Example 11 – Helical gear

$n_p = 18000$ rpm; $n_g = 3000$ rpm

$N_p = 21$

$N_g = 126$

$P = 14''$

$b_W = 45,72$ mm

$\phi = 20°$

a) $C_r = ?$

b) $C_{r\Psi} = ?$ $\Psi = 30°$

c) $\Psi_a = ?$ for $C_r = 3$

d) $F_a, F_r; F_N = ?$

a) circular pitch is

$$\boxed{p_c = \frac{\pi}{P_d} = \frac{\pi}{14} = 0,224\,[in] = \underline{5,7\text{ mm}}}$$

Module $m = \frac{p}{\pi} = \frac{5,7}{\pi} = 1,814$ mm

For spur gear $\Psi_s = \phi \Rightarrow \cos\Psi = 1$

$$C_d = \frac{D_p + D_g}{2} = \left(\frac{N_p + N_g}{2}\right) m = \frac{21+126}{2} \cdot 1,814$$

$D_p = m \cdot N_p;\ D_g = m \cdot N_g$

$$\boxed{C_d = 133,4\text{ mm}}$$

Pitch radii

$$r_p = \frac{m \cdot N_p}{2} = \frac{1,814 \cdot 21}{2} = 19,047\text{ mm}$$

$$r_g = \frac{m \cdot N_g}{2} = \frac{1,814 \cdot 126}{2} = 114,282\text{ mm}$$

Radii of base circles

$$r_{bp} = r_p \cdot \cos\phi = 19.047 \cdot \cos 20° = 17,9\text{ mm}$$

$$r_{bg} = r_g \cdot \cos\phi = 114,282 \cdot \cos 20° = 107,4\text{ mm}$$

addendum = module $a = m$

$r_{op} = r_p + a = 19,047 + 1,814$ $r_{op} = 20,86$ mm

$r_{og} = r_g + a = 114,282 + 1,814$ $r_{og} = 116.1$ mm

$$C_r = \frac{\sqrt{r_{op}^2 + r_{bp}^2} + \sqrt{r_{og}^2 + r_{bg}^2} - C_d \cdot \sin\phi}{p_c \cdot \cos\phi} =$$

$$= \frac{\sqrt{20,86^2 + 17,9^2} + \sqrt{116,1^2 + 107,4^2} - 133,4 \cdot \sin 20°}{5,7 \cdot \cos 20°}; \quad C_r = 1,712$$

b) $$C_{r\Psi} = C_r + C_{ra} = C_r + \frac{b_W \cdot \tan\Psi}{p_c} = 1,712 + \frac{45,72 \cdot \tan 30°}{5,7}$$

$$\boxed{C_{r\Psi} = 6,343}$$

(20)

c) To have $Cr_{an} = 3$:

$$Cr_a = Cr_2 - Cr \Rightarrow \frac{b_w \cdot \tan \gamma_2}{p_c} = 3 - 1,712 = 1,288 \text{ mm}$$

$$\tan \gamma_2 = \frac{1,288 \cdot p_c}{b_w} = \frac{1,288 \cdot 5,7}{45,72} = 0,160577$$

$$\boxed{\gamma_2 = 9,122^\circ}$$

d) $F_{a_1} = F_t \cdot \tan \gamma_1$

$F_{r_1} = F_t \cdot \tan \phi$

$$F_N = \frac{F_t}{\cos \phi \cdot \cos \gamma}$$

$Pow = \omega_g \cdot T_g \Rightarrow$

$$\boxed{T_g} = \frac{Pow}{\omega_g} = \frac{Pow \cdot \cancel{30}}{n_g \cdot 3\cancel{\pi}} = \frac{10 \cdot 10^3 \cdot 30}{3000 \cdot 3\pi} = 31,83 \text{ Nm}$$

$$\boxed{F_t} = \frac{T_g}{r_g} = \frac{31,83}{0,114282} = \boxed{278,5 \text{ N}}$$

$$\boxed{\begin{aligned} F_{a_1} &= 278,5 \cdot \tan 30 = 160,8 \text{ N} \\ F_{r_1} &= 278,5 \cdot \tan 20 = 101,4 \text{ N} \end{aligned}}$$

$\cancel{F_N = 278,5 / (\tan 30 \cdot \tan 20) = 1325}$

$$\boxed{F_{N_1}} = \frac{F_t}{\cos\phi \cdot \cos\gamma} = \frac{278,5}{\cos 20 \cdot \cos 30} = \boxed{342,2 \text{ N}}$$

