

University of Tuzla,  
Faculty for Mechanical Engineering, Postgraduate study  
Modelling, Simulation, Optimisation

## Design Integration for Screw Compressors

Part 1:  
Methods and Tools in Screw Compressor Design

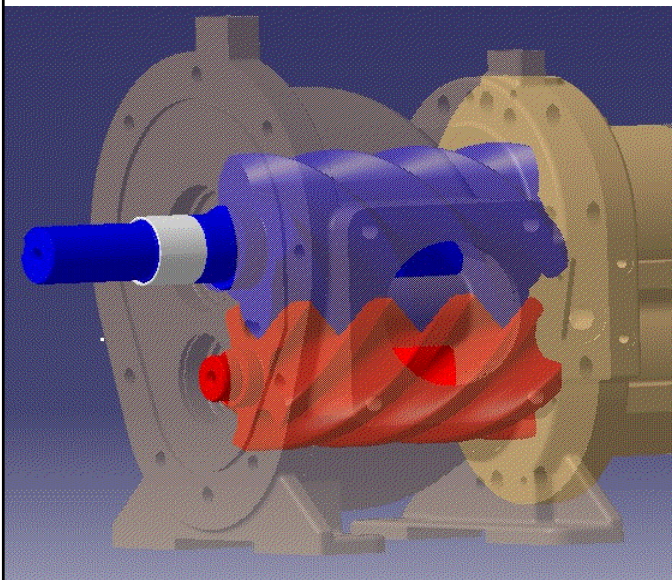
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[www.city-compressors.co.uk](http://www.city-compressors.co.uk)

## Positive displacement rotary machine



### Brief history:

The operating principle known for more than 100 years

1942 *Lysholm* of SRM – patent on working principle and profile.

1952 *Nilsen* - patent on circular profile, SRM designed first commercial screw compressor.

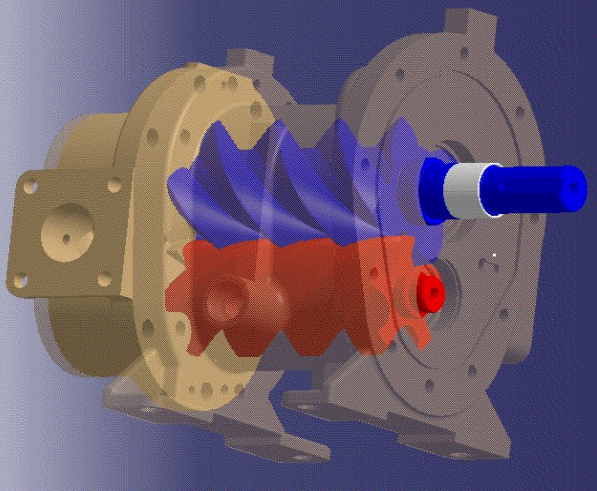
Until 1990's SRM was practically the only licensor of screw compressor technology

1985 Independent development of screw compressors begins with rack generated profiles from Rinder, Stosic and some Japanese companies.


