OPTIMIZATION OF SCREW COMPRESSOR DESIGN

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SYNOPSIS

Ever increasing demands for efficient screw compressors require that compressor designs are tailored upon their duty, capacity and manufacturing capability. A suitable procedure for optimisation of screw compressor shape, dimension and operating parameters is described here, which results in the most appropriate design for a given compressor duty. It is based on a rack generation algorithm for rotor profile combined with a numerical model of the compressor fluid flow and thermodynamic processes. Compressors thus designed achieve higher delivery rates and better efficiencies than those using traditional approaches. Some optimization issues of the rotor profile and compressor parts are discussed, using a 5/6-106 mm screw compressor to illustrate the results. It is shown that the optimum rotor profile, compressor speed, oil flow rate and temperature may significantly differ when compressing different gases or vapours.

Key Words: Screw compressor design, rotor lobe optimal profiling, numerical modelling

1. INTRODUCTION

The screw compressor is a positive displacement rotary machine. It consists essentially of a pair of meshing helical lobed rotors, which rotate within a fixed casing that totally encloses them, as shown in Fig 1. The space between any two successive lobes of each rotor and its surrounding casing forms a separate working chamber of fixed cross sectional area. The length of this chamber varies as rotation proceeds due to displacement of the line of contact between the two rotors. It is a maximum when the entire length between the lobes is unobstructed by meshing contact with the other rotor. It has a minimum value of zero when there is full meshing contact with the second rotor at the end face. The two meshing rotors effectively form a pair of helical gear wheels with their lobes acting as teeth.

As shown in right top side of the figure, gas or vapour enters from the front and on top, through an opening, mainly in the front plane of the casing which forms the low pressure or inlet port. It thus fills the spaces between the lobes, starting from the ends corresponding to A and C in the lightly shaded area. As may be seen, the trapped volume in each chamber increases as rotation proceeds and the contact line between the rotors recedes. At the point where the maximum volume is filled, the inlet port terminates and rotation proceeds without any further fluid admission in the region corresponding to the darkly shaded area.

