Mechanical Analysis and Design
ME 2104

Lecture 1

Mechanical Analysis
Shafts & keyways

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Plan for the analysis of mechanical elements

Objective:

Procedures for design and selection of mechanical elements

- Week 1 – Shafts and keyways
- Week 2 – Bearings and screws
- Week 3 – Belt and chain drives
- Week 4 – Gears and gear trains
- Week 5 – Design Project Review

Plan for this week

- Principles of mechanical design
- Shafts & shaft elements (with examples)

Literature:

Mechanical Engineering Design, 7th edition, Shigley, Mischke, Budynas, McGraw Hill, 2004, ISBN 007-252036-1

Fundamentals of Machine Elements, Hamrock, Jacobson, Smchnmidt, McGraw Hill, 1999, ISBN 0-256-19069-0

Mechanical Design Process

<u>Use of scientific principles and technical information</u> along with innovations, ingenuity or imagination in the <u>definition of a machine, mechanical device or system</u> to perform pre specified functions with maximum economy and efficiency.

(Engineering Design Council, UK)

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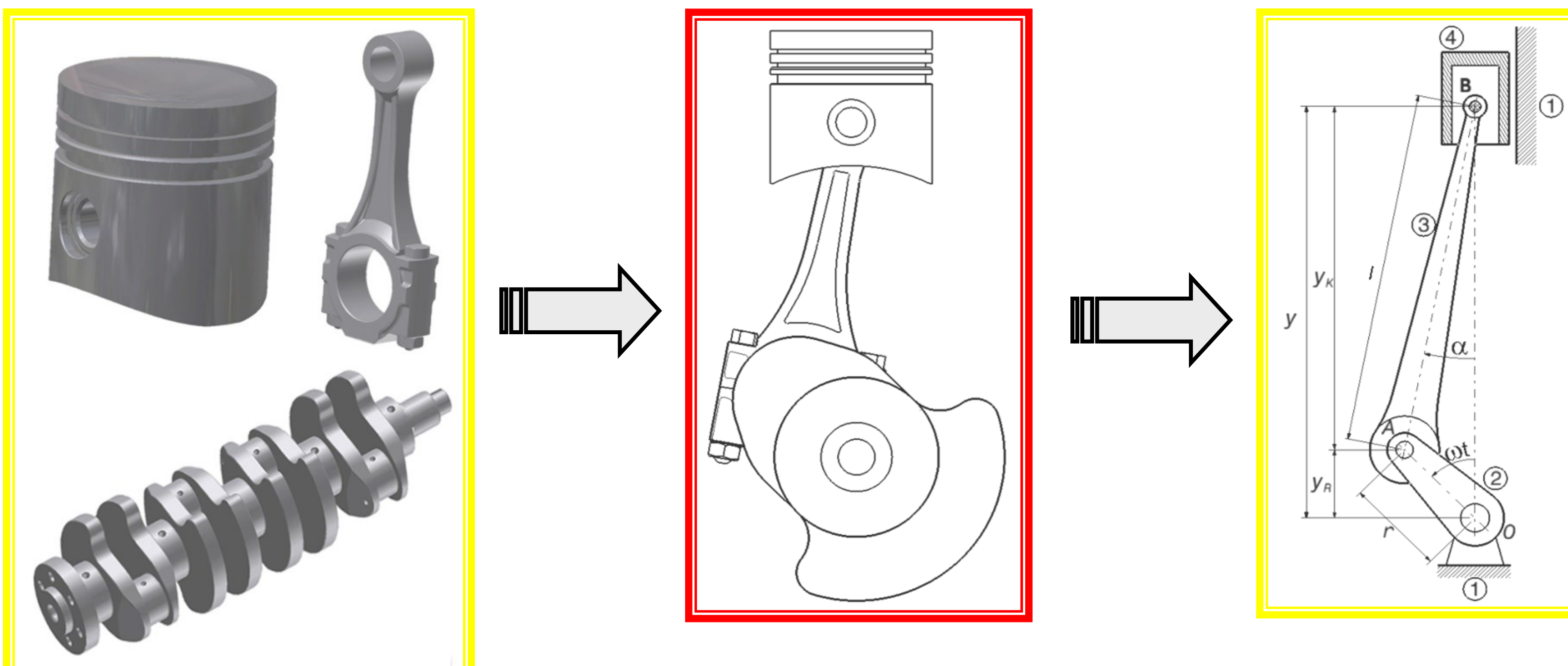
Mechanical Design Procedure

A component is usually designed in the following sequence:

1. At first *a design scheme* (lay out) is drawn in which the shape of the part being designed and the nature of its connection with other elements are presented in a simplified form while the forces acting on the part are assumed to be either concentrated or distributed in conformity with some simple law;



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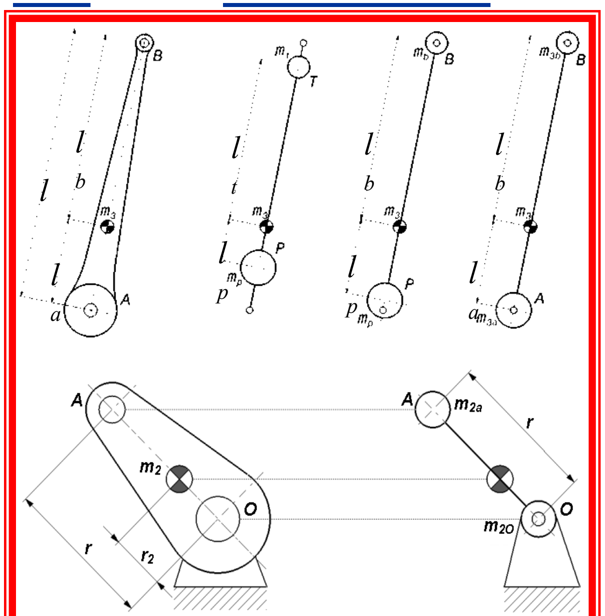

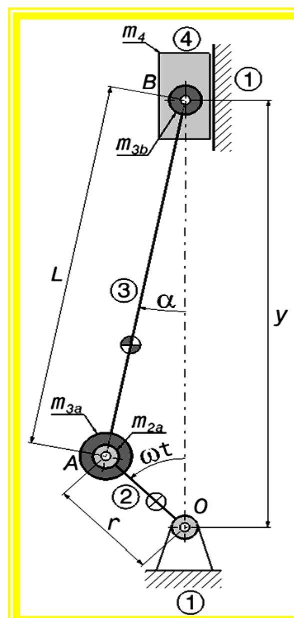

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2. *The forces* acting on the part in the process of machine operation *are* then *determined*;
3. *Material is selected* and the allowable *stresses are found* accounting for all the factors that affect the strength of the part;
4. *The dimensions of the part*, required by the design criteria (strength, rigidity, wear resistance etc.) corresponding to the accepted design scheme, are determined;
5. Finally *the drawing of the part* is made indicating all dimensions, accuracy of manufacture, surface finish and other information necessary for the manufacture of the part.