1995 Significant increase in number of screw compressor manufacturers.

2005 Screw compressor industry needs tool for concurrent engineering of screw compressors.

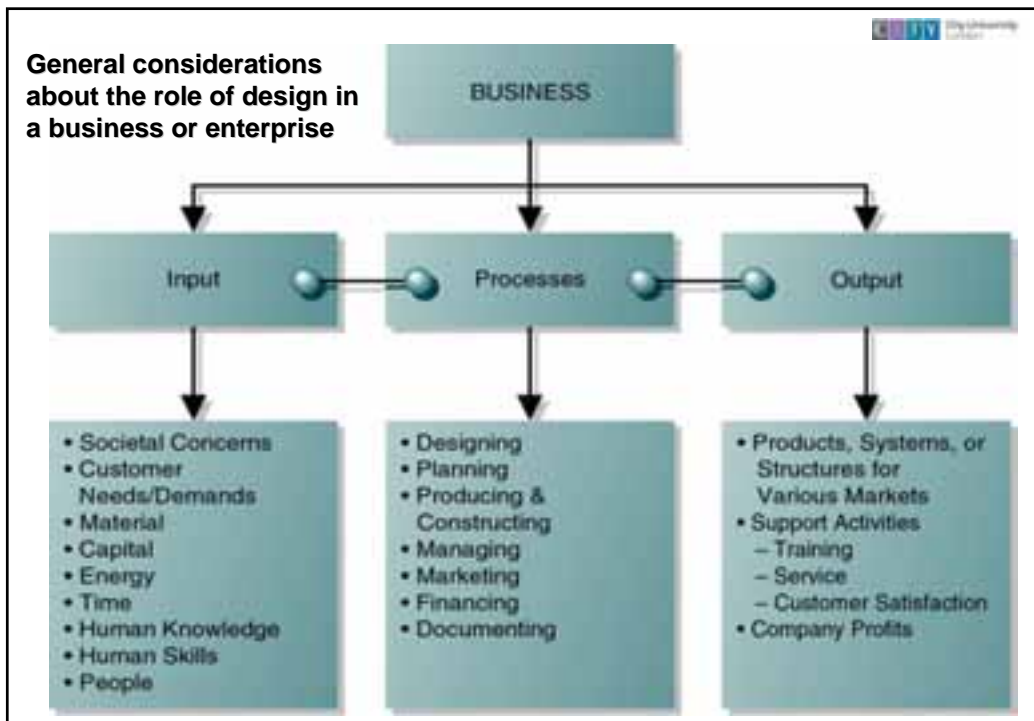
## Screw compressor applications

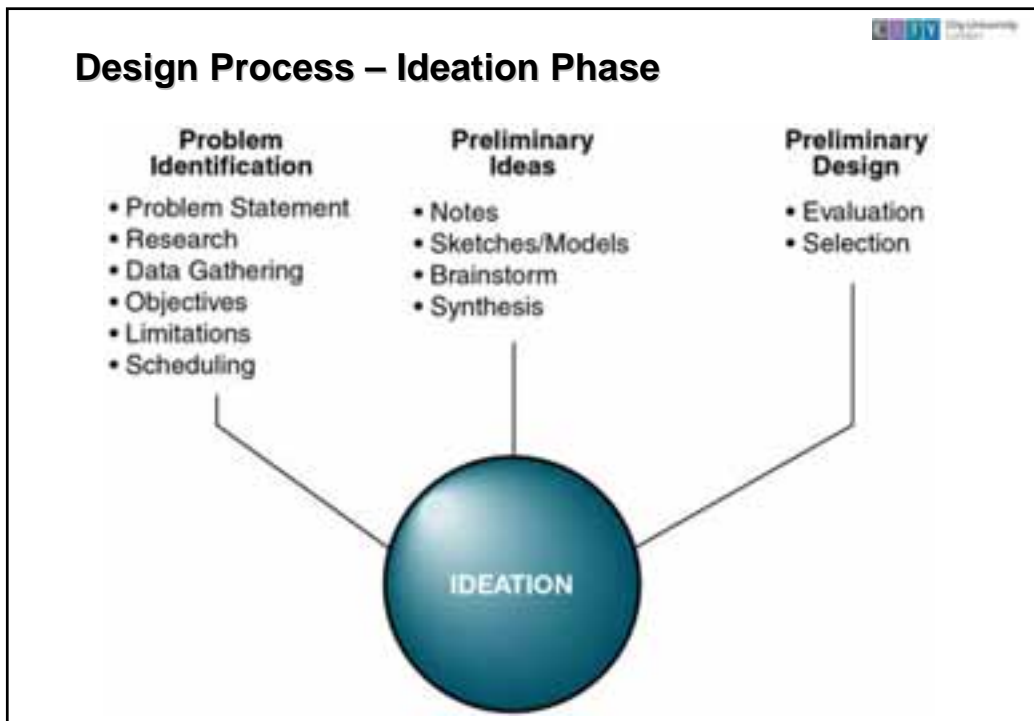
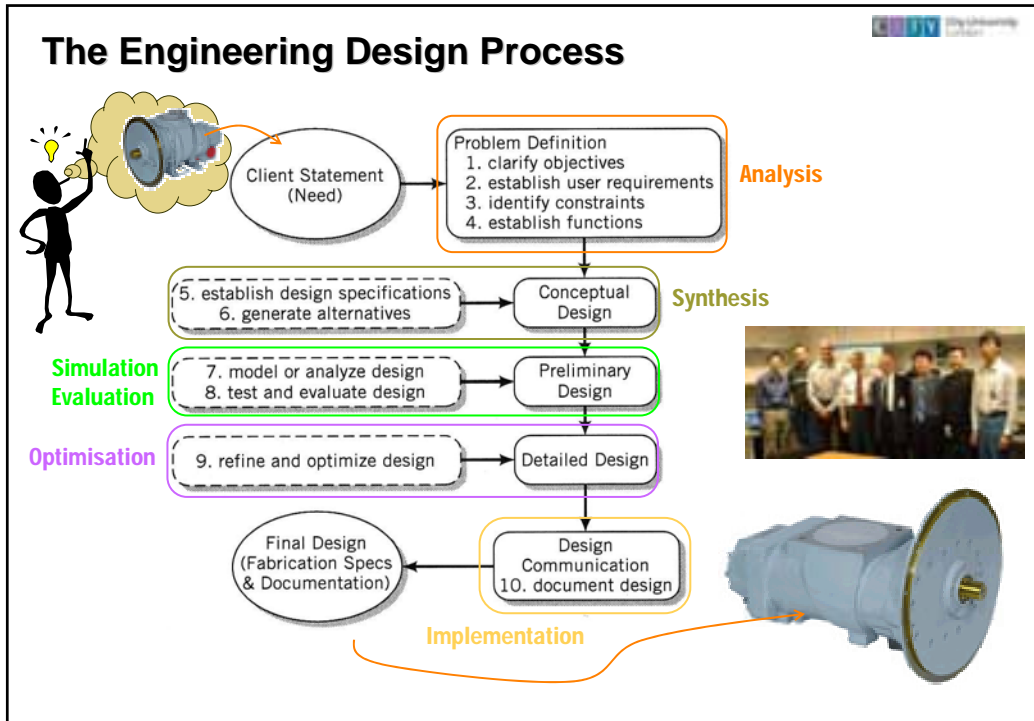