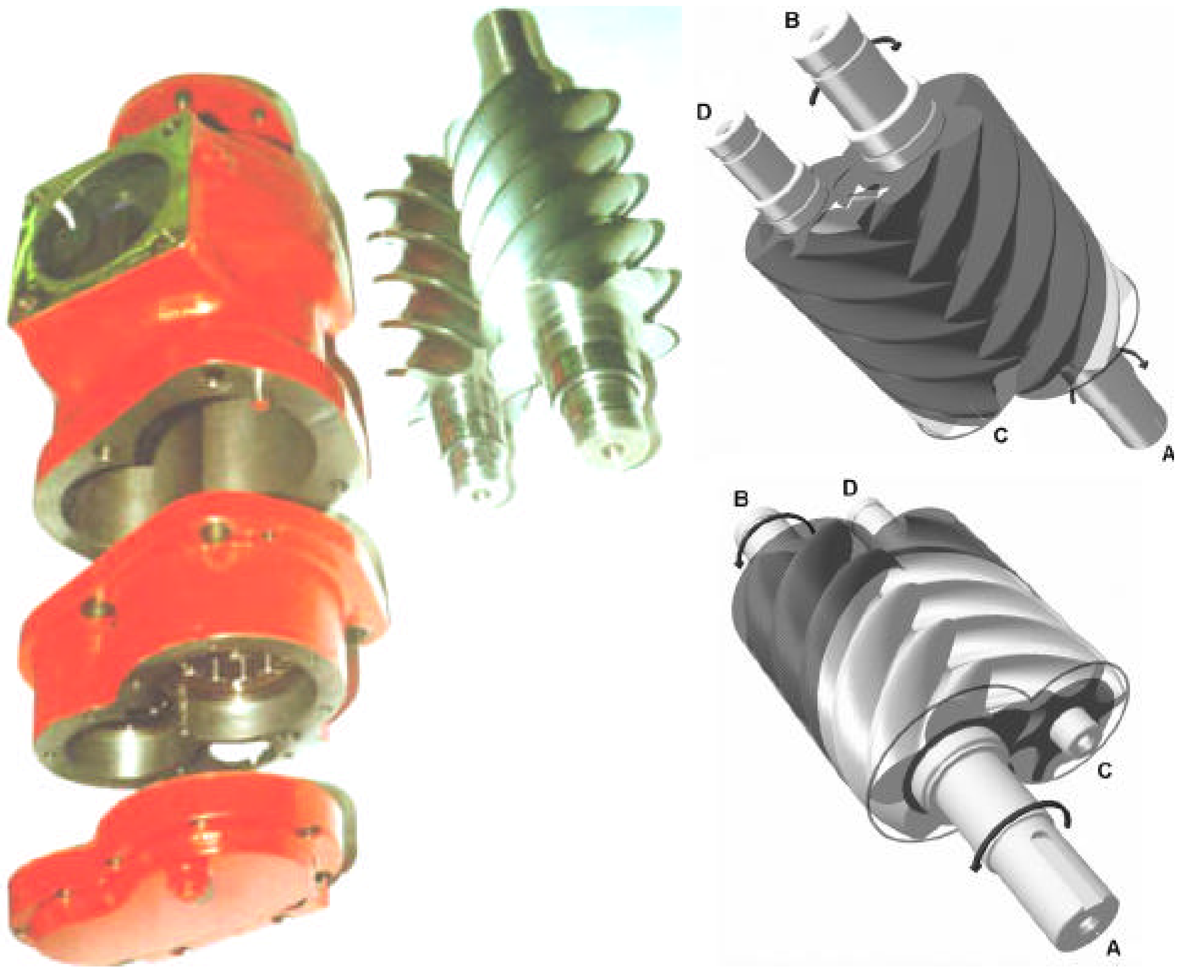

Fig. 1 Screw compressor principal mechanical parts

Viewed from the bottom, it may be seen that the darkly shaded area begins, from the end corresponding to A and C, at the point where the male and female rotor lobes start to reengage on the underside. Thus, from that position, further rotation reduces the volume of gas or vapour trapped between the lobes and the casing. This causes the pressure to rise. At the position where the trapped volume is sufficiently reduced to achieve the required pressure rise, the ends of the rotors corresponding to D and B are exposed to an opening on the underside of the casing, which forms the high pressure or discharge port. This corresponds to the lightly shaded area at the rear end. Further rotation reduces the trapped volume causing the fluid to flow out through the high pressure port at approximately constant pressure. This continues until the trapped volume is reduced to virtually zero and all the gas trapped between the lobes at the end of the suction process, is expelled. The process is then repeated for each chamber. Thus there is a succession of suction, compression and discharge processes achieved in each rotation, dependent on the number of lobes in the male and female rotors. If the direction of rotation of the rotors is reversed, fluid will flow in to the machine at the high pressure end and out at the low pressure end and it will act as an expander.

Screw compressor rotors of various profiles can be flexibly manufactured with small clearances at an economic cost. Internal leakages have been reduced to a small fraction of their values in earlier designs. Screw compressors are therefore efficient, compact, simple and reliable. Consequently, they have largely replaced reciprocating machines in industrial applications and in refrigeration systems.

Recent advances in mathematical modelling and computer simulation may be used to form a powerful tool for the screw compressor process analysis and design optimisation. Such models have evolved greatly during the past ten years and, as they are better validated, their value as a design tool has increased. Their use has led to a steady evolution in screw rotor profiles and compressor shapes which should continue in future to lead to further improvements in machine performance. Evidence of this may be seen in the publications by *Sauls, 1994* and *Fujiwara and Osada, 1995*. In order to make such computer models more readily accessible to designers and engineers, as well as specialists, the authors have developed a suite of subroutines for the purpose of screw machine design, *Hanjalic and Stosic, 1997*.

There are several criteria for screw profile optimization which are valid irrespective of the machine type and duty. Thus, an efficient screw machine must admit the highest possible fluid flow rates for a given machine rotor size and speed. This implies that the fluid flow cross-sectional area must be as large as possible. In addition, the maximum delivery per unit size or weight of the machine must be accompanied by minimum power utilization for a compressor and maximum power output for an expander. This implies that the efficiency of the energy interchange between the fluid and the machine is a maximum. Accordingly unavoidable losses such as fluid leakage and energy losses must be kept to a minimum. However, increased leakage may be more than compensated by greater bulk fluid flow rates. However, specification of the required compressor delivery rate requires simultaneous optimisation of the rotor size and speed to minimise the compressor weight while maximising its efficiency. Finally, for oil-flooded compressors, the oil injection flow rate, inlet temperature and position needs to be optimised. It follows that a multivariable minimization procedure is needed for screw compressor design with the optimum function criterion comprising a weighted balance between compressor size and efficiency or specific power.

A box simplex method was used here to find the local minima, which were input to an expanding compressor database. This finally served to estimate a global minimum. The database may be used later in conjunction with other results to accelerate the minimization.

2. GEOMETRY OF SCREW COMPRESSOR ROTORS

Screw machine rotors have parallel axes and a uniform lead and they are a form of helical gears. The rotors make line contact and the meshing criterion in the transverse plane perpendicular to their axes is the same as that of spur gears. A procedure to get the required meshing condition as described in *Stosic, 1998*. More detailed information on the envelope method applied to gears can be found in *Litvin, 1994*.

To start the procedure of rotor profiling, the profile point coordinates in the transverse plane of one rotor, and their first derivatives, must be known. This profile can be specified on either the main or gate rotors or in sequence on both. Also the primary profile may also be defined as a rack as shown in Fig 2.