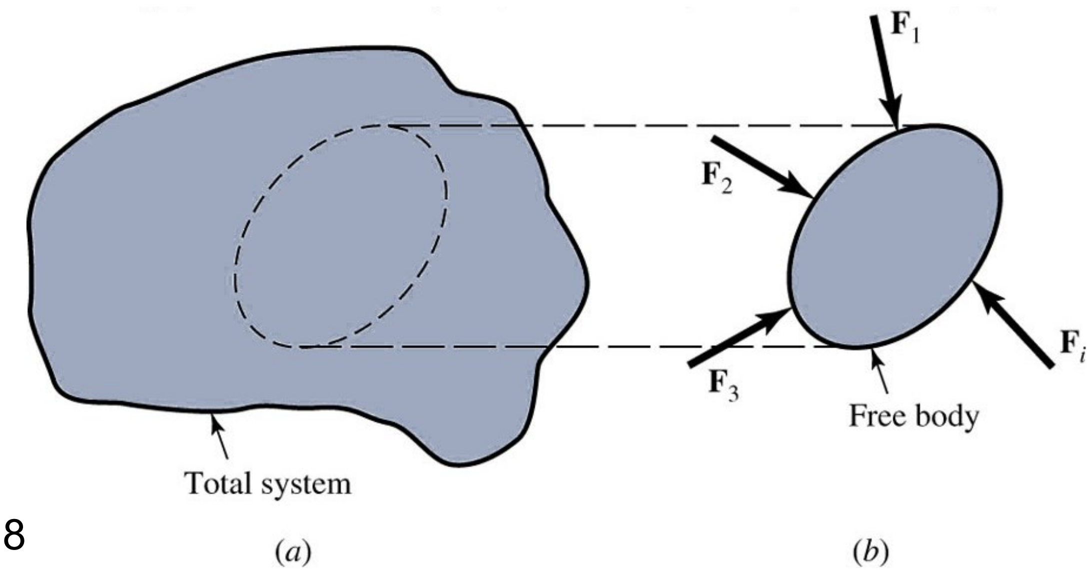
System, Equilibrium and Free-Body Diagrams

System is any isolated part or portion of a machine or structure to be studied. If the system is motionless or has constant velocity it is said the system is in *equilibrium*.

For such a system *all forces and moments* acting on the system *balance*:

$$\Sigma \mathbf{F}=0 \qquad \Sigma \mathbf{M}=0$$

The isolated system together with all forces and moments due to any external effects and the reactions with the main system is called *free-body diagram*.



An analysis of any structure can be greatly simplified by successively isolating each element and studying and analysing it by the use of free-body diagram.

Stress and Strength

The quality of a mechanical system depends on the relationship of the maximum stress to the component strength

1. **Stress** is a *state property of a body* which is a function of :
   load, geometry, temperature and manufacturing process.

   Stress is denoted with Greek letters and has unit [N/m^2]

   σ = F/A – normal stress (tensile or compressive)

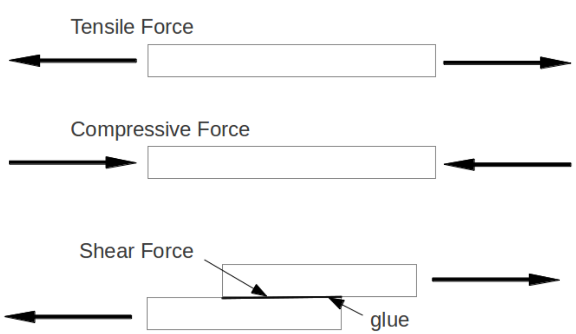
   τ = F_p/A – shear stress

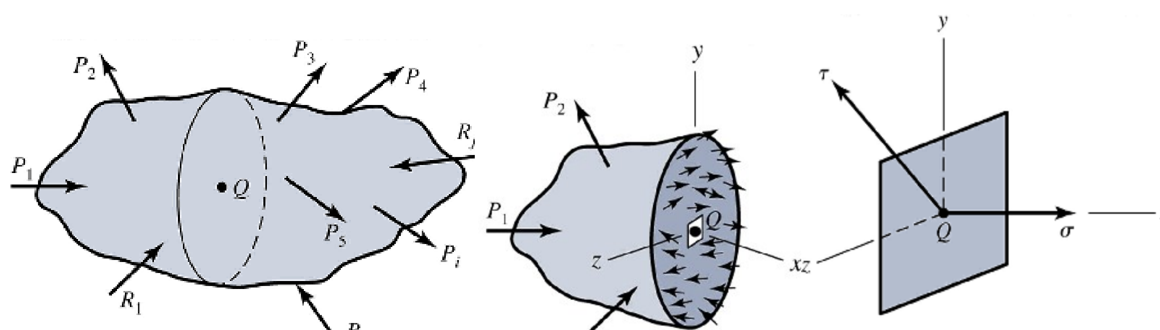
   Various marks denote various kinds of stress:

   σ_1 – principal stress – stress in the direction of a principal axis

   σ_y – coordinate stress component

   σ_r – radial stress component





(a) Structural member (b) Isolated section (c) Normal and shear stresses on an infinitesimal surface at Q

Stress, Strain and Strength

2. **Strain** is defined as *deformation of a solid due to stress* in terms of displacement of material

Strain is denoted with Greek letters:

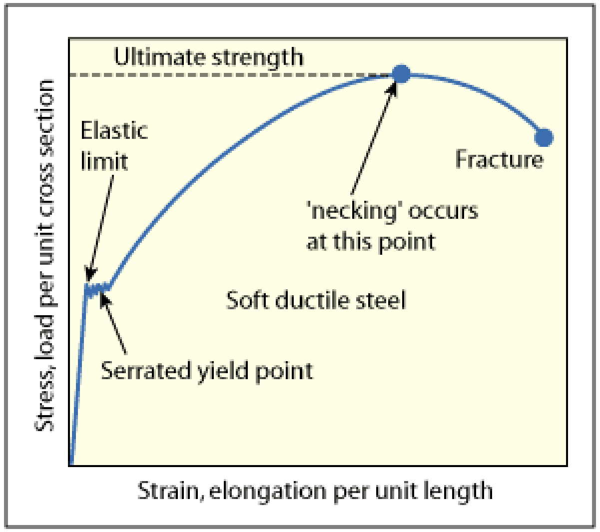
$\varepsilon = dl / l_o$ [m/m]

$\gamma = ds / s_o$ [m/m]

Elastic Moduli are ratio of stress / strain:

$E = \sigma / \varepsilon$ [N/m²] - *Young's modulus*
Modulus of Elasticity

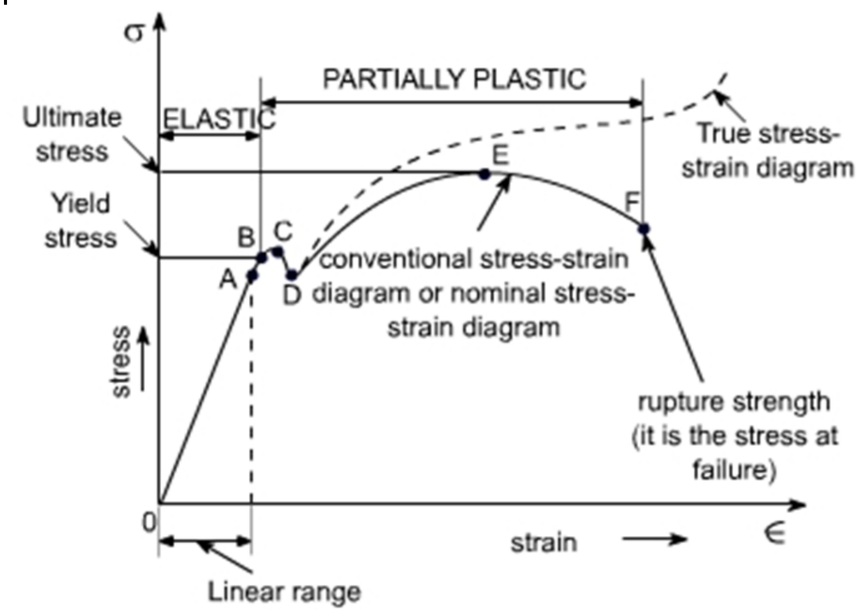
$G = \tau / \gamma$ [N/m²] - *Shear modulus*
Modulus of Rigidity