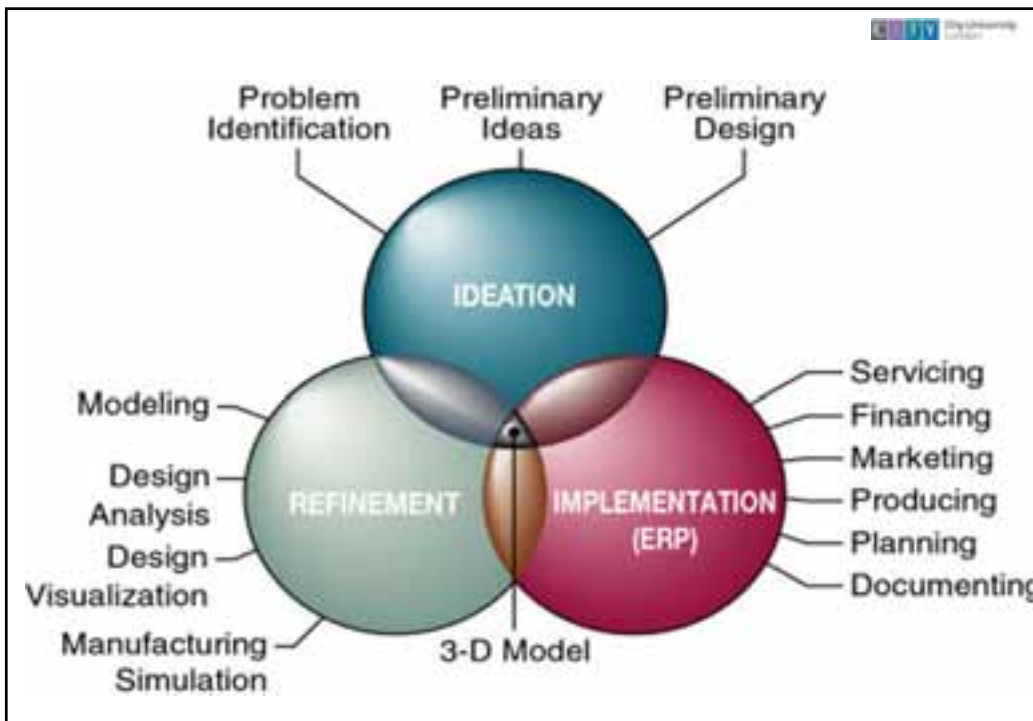
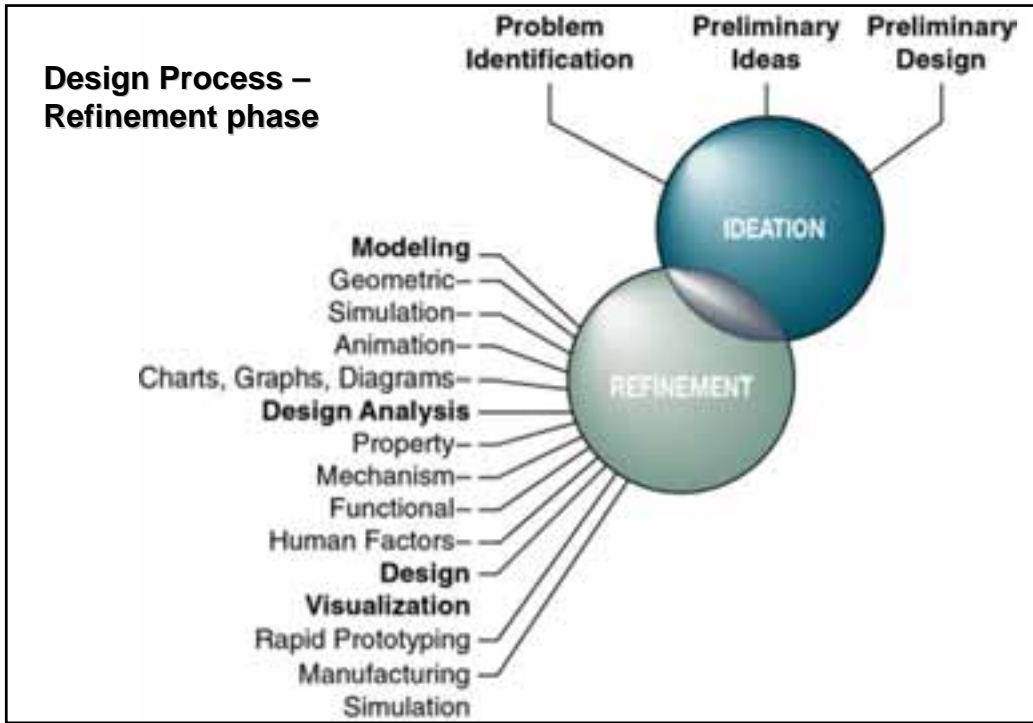
- Dry & oil injected:
  - Air compression
  - Refrigeration
  - Process fluids
- (35) 50 – 800 mm dia
- 0.3 – 800 m<sup>3</sup>/min
- High Volumetric efficiency
- 80% of industrial compressors produced are screw compressors