A helicoid surface and its derivatives for the given rotor profile can be found from the transverse plane rotor coordinates,. The envelope meshing condition for screw machine rotors gives the meshing condition either numerically, if the generating curves are given on the compressor rotors, or directly, if the curves are given on the rotor rack. This enables a variety of primary arc curves to be used and basically offers a general procedure. Moreover, numerical derivation of the primary arcs permits such an approach even when only the coordinates of the primary curves are known, without their derivatives.

The following are the elements of the rack-generated 'N' profile. The primary curves are specified on the rack: D-C is a circle with radius r_3 on the rack, C-B is a straight line, B-A is a parabola constrained by radius r_1, A-H-G are trochoids on the rack generated by the small circles of radii r_2 and r_4 from the main and gate rotors respectively, G-E is a straight line and E-F and E-D are circles on the rack. A full description of the rack generation procedure and rotor geometry is given in *Stosic* and *Hanjalic, 1997*. Three rotor radii, r_1-r_3 and the gate rotor addendum r_0 are used as variables for the rotor optimisation.

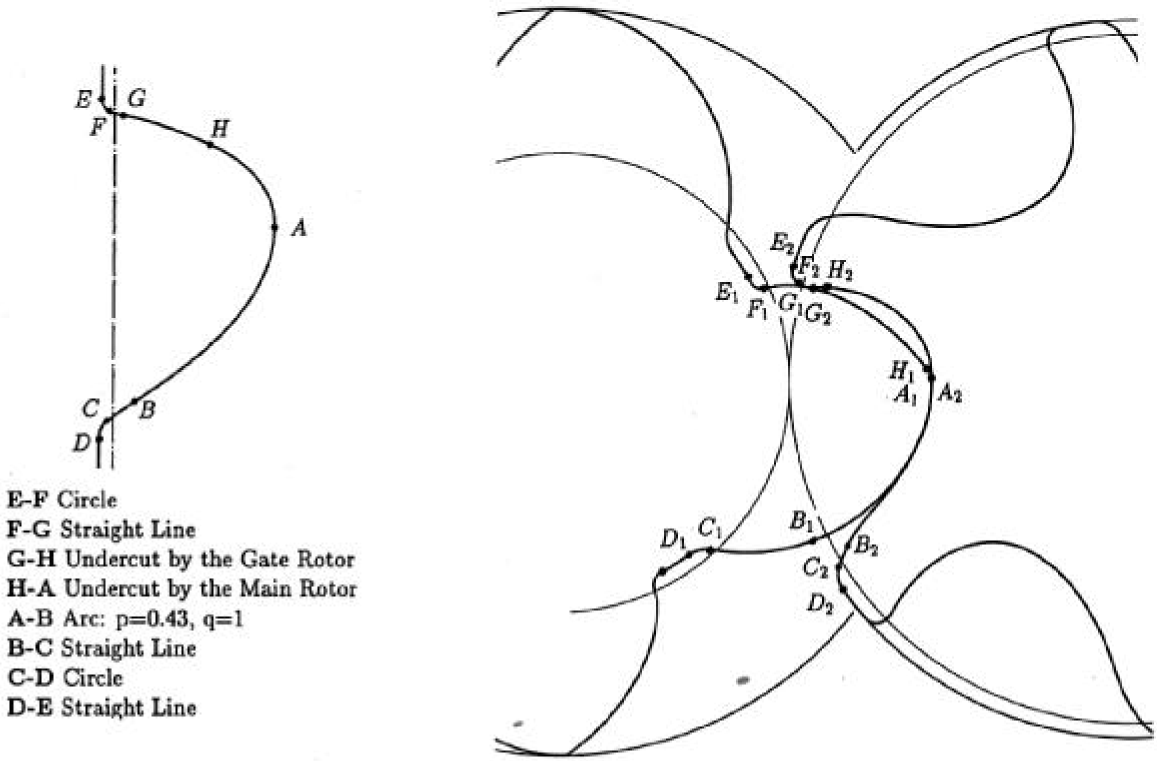

Fig 2. Distribution of generating profile curves on the rack for 'N' rotors

Full rotor and compressor geometry, like the rotor throughput cross section, rotor displacement, sealing lines and leakage flow cross section, as well as suction and discharge port coordinates are calculated from the rotor transverse plane coordinates and rotor length and lead. They are later used as input parameters for calculation of the screw compressor thermodynamic process. For any variation of input parameters r_0 to r_3, the primary arcs must be recalculated and a full transformation performed to obtain the current rotor and compressor geometry. The compressor built-in volume ratio is also used as an optimisation variable.