Stress and Strength

The quality of a mechanical system depends on the relationship of the maximum stress to the component strength

1. **Strength** is an *inherent property of a material* built into the part because of the use of a *particular material and process*.
   Strength is denoted with capital letter S *[N/m2]*
   S_E – Yield strength (lowest stress that produces permanent deformation)
   S_G – Shear strength (lowest shear stress that produces permanent deformation)
   S_τ – Tensile or ultimate strength
   (limit state of tensile stress)
   S_d – Fatigue strength – dynamic loading
   (in a stress range $\Delta\sigma=\sigma_{max}-\sigma_{min}$)
   S_i – Impact strength



Selection of material

$$\sigma = E\varepsilon \qquad \tau = G\gamma \qquad \nu = -\frac{lateral\ strain}{axial\ strain} \qquad E = 2G(1+\nu)$$

Material	Modulus of Elasticity E		Modulus of Rigidity G		Poisson's Ratio ν	Unit Weight w		
	Mpsi	GPa	Mpsi	GPa		lbf/in^3	lbf/ft^3	kN/m^3
Aluminum (all alloys)	10.4	71.7	3.9	26.9	0.333	0.098	169	26.6
Beryllium copper	18.0	124.0	7.0	48.3	0.285	0.297	513	80.6
Brass	15.4	106.0	5.82	40.1	0.324	0.309	534	83.8
Carbon steel	30.0	207.0	11.5	79.3	0.292	0.282	487	76.5
Cast iron (gray)	14.5	100.0	6.0	41.4	0.211	0.260	450	70.6
Copper	17.2	119.0	6.49	44.7	0.326	0.322	556	87.3
Douglas fir	1.6	11.0	0.6	4.1	0.33	0.016	28	4.3
Glass	6.7	46.2	2.7	18.6	0.245	0.094	162	25.4
Inconel	31.0	214.0	11.0	75.8	0.290	0.307	530	83.3
Lead	5.3	36.5	1.9	13.1	0.425	0.411	710	111.5
Magnesium	6.5	44.8	2.4	16.5	0.350	0.065	112	17.6
Molybdenum	48.0	331.0	17.0	117.0	0.307	0.368	636	100.0
Monel metal	26.0	179.0	9.5	65.5	0.320	0.319	551	86.6
Nickel silver	18.5	127.0	7.0	48.3	0.322	0.316	546	85.8
Nickel steel	30.0	207.0	11.5	79.3	0.291	0.280	484	76.0
Phosphor bronze	16.1	111.0	6.0	41.4	0.349	0.295	510	80.1
Stainless steel (18-8)	27.6	190.0	10.6	73.1	0.305	0.280	484	76.0
Titanium alloys	16.5	114.0	6.2	42.4	0.340	0.160	276	43.4

Mechanical Design Criteria

specific characteristics which influence the design of the element or the entire system

1. **Strength**: A component *should not fail or have residual deformations* under the effect of the forces that act on it.
This is satisfied if the *induced stress* is smaller then the *material strength*

$$\sigma \leq [S]$$

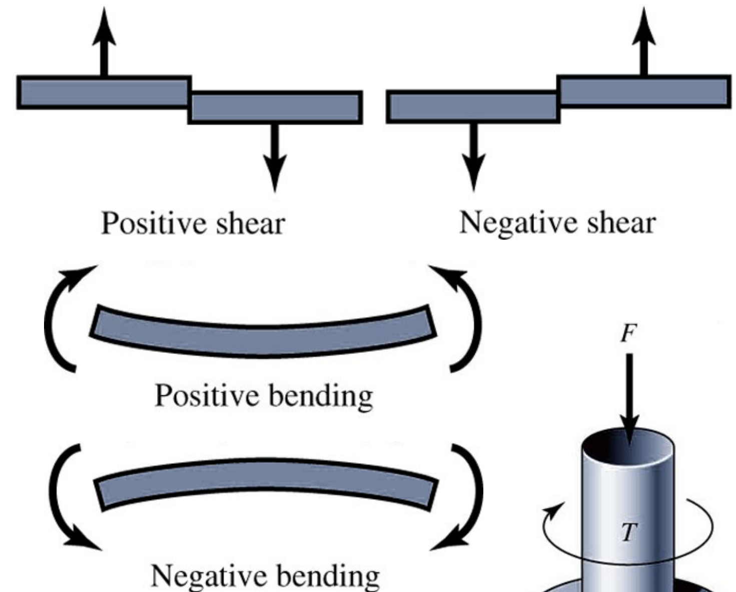
The necessary and sufficient strength of the part of a selected material under a given load is ensured by *dimensions and shapes*, which preclude damage and residual deformations.
A component can also fail because of damaged working surfaces induced by the very high stress or very small area.

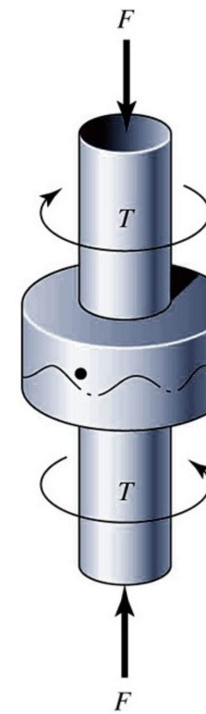
2. **Rigidity:** is the *ability of parts to resist deformations* under the action of forces.
Proper rigidity is necessary to ensure that machine as a whole and its elements operate effectively.
In many cases this parameter of operating capacity proves to be most important.
Therefore apart from the strength calculations, rigidity of a number of parts is also calculated as ratio of *the actual displacements* (deflections, angles of turn, angles of twist) with *allowable (rated) displacements*.

Mechanical Design Criteria (Cont.)

3. **Wear Resistance**: Wear is the principal cause of putting machine elements out of commission. Problems: frequent stops, loss of machine accuracy etc. *Calculations of wear* are usually of an arbitrary nature and carried out *together with calculations of strength*.
4. **Heat resistance**: The liberation of heat involved in the working process or some times due to friction between moving surfaces, causes the components of some machines to operate under conditions of increased temperature. An increased temperature (> 100^o C) impairs the lubricating ability of oils; Continuous operations involving temperatures > 300-400^o C causes slow plastic deformations called *creep.* Thermal deformations may reduce the accuracy of a machine.
   Effective cooling and special calculations for heat to *find the working temperature* of the machine elements, *evaluate the working stresses* and *compare* them *with the creep limits* for the material of the part.
5. **Resistance to vibrations**: The term implies the ability of a machine to operate at the assigned speeds and loads without impermissible oscillations Dynamic analysis after finalizing the design to *avoid inherent unbalances*.