17% energy produced in developed countries used for compression  
25% energy in USA during summer is used for refrigeration and air-conditioning









## Methods and Tools in Screw compressor design

- **2-D design tools** - Conventional approach
  - **SCORPATH** (Screw Compressor Rotor Profiling and THERMODYNAMICS)
  - **2-D CAD Software: AutoCad 2000+**
- **3-D design tools** - More modern approach
  - **SCORPATH** (Screw Compressor Rotor Profiling and THERMODYNAMICS)
  - **3-D CAD** MDT, Inventor, Catia, Solid Works
  - **SCORG** (Screw Compressor Rotor Grid)
  - **CFD (CCM)** Comet, Star, CFX, Fluent
- **3-D design management** - Concurrent approach
  - **DISCO** (Design Integration for Screw COMPRESSORS)

**60 %**

**35 %**

**Only rear users**



## SCORPATH MMI

- **METHODS:** Envelope meshing, 1D Thermodynamics, Multivariable optimisation
- **TOOL:** SCORPATH

- Profile Generation
- Tool Calculation
- Thermodynamics
- Forces Calculation
- CAD Interface
- Property Calculations
- Optimisation

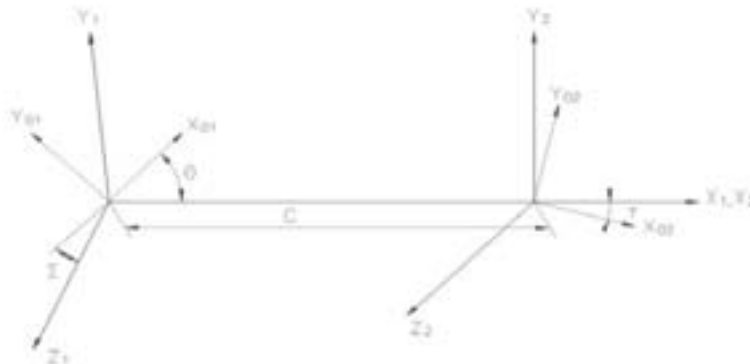
Screw Compressor Optimal Rotor Profiling and Thermodynamics

## Basic Screw Compressor Geometry

Rotor lobe profiles must be defined together with the remaining parameters with which the rotor and housing geometry can be fully specified.

- Rotor profile: x and y coordinates, pressure angle
- Helix/lead angle, rotor length
- Interlobe, end and radial clearance
- Suction/discharge ports

## General case: non-parallel and non-intersecting axes



Given profile

$$\mathbf{r}_1 = \mathbf{r}_1(t, \theta) = [x_1, y_1, z_1] = [x_{01} \cos \theta - y_{01} \sin \theta, x_{01} \sin \theta + y_{01} \cos \theta, p_1 \theta]$$

$$\frac{\partial \mathbf{r}_1}{\partial t} = \left[ \frac{\partial x_1}{\partial t}, \frac{\partial y_1}{\partial t}, 0 \right] = \left[ \frac{\partial x_{01}}{\partial t} \cos \theta - \frac{\partial y_{01}}{\partial t} \sin \theta, \frac{\partial x_{01}}{\partial t} \sin \theta + \frac{\partial y_{01}}{\partial t} \cos \theta, 0 \right]$$

$$\frac{\partial \mathbf{r}_1}{\partial \theta} = \left[ \frac{\partial x_1}{\partial \theta}, \frac{\partial y_1}{\partial \theta}, 0 \right] = [-y_{01}, x_{01}, 0]$$

### General case: non-parallel and non-intersecting axes

#### Meshed profile

$$\mathbf{r}_2 = \mathbf{r}_2(t, \theta, \tau) = [x_2, y_2, z_2] = [x_1 - C, y_1 \cos \Sigma - z_1 \sin \Sigma, y_1 \sin \Sigma + z_1 \cos \Sigma] = [x_{02} \cos \tau - y_{02} \sin \tau, x_{02} \sin \tau + y_{02} \cos \tau, p_2 \tau]$$

$$\frac{\partial \mathbf{r}_2}{\partial \tau} = [-y_2, x_2, p_2] = [x_{02} \sin \tau + y_{02} \cos \tau, x_{02} \cos \tau - y_{02} \sin \tau, p_2] = [p_1 \theta \sin \Sigma - y_1 \cos \Sigma, p_2 \sin \Sigma + (x_1 - C) \cos \Sigma, p_2 \cos \Sigma - (x_1 - C) \sin \Sigma]$$