3. COMPRESSOR THERMODYNAMICS IN OPTIMISATION CALCULATIONS

The algorithm of the thermodynamic and flow processes used is based on a mathematical model comprising a set of equations which describe the physics of all the processes within the screw compressor. The mathematical model describes an instantaneous operating volume, which changes with rotation angle or time, together with the equations of conservation of mass and energy flow through it, and a number of algebraic equations defining phenomena associated with the flow. These are applied to each process that the fluid is subjected to within the machine; namely, suction, compression and discharge. The set of differential equations thus derived cannot be solved analytically in closed form. In the past, various simplifications have been made to the equations in order to expedite their numerical solution. The present model is more comprehensive and it is possible to observe the consequences of neglecting some of the terms in the equations and to determine the validity of such assumptions. This provision gives more generality to the model and makes it suitable for other applications.

A feature of the model is the use energy equation in the form which results in internal energy rather than enthalpy as the derived variable. This was found to be computationally more convenient, especially when evaluating the properties of real fluids because their temperature and pressure calculation is not explicit. However, since the internal energy can be expressed as a function of the temperature and specific volume only, pressure can be calculated subsequently directly. All the remaining thermodynamic and fluid properties within the machine cycle are derived from the internal energy and the volume and the computation is carried out through several cycles until the solution converges. A full description of the model is given in *Hanjalic and Stosic, 1997*.

The following forms of the conservation equations have been employed in the model. The conservation of internal energy is:

$$w\left(\frac{dU}{dq}\right)=\dot{m}_{in}h_{in}-\dot{m}_{out}h_{out}+Q-w\,p\frac{dV}{dq}$$

where θ is angle of rotation of the main rotor, h=h(θ) is specific enthalpy, $\dot{m}=\dot{m}(q)$ is mass flow rate p=p(θ), fluid pressure in the working chamber control volume, $\dot{Q}=\dot{Q}(q)$, heat transfer between the fluid and the compressor surrounding, $\dot{V}=\dot{V}(q)$ local volume of the compressor working chamber. In the above equation the index in denotes inflow and the index out the fluid outflow.

The mass continuity equation is:

$$w\frac{dm}{dq} = \dot{m}_{in} - \dot{m}_{out}$$

The instantaneous density ρ=ρ(θ) is obtained from the instantaneous mass m trapped in the control volume and the size of the corresponding instantaneous volume V as ρ=m/V.

The suction and discharge port flow is defined by the velocity through them and their cross section area. The cross-section area A is obtained from the compressor geometry and it was considered as a periodical function of the angle of rotation θ.

Leakage in a screw machine forms a substantial part of the total flow rate and plays an important role because it affects the delivered mass flow rate and hence both the compressor volumetric and adiabatic efficiencies.

$$\dot{m}_l = r_l w_l A_g = \sqrt{\frac{p_2^2 - p_1^2}{a^2\left(z + 2\ln\frac{p_2}{p_1}\right)}}$$

Injection of oil or other liquids for lubrication, cooling or sealing purposes, modifies the thermodynamic process in a screw compressor substantially. Special effects, such as gas or its condensate mixing and dissolving in or coming out of the injected fluid should be accounted for separately if they are expected to affect the process. In addition to lubrication, the major purpose for injecting oil into a compressor is to cool the gas.

Flow of the injected oil, oil inlet temperature and injection position are additional optimisation variables if the oil-flooded compressors are in question.

The solution of the equation set in the form of internal energy U and mass m is performed numerically by means of the Runge-Kutta 4th order method, with appropriate initial and boundary conditions. As the initial conditions were arbitrary selected, the convergence of the solution is achieved after the difference between two consecutive compressor cycles is sufficiently small.

Numerical solution of the mathematical model of the physical process in the compressor provides a basis for a more exact computation of all desired integral characteristics with a satisfactory degree of accuracy. The most important of these properties are the compressor mass flow rate $\dot{m}$ [kg/s], the indicated power P_{ind} [kW], specific indicated power P_s [kJ/kg], volumetric efficiency η_v, adiabatic efficiency η_a, isothermal efficiency η_t and other efficiencies, and the power utilization coefficient, indicated efficiency η_i.

4. OPTIMIZATION OF THE ROTOR PROFILE AND COMPRESSOR DESIGN

The power and capacity of contemporary computers is only just sufficient to enable a full multivariable optimisation of both the rotor profile and the whole compressor design to be performed simultaneously in one pass. Nine optimization variables were used in the calculation presented, radii r_0, r_1, r_2, and r_3 were four rotor profile parameters, built–in volume ratio is another compressor geometry variable, compressor speed is an operating variable and oil flow, temperature and injection position are oil optimisation parameters.

A box constrained simplex method was used here to find the local minima. The box method stochastically selects a simplex, which is a matrix of independent variables and calculates the optimisation target. This is later compared with those of previous calculations and then their minimization is performed. One or more optimisation variables may be limited by the calculation results in the constrained Box method. This gives additional flexibility to the compressor optimisation.