Design for Strength





- **Basic assumptions**
  - » Material is continuous, homogenous and isotropic
  - » Material is linearly elastic or Hook's law is valid
  - » There are no internal stresses prior to loading
  - » Load is static
- **Static loading – any combination of:**
  - » *Axial loading, either tension or compression*
  - » *Shear loading*
  - » *Transverse loading which causes bending*
  - » *Torsional loading*

  Valid if time of application of load > **three times** its natural period.
  - » Apply ***yield*** criteria to the **maximum *expected* loading – the limit loading**
  - » Apply ***ultimate*** criteria to a possible ***overload -*** should survive without material rupture
- **Dynamic loading**
  - » *Apply fatigue criteria ($\Delta\sigma=\sigma_{max}-\sigma_{min}$)*
  - » *Depend on a type of loading*
  - » *Use empirical factors (reliability….)*

Design for Strength – Static Load

- **The part to be designed must be capable of**
  - » *transmitting* the necessary *forces* and performing necessary motions efficiently and economically
  - » failure must not occur in it before a predetermined span of operating life has elapsed
  - » the part must *perform* its *function* without interfering with any other part of the machine

Strength **is the primary criteria for the design. The *relation between the strength* of the part *and the stress* induced on it due to the anticipated static loading must also be considered in order to select the optimum material and dimensions for satisfying above requirements**

- **Two distinct and separate approaches**

1. *The deterministic, or factor-of-safety approach.* In this method, the maximum stress or stresses in a part are kept below the minimum strength by a suitable design factor or margin of safety, to ensure that the part will not fail.
2. *The stochastic, or reliability approach*. This method involves the selection of materials, processing and dimensions such that the probability of failure is always less than a pre selected value

Factor of Safety Method

- **Factor of safety method,** the classical method of design, employs reduced values of strengths that are used in the design to determine the geometrical dimensions of the parts.
- A design factor of safety N_d, some times called simply design factor N, is defined by the relation

$$N = \frac{Loss\ of\ \ function\ Load}{Allowable\ Load} = \frac{Strength}{Stress} = \frac{S}{\sigma}$$

- The failure stress (strength) can be anything the designer chooses it to be. Often such strengths as minimum, mean, yield, ultimate, shear, fatigue as well as others are used; of course it must correspond in type and units to the induced stress.

Material	Load	Factor of safety value N
Exceptionally reliable	Certainly known	1.25 to 1.50
Well known	Known	1.50 to 2.00
Known	Well known	1.50 to 2.00
Less tried	Known orAverage	2.00 to 2.50

Reliability Method

- The **reliability method,** is the method in which statistical distribution of stresses and strength are obtained and related to each other.
- The statistical measure of the probability that a mechanical element will not fail in use is called the *reliability* of that element. It can be quantified as:

$$0 \le R \le 1$$

- If in the above equation R=0.90 that means that there is 90% chance that the part will perform its proper function without failure.
- In the reliability method, designer should select materials, processes and geometry in order to achieve a reliability goal.
- Analyses that lead to an assessment of reliability address uncertainties, or their estimates, in parameters that describe the situation. Stochastic variables such as stress, strength, load, or size are described in terms of their means, standard deviations and distributions.

Shafts
and
Keyways

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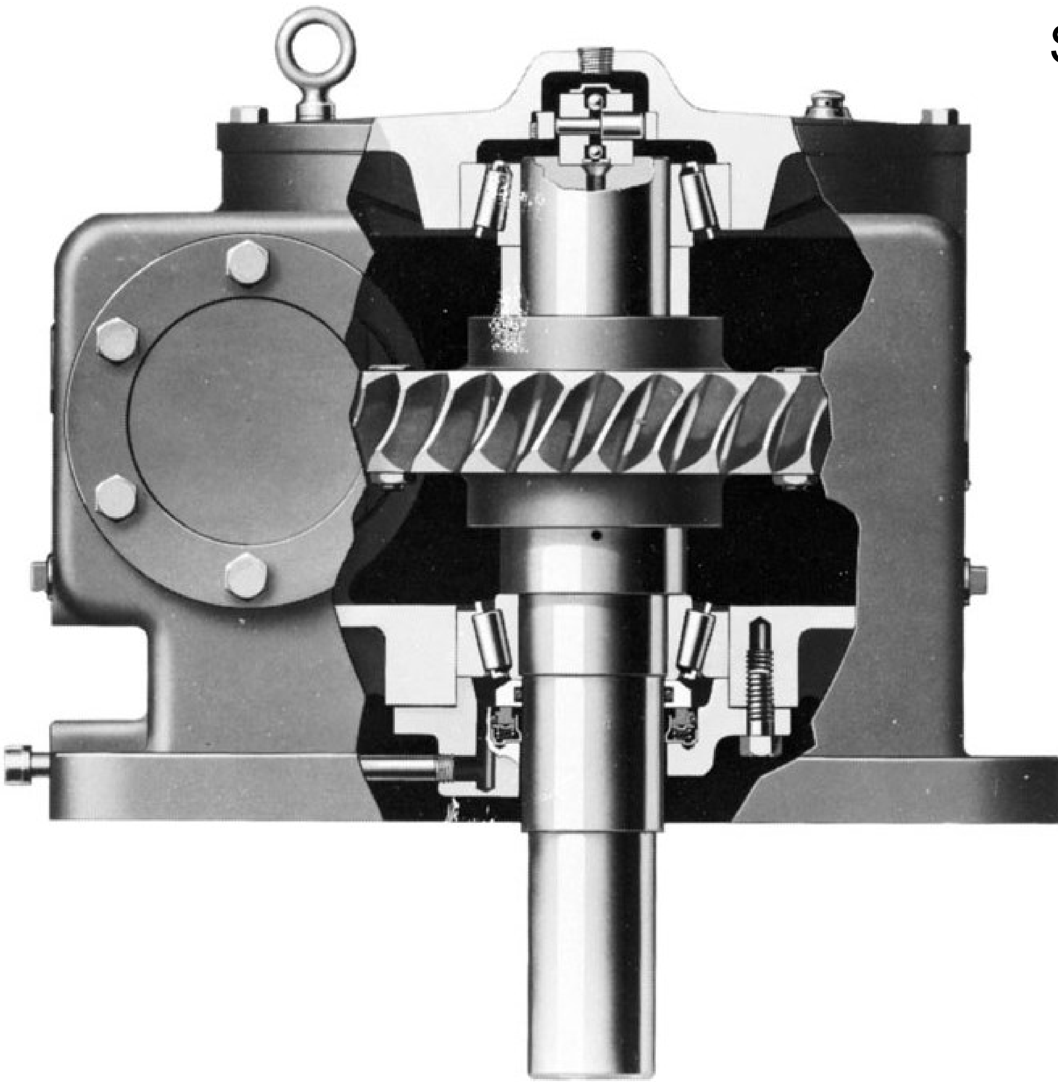
Steps in shaft design

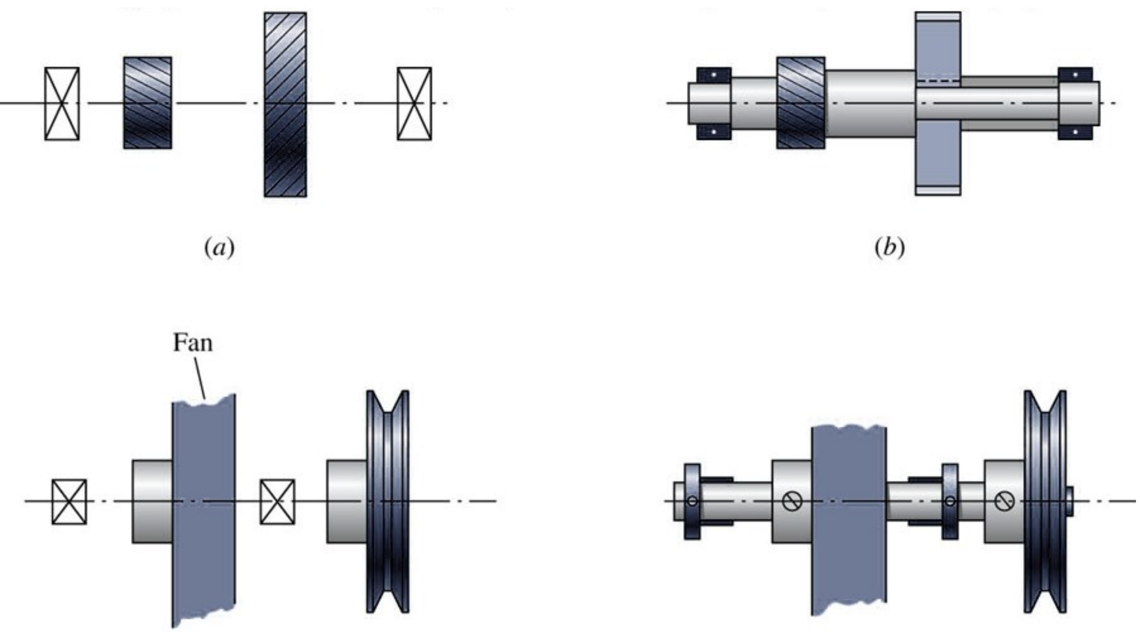


Steps in the shaft design are:

1. Define shaft topology
2. Specify driving elements
3. Free body diagram
4. Select bearings
5. Consider shaft deflection and stress
6. Check for critical speed
7. Specify connections
8. Dimensions

Steps in shaft design

1. Shaft topology

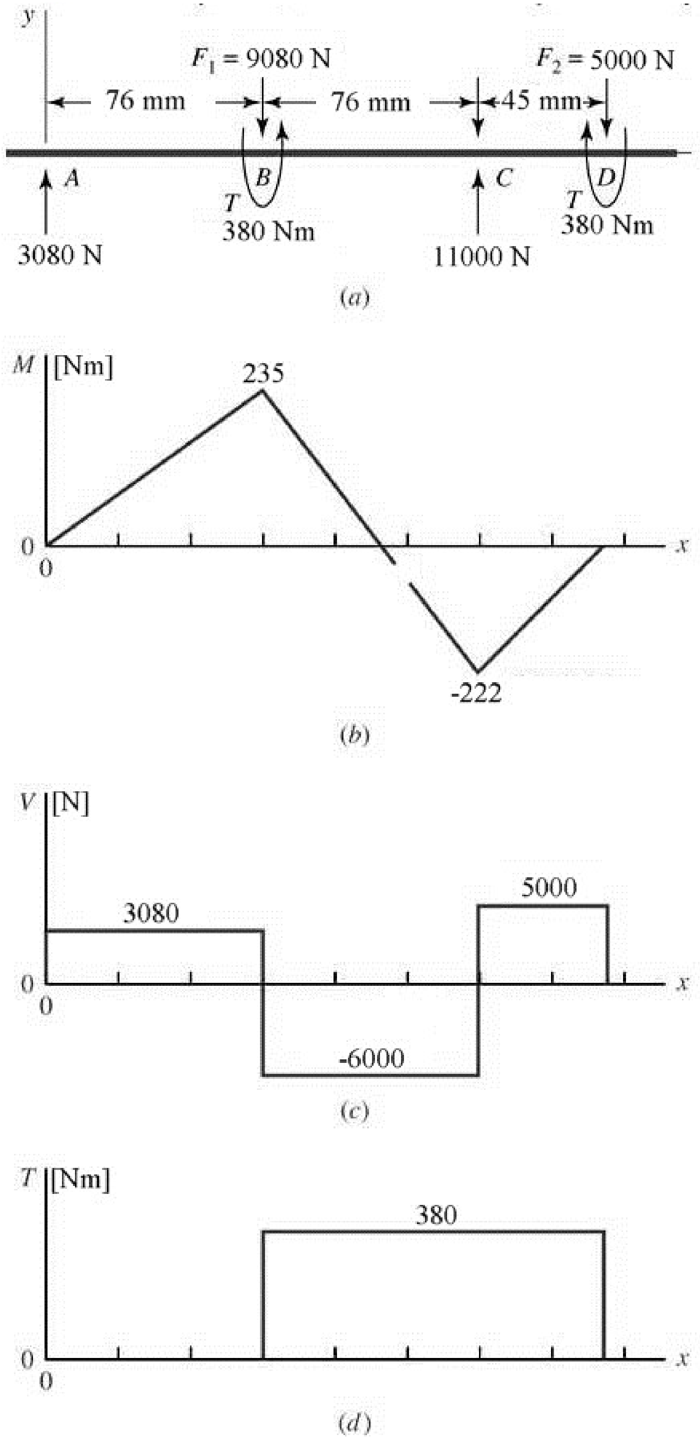


(a) (b) (c) (d)

2. Driving elements

Gears, pulleys, sprockets …

Shaft design depends on the size of rotating element and forces acting on it



Steps in shaft design

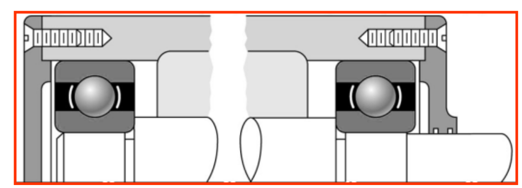
3. ***Free body diagram***

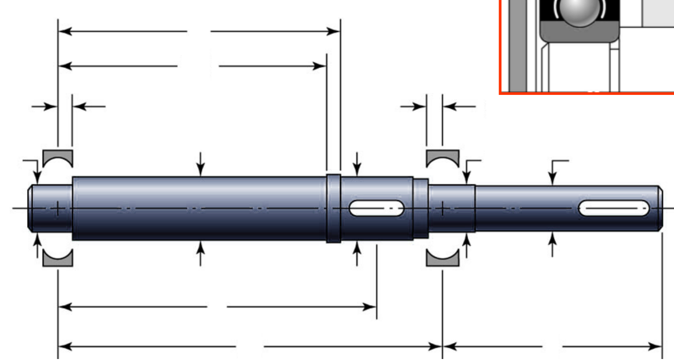
   The system of interest is separated from the surrounding and connections are replaced by forces

   - Reactions in bearings & force diagram
   - Bending moment
   - Torsional moment ($P=\omega T$)

4. ***Bearing selection and position***

   - Equivalent load (forces)
   - Bearing rating life based on the size
   - Positioning and lubrication





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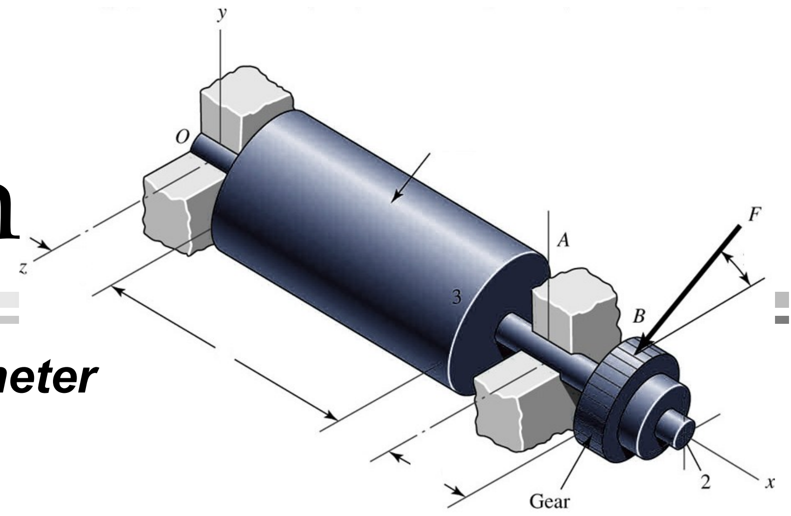
Steps in shaft design



5. ***Shaft deflection and stress – minimum diameter***

Difficult to calculate exactly. Reasons for complexity:

- Variable shaft diameter
- Undercuts and grooves – stress concentration points
- Type of load – axial, radial, torsional, bending, static, dynamic …

Static load:

