Identification of Constraints in the Optimal Generation of Screw Compressor Rotors by the Pressure Angle Method

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ABSTRACT

The profile gradient method has been recently introduced as a means of generating screw compressor rotor profiles. As a single parameter method, this procedure is convenient for the optimisation of screw compressor rotors and evaluating their quality. In this paper, the procedure is modified to adopt the pressure angle instead the profile gradient and applied to a screw compressor rotor rack. Optimisation constraints are analysed and an example of optimal profile generation is presented.

NOMENCLATURE

C	Rotor centre distance
c	Parameter in Box optimisation method
k	Constant in Box optimisation method
n	Constant in Box optimisation method
R	Random number
r	Radius
x	Coordinate, Box variable
y	Coordinate
α	Line slope angle, Box reflection constant
φ	Profile parameter, pressure angle
θ	Meshing Angle

Subscripts

1	Main rotor
2	Gate rotor
i	Index
l	Index
r	Rack, index
w	Pitch circle

Superscripts

g	Explicit constraints
h	Implicit constraints

1 INTRODUCTION

New procedures for screw compressor rotor profile generation are continually being published. One of the more recent is to use the profile gradient, which was introduced to the screw compressor rotor profiling by Rinder and Grafinger, 2002, both to generate a profile and to evaluate it. As a single parameter procedure, this method is convenient for the optimisation of screw compressor rotors, if used together with the constraints required to match the rotor geometry to its functional requirements. The pressure angle, which is the angle of the profile normal and which is the arc tangent of the profile gradient dx/dy, is used in this paper for the same purpose. It is applied here to the rack form, from which the rotor profiles are generated, rather than to the actual rotor profiles. By this means, pressure angle profiling can be combined with previously published optimisation procedures, to design screw compressors with the lowest manufacturing and running costs.

In industrial practice, rotor profiles are defined by Cartesian coordinates in the plane normal to their axes of rotation and by the gradient of these coordinates in that plane. These can be expressed in term of x and y and their derivatives, either dy/dx or dx/dy. Alternatively they may be defined by use of polar coordinates, as functions of the angular position, φ, which in majority of cases corresponds to the pressure angle. The gradient values are obtained from the profile curves, expressed in analytical form, or numerically, from the rotor coordinates. Specification of both the coordinate values and their gradients results in redundant information. Therefore, it may be possible to reduce the information required by defining a profile gradient as a generic function from which all the information required can be derived. Thus starting with the profile gradient and one of the coordinates, any other coordinate can be estimated by:

$$x = x_0 + \int_{y_0}^{y} \frac{dx}{dy} dy \qquad (1)$$

If dx/dy is given in analytical form and its integral is known, x may be found by direct integration. The integration can also be performed numerically if the integral is not known, or if the gradient is specified in a discrete form.

When generating screw compressor rotors, the pressure angle method has the advantage of easily maintaining profile continuity not only of the profile curves, but also of their gradients. Thus, if a succession of curves is used, the final coordinates of one curve serve as the initial condition for the next curve and therefore the profile continuity is retained. Moreover, the continuity of the profile gradient will be maintained even if the pressure angle value is retained only at the profile connections.

Since the profile curvature is a derivative of the profile gradient, it can be used to advance the coordinate step for a smooth profile, as well as a constraint required by the tool. Moreover, since the contact line between the rotors is defined by the profile gradient, its shortest distance from the rotor cusp is a good measure of the blow-hole area. Finally the profile gradient directly defines the rotor meshing condition θ.

2 ANALYSIS OF THE PRESSURE ANGLE PROCEDURE IN PROFILING SCREW COMPRESSOR ROTORS

The majority of rotor profiles currently used by industry are generated from curves based on the actual rotor of interest. The majority of these curves are circular. Trochoids are then generated from them to obtain their counterpart profile on the meshing rotor. Circles are very suitable functions for the pressure angle method, because the pressure angle is the same as their profile parameter φ and the profile gradient is a simple arc tangent function of the angle parameter, which is usually the same as pressure angle. Thus, $x=r\cos\varphi$ and $y=r\sin\varphi$, while dx/dy=-tanφ. The situation is very similar with straight lines, which have constant derivatives. Other generating curves, like ellipses, parabolae and hyperbolae have more elaborate, but still simple analytical forms, usually expressed as functions of the profile coordinates, x or y. Moreover, if properly distributed, even the discrete point values of the profile gradient can be sufficient for generating the profile.