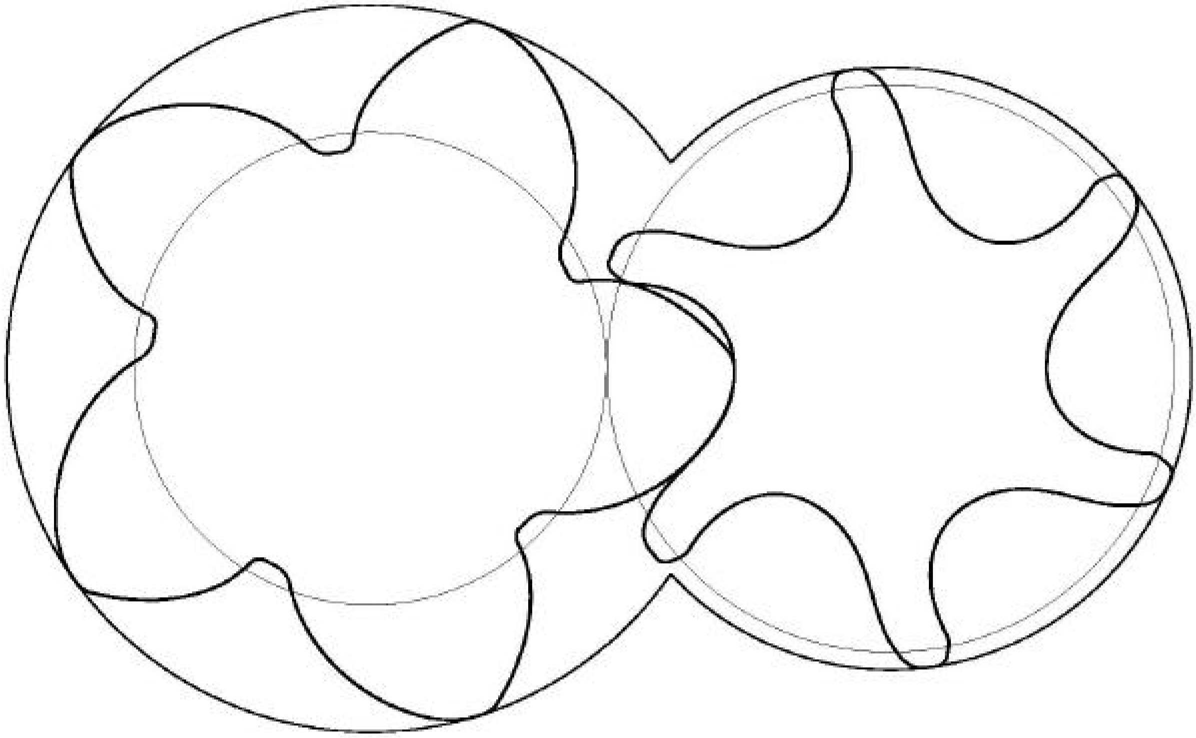

Fig. 3 Rotor profile optimized for an oil-free air compressor duty

The optimisation results, after being input to an expandable compressor database, finally served to estimate a global minimum. The database may be used later in conjunction with other results to accelerate the minimization.

The suction and discharge pressures were 1 – 3 bar for the dry air compressor and 1 - 8 bar for the oil flooded compressor, while the evaporation and condensation temperatures were 5 and 40 ^{o}C for R-134A. The centre distance and male rotor outer diameters were kept constant for all compressors, 90 and 128.450 mm respectively.

The optimisation criterion was the lowest compressor specific power. As a result, three distinctively different rotor profiles were calculated, one for oil-free compression and the other two for oil-flooded air and refrigeration compression. They are presented in Figs. 4-6.

Although the profiles somewhat look alike, there is a substantial difference between their geometry which is given in the following table as well as further results of the compressor optimisation.