Bending stress

$$\sigma_z = \frac{M}{Z} = \frac{Mc}{I} = \frac{32M}{\pi d^3}$$

Torsional stress

$$\tau_{zy} = \frac{T}{S} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$$

Minimum diameter (distortion energy) theory

$$d = \sqrt[3]{\frac{32 f_s}{\pi S_y} \sqrt{M^2 + \frac{3}{4} T^2}}$$

$c=d/2$	- maximum span
$I=\pi d^4/64$	- second moment of area
$Z=c/I$	- section modulus
$J=\pi d^4/32$	- second polar moment of area
$S=c/J$	- polar section modulus
f_s	- factor of safety

Dynamic loading – shaft design

6. Dynamic load:

$$\sigma_a = K_f \frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3}$$

$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3}$$

K_f - Fatigue stress concentration factor for bending

K_{fs} - Fatigue stress concentration factor for torsion

Indices:

a - alternating moment or torque $M_a = \frac{M_{max} - M_{min}}{2}$

m - midrange moment or torque $M_m = \frac{M_{max} + M_{min}}{2}$

$$\sigma'_{\max} = \left[(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2\right]^{1/2}$$

$$= \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2}$$

Von-misses stresses

for rotating round solid shaft neglecting axial load

$$n_y = \frac{S_y}{\sigma'_{\max}}$$

Factor of safety:

Compares maximum yielding stress with yielding strength

Dynamic loading – shaft design

$$K_f = 1 + (K_c - 1)q_n \qquad K_f = \frac{Endurance\ limit\ for\ notch - free\ shaft}{Endurance\ limit\ for\ notched\ shaft}$$

Fatigue stress concentration factor

K_c – Stress concentration factor (function of geometry)

$q_n = \dfrac{K_f - 1}{K_c - 1}$ – Notch sensitivity factor

$$S'_e = 0.5\ S_u \quad bending$$
$$S'_e = 0.45\ S_u \quad tension$$
$$S'_e = 0.29\ S_u \quad torsion$$

Stress endurance limit for steels under idealised conditions (high-cycles fatigue)

$$S_e = k_f k_s k_r k_t k_m S'_e$$

Modified endurance limits

$k_f = 0.65 - 0.85$

$k_s = 1.189\ d^{-0.112}$

k_r: 1 for 50%

0.9 for 90%

0.82 for 99%

0.7 for 99.99%

$k_t = \dfrac{S_{ut}}{S_u}$

k_f – surface finish factor

k_s – size factor

k_r – reliability factor

k_t – temperature factor

k_m – miscellaneous factor

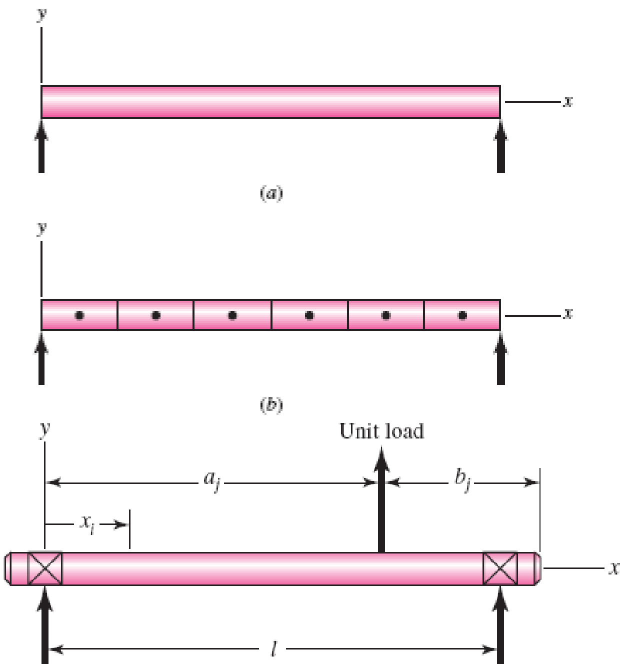
$$d = \sqrt[3]{\frac{32 f_s}{\pi S_y}\sqrt{\left(M_m + \frac{S_y}{S_e}K_f M_a\right)^2 + \left(T_m + \frac{S_y}{S_e}K_{fs} T_a\right)^2}}$$

Minimum diameter for cyclic loading of a shaft

Propertied of steel

Material	Density, kg/m^3	Modulus of elasticity, $psi \times 10^6$ (GPa)	Yield strength, ksi (MPa)	Ultimate strength, ksi (MPa)	Ductility, %EL in 2 in.	Poisson's ratio	Thermal conductivity, W/m-°C	Coefficient of thermal expansion, $(°C)^{-1} \times 10^{-6}$
Iron	7 870	30 (207)	19 (130)	38 (260)	45	0.29	80	11.8
Gray cast iron	7 150	Variable	—	18 (125)	—	Variable	46	10.8
Nodular cast iron	7 120	24 (165)	40 (275)	60 (415)	18	0.28	33	11.8
Malleable cast iron	7 200–7 450	25 (172)	32 (220)	50 (345)	10	0.26	51	11.9
Low-carbon steel (AISI 1020)	7 860	30 (207)	43 (295)	57 (395)	37	0.30	52	11.7
Medium-carbon steel (1040)	7 850	30 (207)	51 (350)	75 (520)	30	0.30	52	11.3
High-carbon steel (AISI 1080)	7 840	30 (207)	55 (380)	89 (615)	25	0.30	48	11.0
Stainless steels								
Ferritic, type 446	7 500	29 (200)	50 (345)	80 (552)	20	0.30	21	10.4
Austenitic, type 316	8 000	28 (193)	30 (207)	80 (552)	60	0.30	16	16.0
Martensitic, type 410	7 800	29 (200)	40 (275)	70 (483)	30	0.30	25	9.9

Critical speed - shaft design



Critical speed of a shaft without load

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{gEI}{A\gamma}} \qquad [\frac{rad}{s}]$$

Rayleigh's equation

$$\omega_1 = \sqrt{\frac{g\sum w_i y_i}{\sum w_i y_i^2}}$$

m – mass per unit length

γ – specific weight

w_i – weight at the location

y_i – deflection at the location

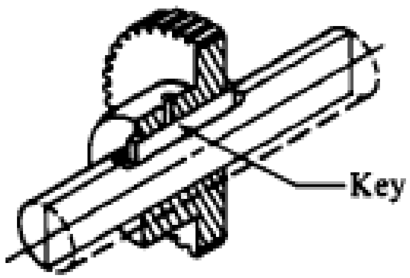
Deflection at point i *due to the load at point* j

$$\delta_{ij} = \begin{cases} \dfrac{b_j x_i}{6EIl}(l^2 - b_j^2 - x_i^2) & x_i \le a_i \\ \dfrac{a_j(l - x_i)}{6EIl}(2l x_i - a_j^2 - x_i^2) & x_i > a_i \end{cases}$$

