D - d)(D + d)}{(1.155 \cdot t + D + d)(D - d)}}}$$

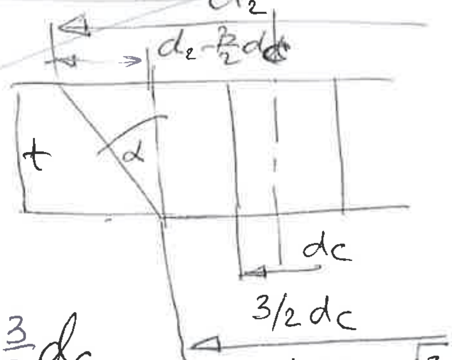
$d_c = 15\ mm$

$D = \frac{3}{2} d_c = 22{,}50\ mm$

Member 1: $E = 207\ GPa$ - steel

$$k_1 = \frac{0{,}5777 \cdot 4 \cdot \pi \cdot 207 \cdot 10^9 \cdot 0{,}015}{\ln \dfrac{(1{,}155 \cdot 0{,}025 + 0{,}0225 - 0{,}015)(0{,}0375)}{(1{,}155 \cdot 0{,}025 + 0{,}0225 + 0{,}015)(0{,}0225 - 0{,}015)}}$$

$$k_1 = \frac{2{,}2541 \cdot 10^{10}}{\ln \dfrac{1{,}110937 \cdot 10^{-2}}{4{,}97812 \cdot 10^{-4}}} =$$



$$tg\alpha = \frac{\dfrac{d_2 - \frac{3}{2} d_c}{2}}{t}$$

$$d_2 = 2t \cdot tg\alpha + \frac{3}{2} d_c$$

$$d_2 = 0{,}02 \cdot 2 \cdot tg\frac{\pi}{6} + \frac{3}{2} \cdot 0{,}015 \qquad tg\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$d_2 = \frac{0{,}04 \cdot \sqrt{3}}{3} + \frac{0{,}045}{2} = 0{,}0302$$

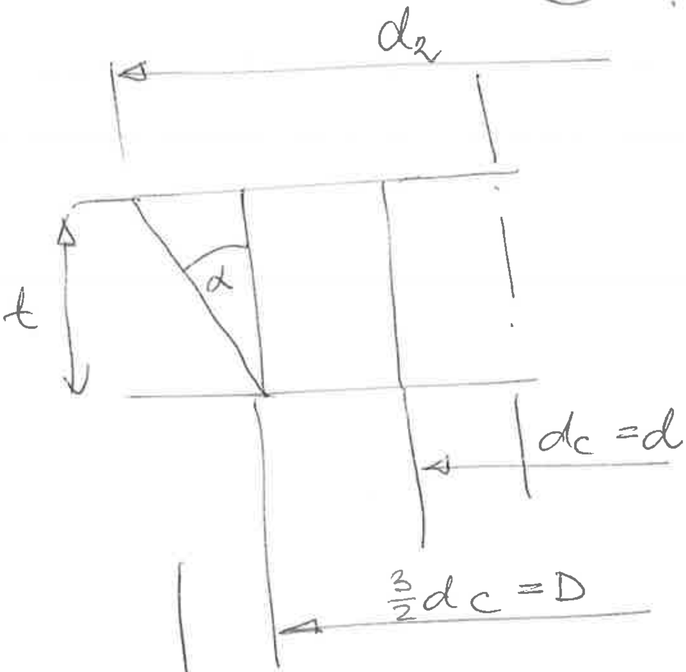
Member 1

$E = 207 \cdot 10^9 \, Pa$

$d = d_c = 0{,}014 \, m$

$D = \frac{3}{2} d_c = 0{,}021 \, m$

$t = 25$



$$k_1 = \frac{4 \cdot \pi \cdot \tan 30 \cdot 207 \cdot 10^9 \cdot 0{,}014}{\ln \frac{(1{,}155t + D - d)(D + d)}{(1{,}155t + D + d)(D - d)}} =$$

$$= \frac{4 \cdot \pi \cdot 0{,}577 \cdot 207 \cdot 10^9 \cdot 0{,}014}{\ln \frac{(1{,}155 \cdot 0{,}025 + 0{,}021 - 0{,}014) \cdot (0{,}021 + 0{,}014)}{(1{,}155 \cdot 0{,}025 + 0{,}021 + 0{,}014)(0{,}021 - 0{,}014)}}$$

$$\frac{\frac{d_2 - D}{2}}{t} \cancel{= \tan\alpha} = \tan\alpha$$

$$\frac{d_2 - D}{2t} = \tan\alpha$$

$$d_2 = 2 \cdot t \cdot \tan\alpha + D$$

$$d_2 = 2 \cdot 0{,}025 \cdot \frac{\sqrt{3}}{3} + \frac{3}{2} \cdot 0{,}14$$

$$\boxed{d_2 = 0{,}04986 \, m}$$

$\cancel{d_c = d}$

$k_1 = 50{.}86 \cdot 10^8 \, N/m$

Member 2: (Al)