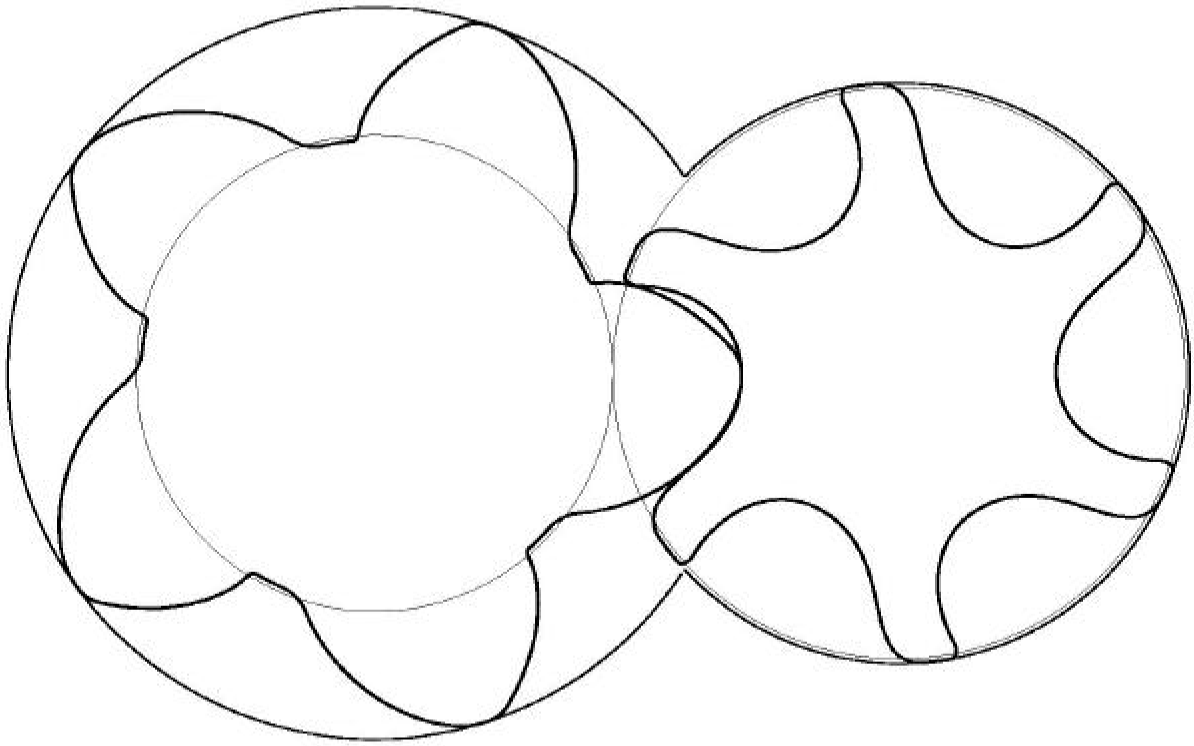

Fig. 4 Rotor profile optimized for an oil-flooded air compressor

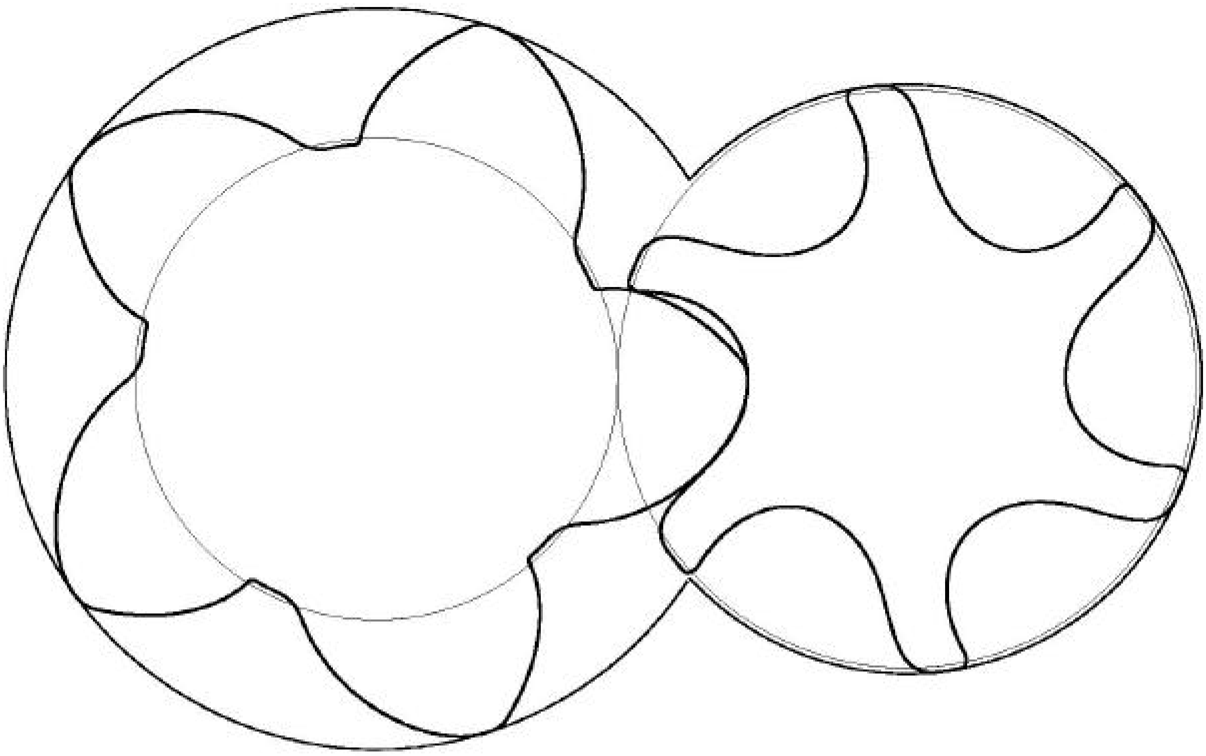

Fig. 5 Rotor profile optimized for a refrigeration compressor duty

Table 1: Results of optimisation calculations for dry and oil flooded air compressors and oil flooded refrigeration compressor