#### Meshing condition (Envelope conjugacy condition)

$$\left( \frac{\partial \mathbf{r}_1}{\partial t} \times \frac{\partial \mathbf{r}_1}{\partial \theta} \right) \cdot \frac{\partial \mathbf{r}_1}{\partial \tau} = - \left( \frac{\partial \mathbf{r}_1}{\partial t} \times \frac{\partial \mathbf{r}_1}{\partial \theta} \right) \cdot \frac{\partial \mathbf{r}_2}{\partial \tau} = 0$$

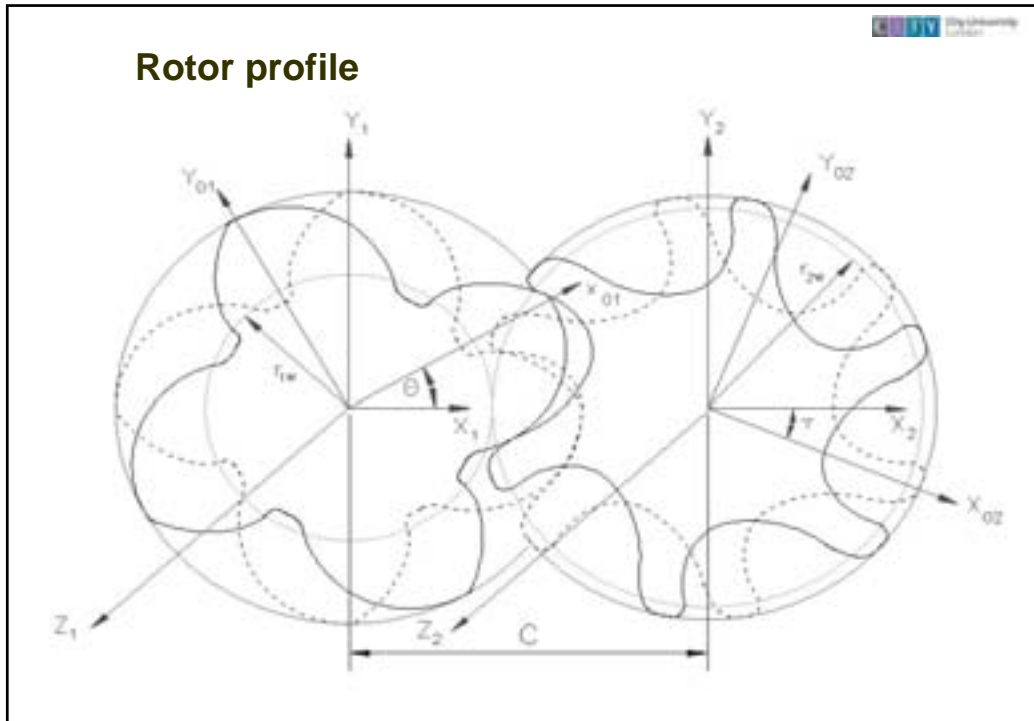
$$[C - x_1 + (p_1 - p_2) \cot \Sigma] \left( x_1 \frac{\partial x_1}{\partial t} + y_1 \frac{\partial y_1}{\partial t} \right) + p_1 \left[ p_1 \theta \frac{\partial y_1}{\partial t} + (p_2 - C \cot \Sigma) \frac{\partial x_1}{\partial t} \right] = 0$$


### General case:

- non-parallel and non-intersecting axes  
rotor – hobbing tool relation

### Special cases:

- $p_2=0$   
rotor - plate milling tool, grinding tool relation
- $\Sigma=0$   
screw compressor rotors





**Given rotor profile**  $\Sigma = 0, \quad i = \frac{p_2}{p_1}, \quad k = 1 - \frac{1}{i}$

**Meshing condition**  $\frac{dy_{01}}{dx_{01}} \left( ky_{01} - \frac{C}{i} \sin \theta \right) + kx_{01} + \frac{C}{i} \cos \theta = 0$