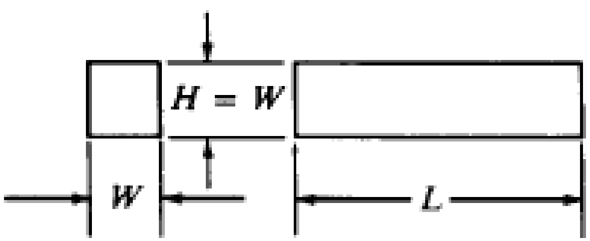
Dunkerley equation

$$\frac{1}{\omega_1^2} \doteq \sum_{1=1}^{n} \frac{1}{\omega_{ii}^2}$$

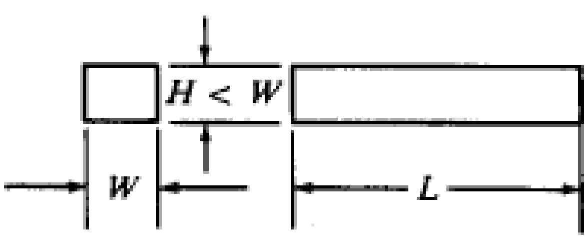
Strength of a key



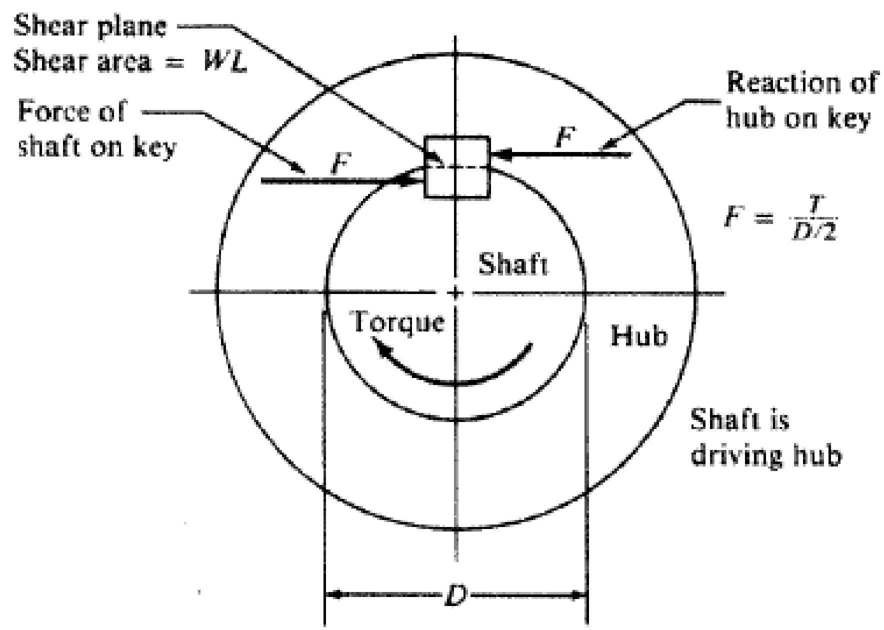
(a) Key and keyseat applied to a gear and shaft



(b) Square key



(c) Rectangular key



$$W \approx D/4$$

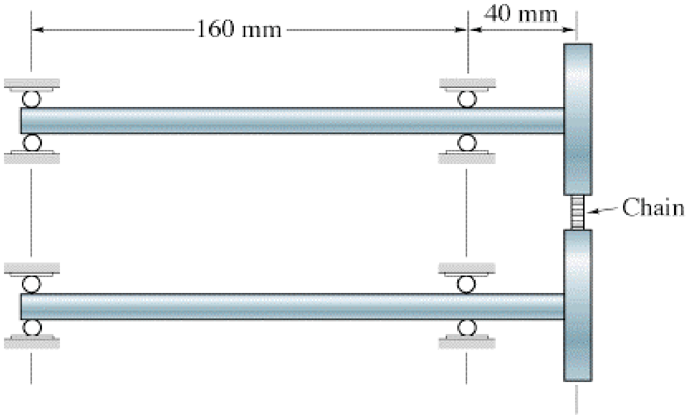
$$T = F\frac{D}{2};$$

Allowed torque on the key:

$$A = WL$$

$$\tau = \frac{F}{A} = \frac{2T}{DWL} \Rightarrow \boxed{T = \frac{DWL\,S_y}{4 f_s}}$$

Example 1 – Shaft design



(a)

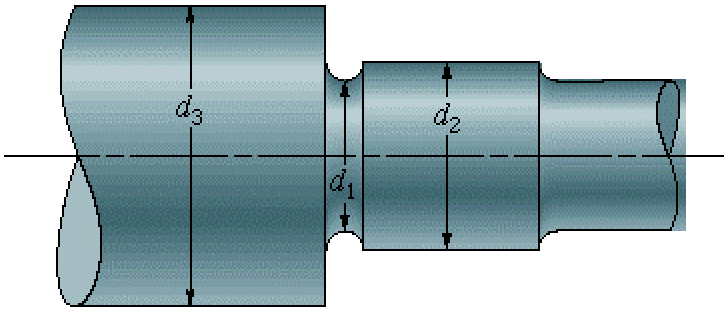
The power from the motor of a front-wheel drive car is transmitted to the gearbox by a chain drive (Figure a). The two chain wheels are the same size. The chain is not pre-stressed and it does exerts no force. The safety factor is 4. The shaft is to be made of ANSI 1080 steel. The chain transmits 100kW of power at the chain speed of 50 m/s when the motor speed is 6000 rpm.

Find the appropriate shaft diameter.

Solution: d=26 mm

Example 2 – Shaft dynamic load

The shaft made of high-carbon steel, in figure, is subjected to reversed bending and steady torsion. A standard ball bearing is to be placed on diameter d_2 and this surface is machined to form a good seat fro the bearing.



The groove between the sections is there to ensure that the large diameter section is not damaged by the grinding operation and is called 'grinding relief'. Assume that standard ball bearing bore sizes are in *5 mm* increments in the range *15* to *50 mm*.

Shaft should be designed so that $d_2=0.75\ d_3$ and $d_1=0.65\ d_3$. The completely reversed bending moment is *70 Nm* and the steady torsion is *45 Nm*. Assume safety factor of *2.5* and size the shaft for infinite life.

Assume stress concentration factor $K_c=1.9$, notch sensitivity $q_n=0.75$, surface factor $k_f=0.75$