	DryAir	Oil-Flooded Air	Refrigeration
r_0 [mm]	2.62	0.74	0.83
r_1 [mm]	19.9	17.8	19.3
r_2 [mm]	6.9	5.3	4.5
r_3 [mm]	11.2	5.5	5.2
Built-in volume ratio	1.83	4.1	3.7
Rotor speed [rpm]	7560	3690	3570
Oil flow [lit/min]	-	12	8
Injection position [^{o}]	-	65	61
Oil temperature [^{o}]	-	33	32

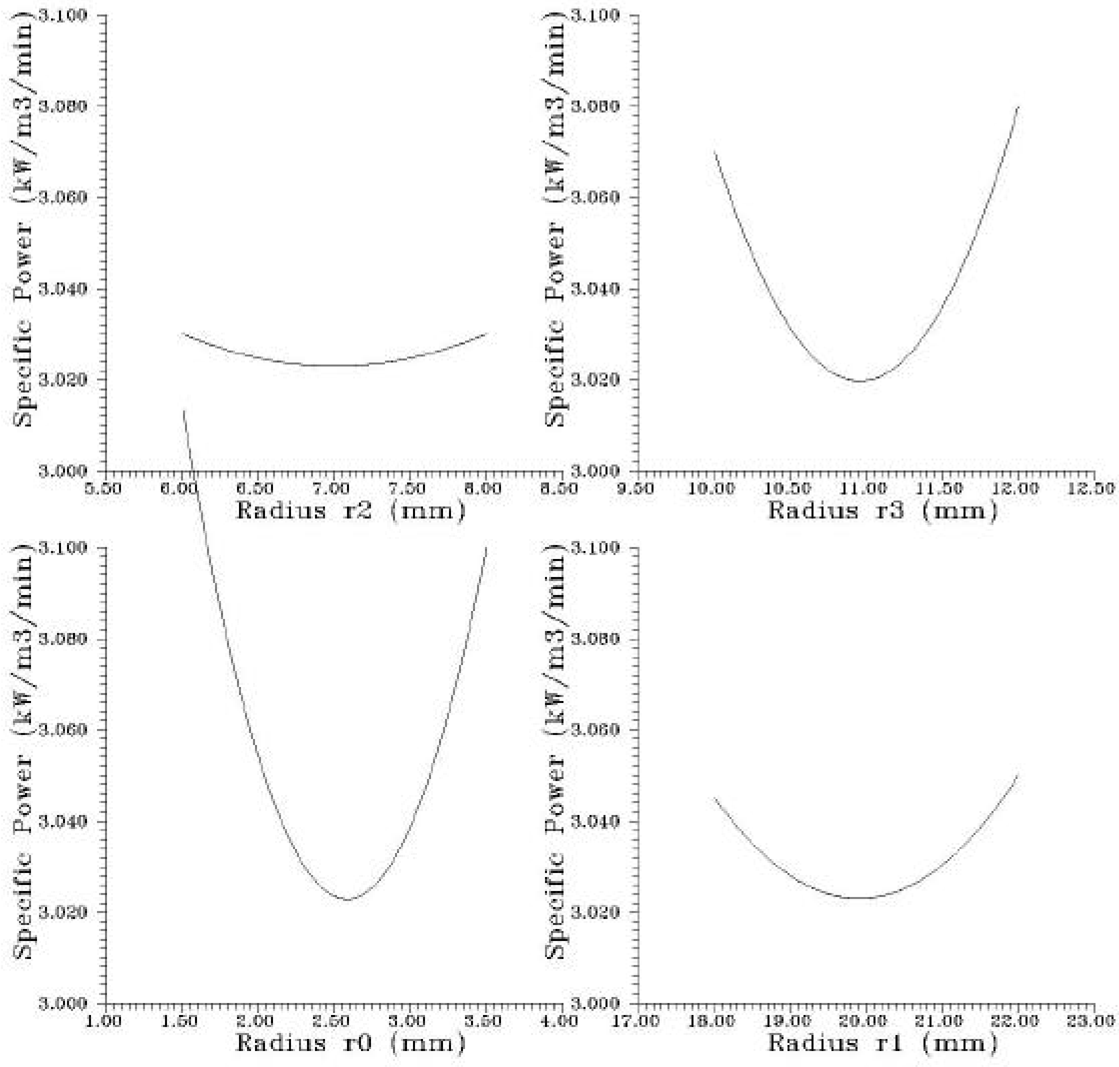

Fig. 6 Variation of the compressor specific power as function of the compressor rotor parameters

As an example, if the gate rotor addendum is analysed in detail, it can be concluded that, the size of the rotor blow-hole area is proportional to the addendum. Therefore r_0 should be made as small as possible in order to minimise the blow-hole. It would therefore appear that ideally, r_0 should be equal to zero or even be 'negative'. However, reduction in r_0 also leads to a decrease the fluid flow cross-sectional area and hence a reduction in the flow rate and the volumetric efficiency. It follows that there is a lower limit to the value of r_0 to obtain the best result. More details of single variable optimisation of screw compressor rotors can be found in *Hanjalic and Stosic, 1994.*

As in the case of any result of multivariable optimization, the calculated screw compressor profile and compressor design parameters must be considered with the extreme caution. This is because multivariable optimisation usually finds only local minima, which may not necessarily be globally the best optimisation result. Therefore, extensive calculations should be carried out before a final decision on the compressor design is made.