**Meshed profile**

$$x_{02} = x_{01} \cos k\theta - y_{01} \sin k\theta - C \cos \frac{\theta}{i}$$


$$y_{02} = x_{01} \sin k\theta + y_{01} \cos k\theta + C \sin \frac{\theta}{i}$$

**Rack profile**

$$x_{0r} = x_{01} \cos \theta - y_{01} \sin \theta$$

$$y_{0r} = x_{01} \sin \theta + y_{01} \cos \theta - r_1 \theta$$






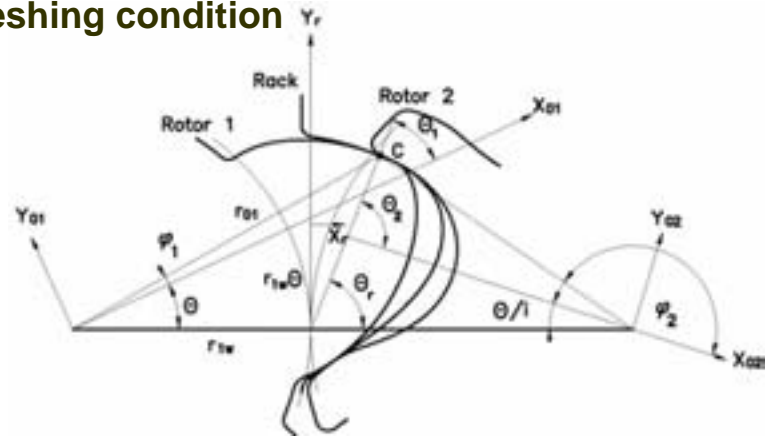
**Given rack profile**       $\Sigma = 0, \quad i = \frac{p_r}{p_1} = \infty, \quad k = 1 - \frac{1}{\infty} = 1$

**Meshing condition**       $\frac{dy_{0r}}{dx_{0r}}(r_{1w}\theta - y_{0r}) - (r_{1w} - x_{0r}) = 0$

**Meshed rotor profile**       $x_{01} = x_{0r} \cos \theta - (y_{0r} - r_{1w}) \sin \theta$   
     $y_{01} = x_{0r} \sin \theta + (y_{0r} - r_{1w}) \cos \theta$




**Willis meshing condition**



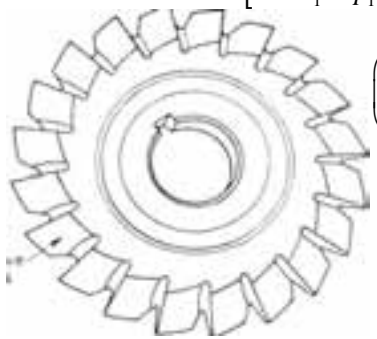
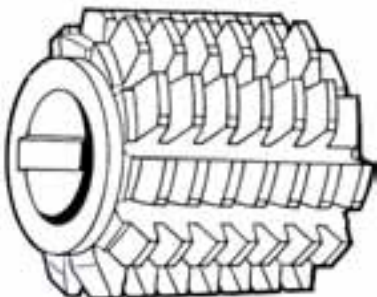
$$\frac{dy_{01}}{dx_{01}} \left( ky_{01} - \frac{C}{i} \sin \theta \right) + kx_{01} + \frac{C}{i} \cos \theta = 0$$

$$\frac{\sin(\theta_1 + \theta)}{r_{01}} = \frac{\sin(\theta_1 - \varphi_1)}{r_{1w}}$$




### Profile of a plate milling tool/grinding tool

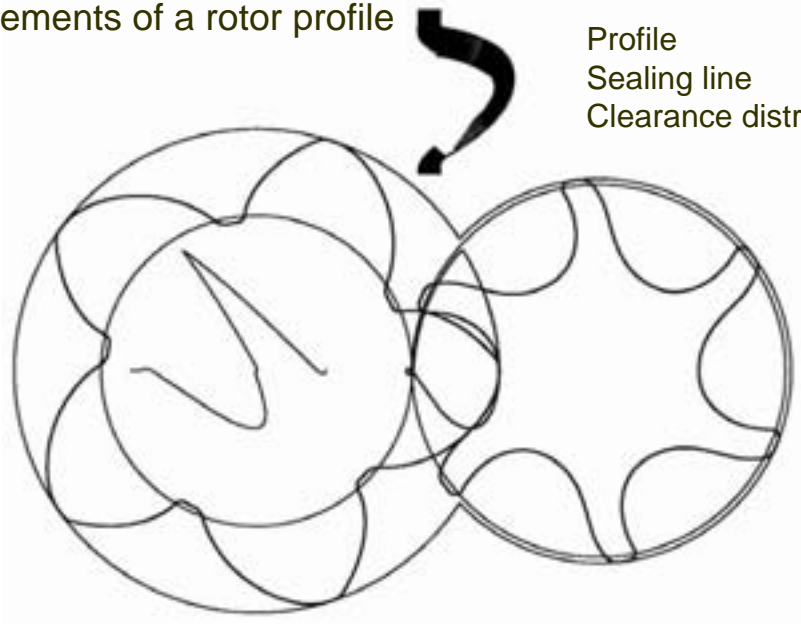
$$[C - x_1 + p_1 \cot \Sigma] \left( x_1 \frac{\partial x_1}{\partial t} + y_1 \frac{\partial y_1}{\partial t} \right) + p_1 \left[ p_1 \theta \frac{\partial y_1}{\partial t} - C \cot \Sigma \frac{\partial x_1}{\partial t} \right] = 0$$

$$\left( R_2 + z_2 \frac{dz_2}{dR_2} \right) \cos \tau + (p_1 + C \cot \Sigma) \frac{dz_2}{dR_2} \sin \tau + p_1 \cot \Sigma - C = 0$$