Determine diameter d_2? Solution: $d_1=25.3\ mm;\ d_2=30mm;\ d_3=40mm$

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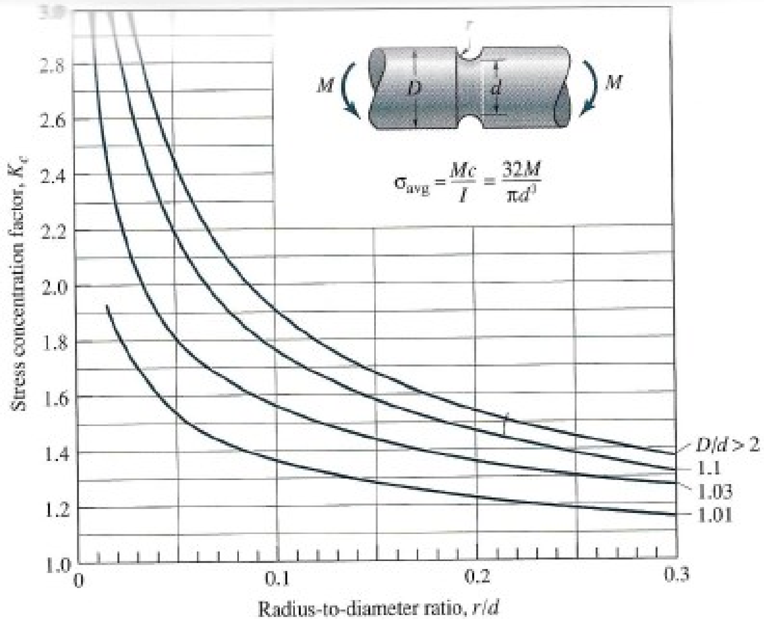
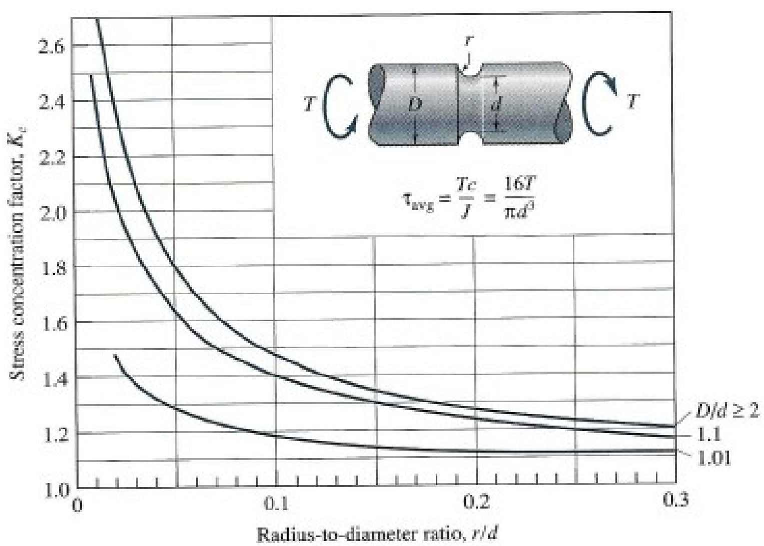
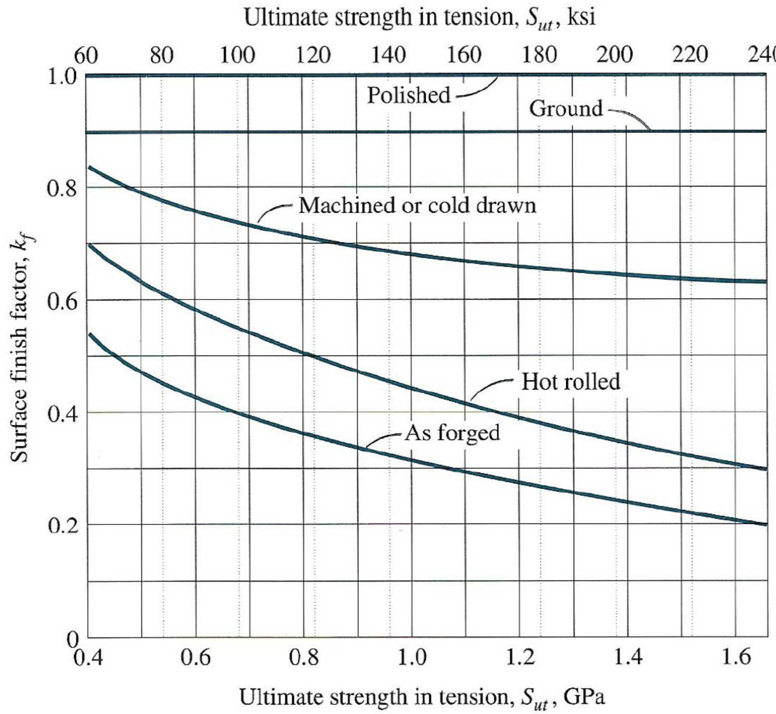
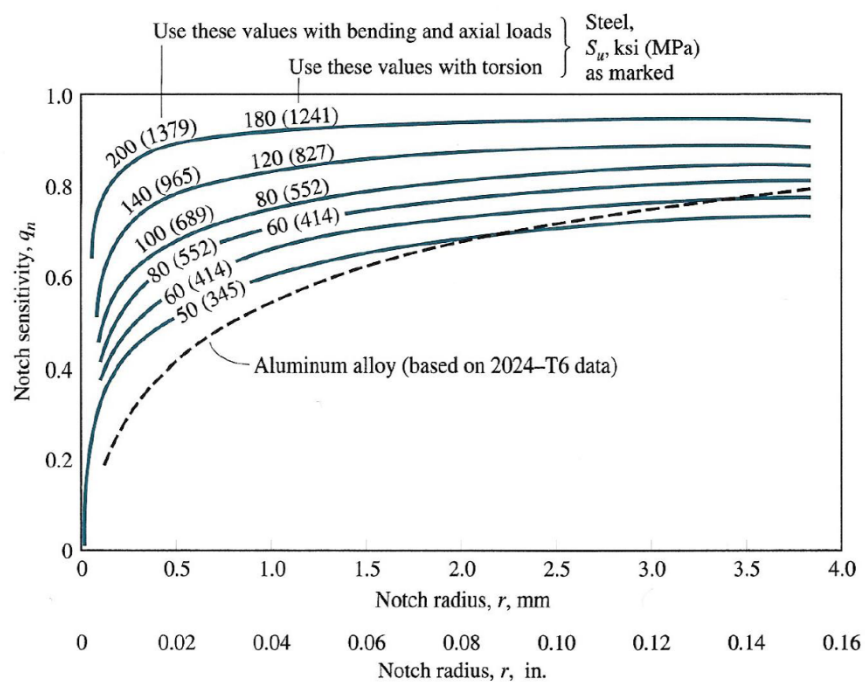


(b)

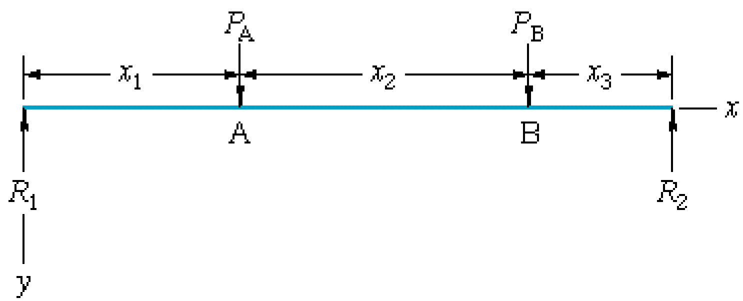


(c)





Example 3 – Shaft critical speed



A simply supported shaft arrangement is shown in the figure.

A solid shaft of 50 mm diameter made of low-carbon steel is used.

Given values are:

x_1=762 mm

x_2=1016 mm

x_3=508 mm

P_A=356 N

P_B=534 N

Determine first critical speeds:

a) Unloaded shaft

b) Rayleigh method

c) Dunkerley method

Solution:

a) 1155 rpm

b) 648 rpm

c) 651 rpm

Example 4 – Key calculation

A 100 mm diameter shaft with a hub is made of high-carbon steel. A square key made of low-carbon steel has a width and height of 25 mm.

Assume allowable stress:

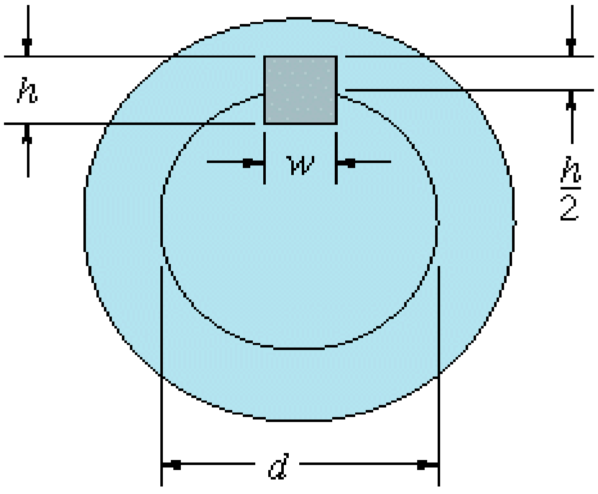
$$\tau_{al} = 0.4\, S_y$$
$$\sigma_{al} = 0.9\, S_y$$

Calculate:

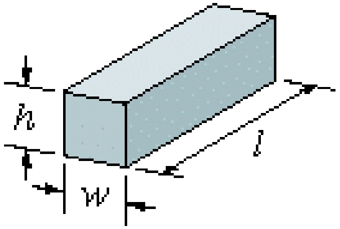
critical length of key while assuming safety factor of 2 and considering both compression and shear

Solution: $L_{cr\tau}$=202.2 mm

$L_{cr\sigma}$=179.7 mm



(a)



(b)