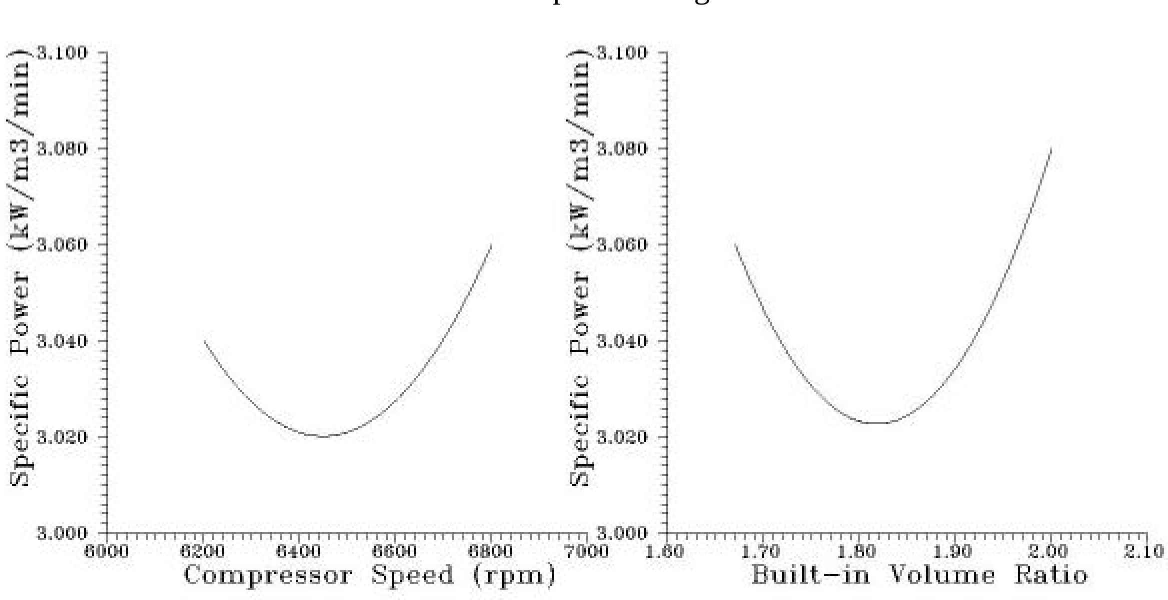

Fig. 7 Specific power as function of the compressor built-in volume and speed

The dry air compressor was chosen for further analysis. This is because the compression process within it is close to that of an ideal gas compressed adiabatically in which γ, the isentropic exponent, has the relatively large value of approximately 1.4. As an example of how the optimisation variables influence the compressor specific power, the radii r_0-r_3 are considered. The influence of the gate rotor tip addendum r_0, and the gate rotor radius r_3 are presented in Fig. 6, as well as the main rotor radii r_1 and r_2. In Fig. 7, the influence of the compressor built-in volume ratio, as well as compressor speed is presented.

5. CONCLUSIONS

A full multivariable optimisation of screw compressor geometry and operating conditions has been performed to establish the most efficient compressor design for any given duty. This has been achieved with a computer package, developed by the authors, which provides the general specification of the lobe segments in terms of several key parameters and which can generate various lobe shapes. Computation of the instantaneous cross-sectional area and working volume could thereby be calculated repetitively in terms of the rotation angle. A mathematical model of the thermodynamic and fluid flow process is contained in the package, as well as models of associated processes encountered in real machines, such as variable fluid leakages, oil flooding or other fluid injection, heat losses to the surrounding, friction losses and other effects. All these are expressed in differential form in terms of an increment of the rotation angle. Numerical solution of these equations enables the screw compressor flow, power and specific power and compressor efficiencies to be calculated.

A rack generated profile in 5/6 configuration rotors of 106 mm was used as an example to show how optimisation may permit both better delivery and higher efficiency for the same tip speed. Several rotor geometrical parameters, namely the main and gate tip radii, as well as the compressor built-in ratio and compressor speed and oil flow and temperature and injection position are used as optimisation variables and applied to the multivariable optimisation of the machine geometry and its working parameters for a defined optimisation target. In the case of the example given, this was minimum compressor specific power. It has thereby been shown that for each application, a different rotor design is required to achieve optimum performance.

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