For $\gamma_2 = 9,122^\circ$

$$\boxed{F_{a_2} = 44,72 \text{ N}}$$

$$F_{N_2} = \frac{278,5}{\cos 20 \cdot \cos 9,122}$$

$$\boxed{F_{N_2} = 300,17 \text{ N}}$$

Example 12 – Compound gear

$\frac{N_2}{N_3} = 6$; $\frac{N_4}{N_5} = 5$; $N_2 + N_3 = N_4 + N_5$

$\phi = 20°$
$m = 3$
$R = 30:1$

From interference equation

$$N_p = \frac{2k}{(1+2m)\sin^2\phi} \cdot \left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) =$$

$$= \frac{2}{(1+6)\cdot\sin^2 20} \cdot \left(3 + \sqrt{m^2 + (1+2m)\cdot\sin^2\phi}\right) = \underline{\underline{15}} \Rightarrow N_3 = 16$$

(use 16)

2,44276

It would not work for $N_3 = 15$
~~$15 \Rightarrow N_5 = 17{,}5$~~

$N_2 = 6 \cdot N_3 = 6 \cdot 16 = 96$

$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$

using $N_4 = 5N_5$

$112 = 5N_5 + N_5 = 6N_5 \Rightarrow N_5 = 18{,}67$

– ~~Since~~ exact gear ratio is required: ~~assume~~ the number can be increased until it works

IT WOULD work with $N_3 = 18 \Rightarrow N_5 = 21$;
$N_2 = 108 \Rightarrow N_4 = 105$

To check calculate e (train value)

$$e = \frac{\text{product of driving tooth no}}{\text{product of driven tooth no}} = \frac{108 \cdot 105}{18 \cdot 21} = 5 \cdot 6 = 30$$

For the inline requirement:

$C_{23} = C_{45}$

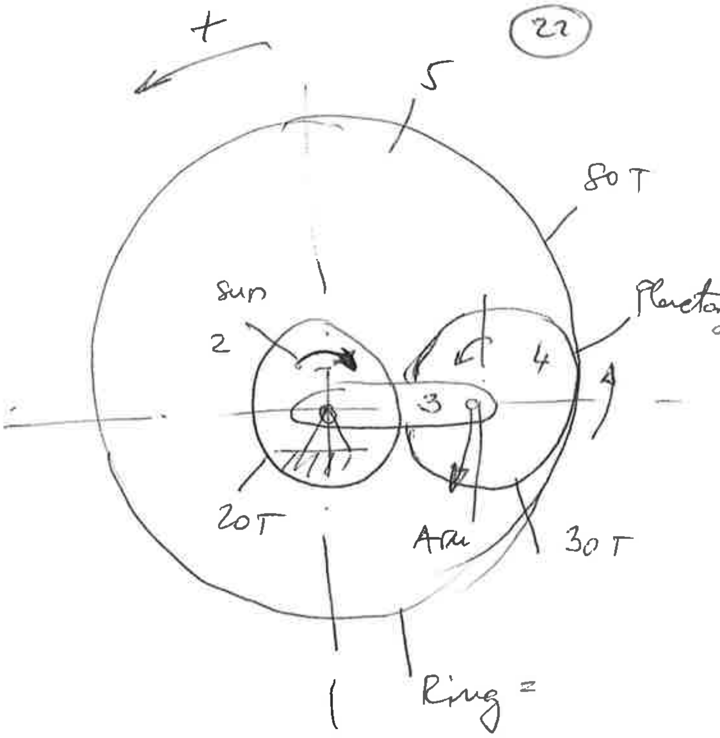
$$\frac{N_2 + N_3}{2\cancel{p}} = \frac{N_4 + N_5}{2\cancel{p}} \Rightarrow 108 + 18 = 105 + 21$$

Example 13: — Planetary gear

$n_2 = -100$ rpm $\cancel{N_{?}} = N_2 = 20$

$\cancel{N_A} = N_4 = 30$ $N_A = ?$

$n_5 = \phi$ $N_5 = N_L = 80$



Imagine unlocking Ring gear and hold the arm

$$e = -\left(\frac{20}{30}\right)\left(\frac{30}{20}\right) = -0{,}25$$

Train value

$$\cancel{e = \frac{N_L - N_A}{N_F - N_a} = \frac{0 - N_A}{(-100) - N_A}}$$

$$e = \frac{n_L - n_a}{n_f - n_a} = \frac{0 - n_a}{(-100) - n_a}$$

$$n_3 = \boxed{N_A = -20 \text{ rev/min}}$$ → ~~gear~~ arm

$n_{43} = n_4 - n_3$; $n_{23} = n_2 - n_3$

$$\frac{n_{43}}{n_{23}} = \frac{n_4 - n_3}{n_2 - n_3} = -\frac{N_{sun}}{N_{planet}} = -\frac{20}{30} = -\frac{2}{3}$$

$$n_4 = -\frac{2}{3}(n_2 - n_3) + n_3 = -\frac{2}{3}(-100+20) \cancel{=} 20$$

$$\boxed{n_4 = \frac{160}{3} - 20 = \frac{100}{3} = 33.3 \text{ rpm}}$$