$E = 71 \cdot 10^9 \, Pa$; $D = d_2 = 0{,}04986 \, m$

$d = d_c = 0{,}014 \, m$

$t = 0{,}0275 - 0{,}025 = 0{,}0025$

$$k_2 = \frac{4 \cdot \pi \cdot 0{,}577 \cdot 71 \cdot 10^9 \cdot 0{,}014}{\ln \frac{(1{,}155 \cdot 0{,}0025 + 0{,}04986 - 0{,}014)(0{,}04987 + 0{,}014)}{(1{,}155 \cdot 0{,}0025 + 0{,}04986 + 0{,}014)(0{,}04987 - 0{,}014)}}$$

$k_2 = 543{.}1 \cdot 10^8 \, N/m$

Example 5 – bolt-joint stiffness.

Member 3: AC: $E = 71 \cdot 10^9\,Pa$; $D = \frac{3}{2} d_c = 0{,}021$

$d = d_c = 0{,}014$

$t = 0{,}0275$

$$k_3 = \frac{4 \cdot \pi \cdot 0{,}577 \cdot 71 \cdot 10^9 \cdot 0{,}014}{\ln \dfrac{(1{,}155 \cdot 0{,}0275 + 0{,}021 - 0{,}014)(0{,}021 + 0{,}014)}{(1{,}155 \cdot 0{,}0275 + 0{,}021 + 0{,}014)(0{,}021 - 0{,}014)}}$$

$$k_3 = 16{,}92 \cdot 10^8\ N/m$$

From $\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$

$$\frac{1}{k_m} = \frac{1}{10^8} \cdot \left(\frac{1}{50.36} + \frac{1}{543.1} + \frac{1}{16.92} \right)$$

$$\boxed{k_m = 12{,}38 \cdot 10^8\ N/m}$$

The dimensionless stiffness of the joint:

$$C = \frac{k_b}{k_b + k_m} = \frac{4{,}703 \cdot 10^8}{4{,}703 \cdot \cancel{10^8} + 12{,}38 \cdot \cancel{10^8}} =$$

$$= \frac{4{,}703}{17{,}083}$$

$$\boxed{C = 0{,}2753}$$

Example 6: – Bolt preloading

From table $S_p = 380\ MPa$ (minimum proof strength)

From table $A_t = 115\ mm^2 = 115 \cdot 10^{-6}\ m^2$ $\quad f_S = n_L = 2{,}5$

$A_d = 104\ mm^2 = 104 \cdot 10^{-6}\ m^2$

$P_{max} = ?$

Proof load

$$F_i = 0{,}75 \cdot F_p = 0{,}75 \cdot S_p \cdot A_t$$ ← Preload force for non-permanent connection

$$F_i = 0{,}75 \cdot 380 \cdot 10^6 \cdot 115 \cdot 10^{-6}$$

$$F_i = 32\,775\ N$$

From ratio of proof load and operating load (factor of safety)

$$f_S = n_L = \frac{S_p \cdot A_t - F_i}{C\,P} \Rightarrow$$

$$P_{b_{max}} = \frac{S_p \cdot A_t - F_i}{n_L \cdot C} = \frac{380 \cdot 115 - 32775}{2{,}5 \cdot 0{,}2753}$$

Maximum external load that bolt can carry is

$$\boxed{P_{b_{max}} = 15.873\ N = 15{,}873\ kN}$$

From factor of safety against joint separation:

$$n_0 = \frac{F_i}{P_{S_{max}}(1-C)} \Rightarrow P_{S_{max}} = \frac{F_i}{n_0(1-C)} = \frac{32775}{2{,}5 \cdot (1-0{,}2753)}$$

Maximum external load before separation:

$$\boxed{P_{S_{max}} = 18090\ N}$$