**Profile of a hobbing tool**  
 Full meshing conditions for:  
 Non-parallel and non-intersecting axes



### Elements of a rotor profile



**Profile**  
**Sealing line**  
**Clearance distribution**

### Rotor parameters

$$C = r_{1w} + r_w$$

$$i = \frac{r_{2w}}{r_{1w}} = \frac{z_2}{z_1}$$

$$\frac{tg\psi}{tg\psi_w} = \frac{r}{r_w}$$

$$p = \frac{h}{2\pi}$$

$$\frac{L}{\varphi} = \frac{h}{2\pi} = p$$

$$tg\psi_w = \frac{\varphi r_w}{h} = \frac{2\pi r_w}{L}$$

### Demonstrator profile

- Rotor generated “N” profile (not patented “N” profile)
- A primary lobe is provided on the male rotor
- Profile consist of series of circles on the main and the gate rotors
- Readily available to try


### Demonstrator profile

- Segment  $A_1B_1$  is a circle of the radius  $r_1$  on the main rotor  
Angular parameter  $t$  varies between  $-\theta_1 < t < 0$
- Segment  $B_1C_1$  is a circle of the radius  $r_3$  on the main rotor,  
Angular parameter  $t$  varies between  $\pi - \psi_1 < t < \pi - \theta_1$
- Segment  $C_1D_1$  is a trochoid on the main rotor generated from the circle of radius  $r_4$  on the gate rotor  
Angular parameter:  $-\theta_4 - \tau_1 < t < -\pi - \tau_1$
- Segment  $A_1D_1$  is a circle of radius  $r_2$  on the main rotor,  
Angular parameter:  $0 < t < \theta_2$ .

### 'N' ROTOR PROFILE

- Rack generation procedure
- Straight line on the rack (involute rotor contact)
- Small torque transmitted
- Large displacement
- Strong gate rotor
- Short sealing line
- Clearance distribution

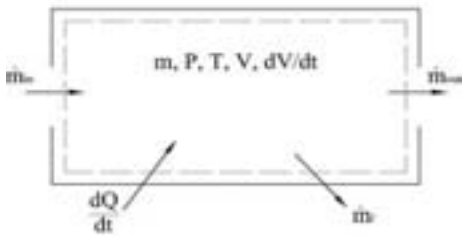
E-F Circle  
 F-G Straight Line  
 G-H Undercut by the Gate Rotor  
 H-A Undercut by the Main Rotor  
 A-B Arc:  $p=0.43, q=1$   
 B-C Straight Line  
 C-D Circle  
 D-E Straight Line




## Thermodynamic calculations

**1-D “chamber” model:**

- **Differential approach:** conservation of continuity, momentum and energy
- **Working fluid:** Ideal or real
- **Heat transfer** between the fluid and solid compressor parts
- **Liquid injection** modelled
- Model is independent on a compressor geometry



- **Limitations:**
  - Fluid flow is assumed “quasi” one-dimensional
  - Kinetic energy neglected in comparison with internal energy
  - The flow through inlet and outlet chambers assumed to be isentropic
  - The flow through clearances assumed to be adiabatic



## Thermodynamic calculations (2)

- **Internal energy**


$$\omega \left( \frac{dU}{d\theta} \right) = \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out} + Q - \omega p \frac{dV}{d\theta}$$

$$\dot{m}_{in} h_{in} = \dot{m}_{suc} h_{suc} + \dot{m}_{l,g} h_{l,g} + \dot{m}_{oil} h_{oil} \quad \dot{m}_{out} h_{out} = \dot{m}_{dis} h_{dis} + \dot{m}_{l,l} h_{l,l}$$
- **Continuity**

$$\omega \frac{d\dot{m}}{d\theta} = \dot{m}_{in} - \dot{m}_{out}$$

$$\dot{m}_{in} = \dot{m}_{suc} + \dot{m}_{l,g} + \dot{m}_{oil} \quad \dot{m}_{out} = \dot{m}_{dis} + \dot{m}_{l,l} \quad \dot{m} = \rho w A$$
- **Leakage flow**

$$\dot{m}_l = \rho_l w_l A_g = \sqrt{\frac{p_2^2 - p_1^2}{a^2 \left( \zeta + 2 \ln \frac{p_2}{p_1} \right)}} w_l dw_l + \frac{dp}{\rho} + f \frac{w_l^2}{2} \frac{dx}{D_g} = 0$$



### Thermodynamic calculations (3)

- **Liquid injection**      $\dot{m}_{oil} = \frac{\dot{m}_{oil}}{\dot{m}_{gas}} \dot{m} \frac{z_1}{2\pi} \quad \frac{dT_o}{d\theta} = \frac{h_o A_o (T_{gas} - T_o)}{\omega m_o c_{oil}}$

$Nu = 2 + 0.6 Re^{0.6} Pr^{0.33}$

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- **Ideal fluid**

$U = (mu)_{gas} + (mu)_{oil} = \frac{mRT_{gas}}{\gamma - 1} + (mcT)_{oil} = \frac{pV}{\gamma - 1} + (mcT)_{oil}$

$u = f(t), \quad \rho = f(p, t), \quad u \neq f(p) \quad T = (\gamma - 1) \frac{(1+k)U - (mcT)_{oil}}{(1+k)mR + (mc)_{oil}}$

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- **Real fluid**      $u = f(t, p), \quad \rho = f(p, t) \quad \text{Iterative}$
- **Wet vapour**      $u = (1-x)u_f + xu_g \quad v = (1-x)v_f + xv_g$



### Compressor integral parameters

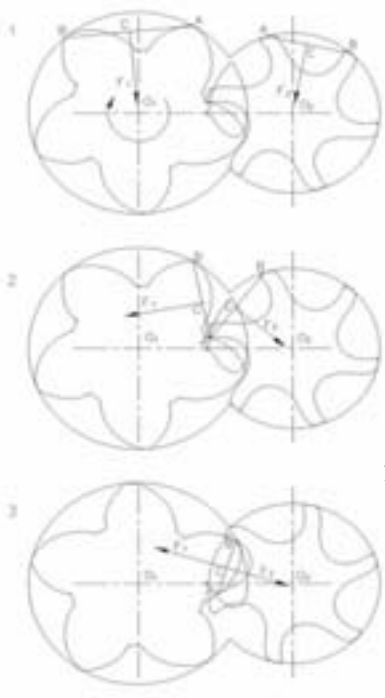
$$m = m_{in} - m_{out} \quad W_{ind} = \int_{cycle} V dp \quad W_{sind} = \int_{cycle} \frac{V}{m} dp$$

$$\dot{m} = m z_1 n / 60 \quad P_{ind} = \frac{W_{ind} z_1 n}{60} \quad P_{sind} = \frac{P}{\dot{V}}$$

$$\dot{V} = 60m / \rho_0 \quad W_t = RT_1 \ln \frac{p_2}{p_1}$$

$$\dot{m}_t = \frac{(F_{1n} + F_{2n}) L n z_1 \rho}{60} \quad W_a = \frac{\gamma}{\gamma - 1} R (T_2 - T_1)$$

$$\eta_v = \dot{m} / \dot{m}_t \quad \eta_t = \frac{W_t}{W_{ind}} \quad \eta_a = \frac{W_a}{W_{ind}}$$



**Pressure loads**

**Radial forces**

$$R_x = -p \int_A^B dy = -p(y_B - y_A)$$

$$R_y = -p \int_A^B dx = -p(x_B - x_A)$$

**Rotor torque**

$$T = p \int_A^B x dx + p \int_A^B y dy - 0.5p(x_B^2 - x_A^2 + y_B^2 - y_A^2)$$

**Rotor deflections**  $\frac{d^2 \delta}{dz^2} = \frac{M}{EI}$



**SCORPATH MMI**

- **METHODS:** Envelope meshing, 1D Thermodynamics, Multivariable optimisation
- **TOOL:** SCORPATH

- Profile Generation
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Screw Compressor Optimal Rotor Profiling and Thermodynamics

SCORPATH 2005  
Profile Calculation Results Utilities: Quit  
Generation

General envelope meshing condition for crossed helical gears used to generate profile points on both rotors from predefined points on either rack or any rotor

1D - Chamber model - Numerically solves conservation equations of mass and energy, equation for instantaneous volume and equations that describe leakage flow, heat transfer, inlet and outlet flows, ...

Integration of resulting forces and calculation of bearing loads

Export shapes of ports, rotors ... for further use with CAD systems

WINDY SYSTEMS Ltd  
Water and Compressed Gas  
Compressor Thermodynamic  
Water and Compressed Gas  
Real Property Rooting  
INTERACTIVE SESSION (TYPE IN Y OR N)  
OPTIMIZATION UNDESIRABLE  
Y/N - OPTIMIZATION TAKES PLACE  
OTHER VALUE - FIXED  
TO ACCEPT, PRESS ENTER, OR TYPE IN NEW  
R1: MAIN ROTOR RADIIUS (M) 0.000000  
R2: MAIN ROTOR TIP RADIIUS (M) 0.000000  
R3: GATE ROTOR TIP RADIIUS (M) 0.000000  
SPEEDS BUILT-IN VOLUME RATIO 0.000000  
MTP: TIP SPEED (m/s) 0.000000  
MOP: OIL-GAS FLOW RATIO 0.000000  
M C200: OIL FREE COMPRESSOR: NO OIL INJECTED 0.000000  
FIDEL: INJECTION POSITION (DEG) 0.000000  
TOLL: INJECTION OIL TEMPERATURE (DEG C) 0.000000  
THE FIRST NUMBER BELOW LARGER THAN 1.00 MEANS THAT THE OPTIMIZATION HAS DIVULGED. CTRL-C AND TRY ANOTHER INPUT  
Step - Program terminated.  
TIME 1 1.5 27.0 4.8 3.2 2.000 70.000 0.000 1 64.705 310.0 1700.2  
TIME - 86.1 1.0 333.7