Alternative approach is to generate the rotor profile on a rack, which is a rotor of infinite radius. This is then equally applicable to both the main and gate rotors. Currently only two rack generated screw rotor profiles are in industrial use. These are that of Rinder, 1984, as adopted by the Trane Company in the USA for their refrigeration compressors, and the author's 'N' profile. The latter is used by a growing number of manufacturers throughout the world in air, refrigeration and process gas compressors and in expanders.

In terms of the gradient profile generation, rack generated curves have a substantial advantage over those which are rotor generated. Namely, the rack rolling coordinate $y_r=r_{1w}\varphi$, unlike that of any rotor generated coordinate, continuously increases with x_r for all known screw rotor profiles. This means that the profile gradient dx/dy is a bounded function, which never goes to infinity and, therefore, the pressure angle is also bounded. Hence it cannot generate discontinuities.

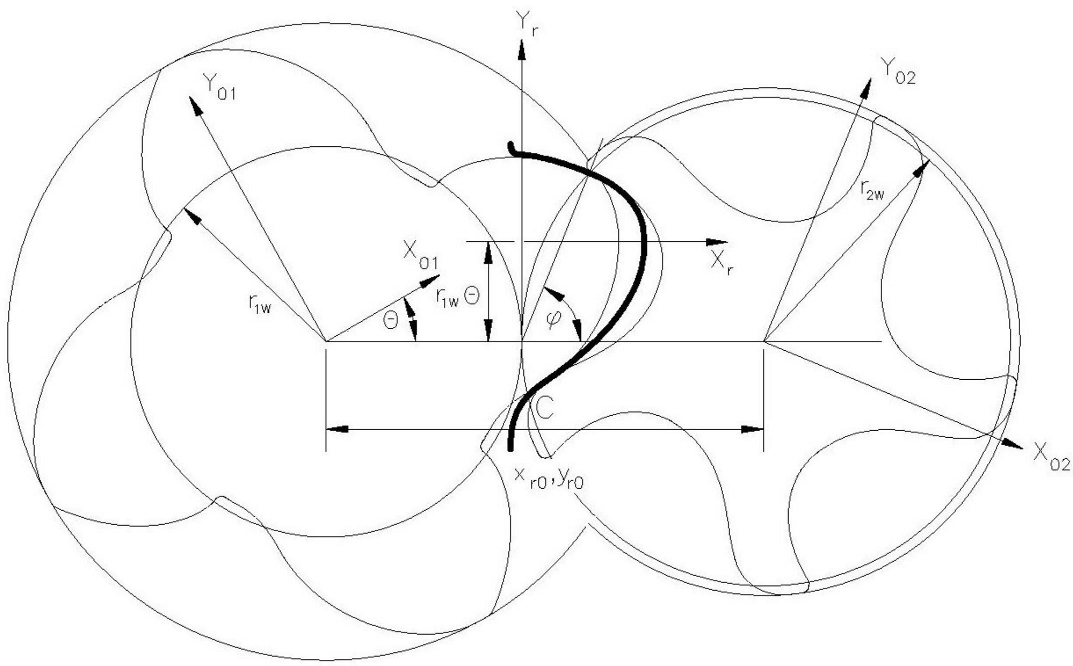

Figure 1 Rotor coordinate systems

Since dx/dy is a bounded function for a rack generated profile, it may be integrated numerically by a simple procedure. However, it has been found out that second order, more accurate methods are advantageous. Therefore, the trapezoidal rule has been applied in this work. A Runge Kutta fourth order method was also used for the integration of y, but the results so obtained were not different to those obtained from a second order solution.

Once generated, either by a fixed geometry approach, or through an optimisation procedure, the rack profile is used for the rotor profile generation. The envelope method was used for that purpose. Its application for the generation of screw compressor rotors was presented by Stosic, 1998.

A pair of screw compressor rotors, together with their rack, is presented in Fig. 1, with the male rotor on the left and the female on the right and the rack in bold between them. The centre line distance between them is $C = r_{1w}+r_{2w}$, where r_{1w} and r_{2w} are the rotor pitch circle radii. Let x_r, y_r and dx_r/dy_r be the rack coordinates and their gradient respectively. The meshing condition θ is a rotation angle of the main rotor at which the rotors contact. It results from the envelope condition:

$$\frac{dy_r}{dx_r}\left(r_{1w}\theta - y_r\right) - \left(r_{1w} - x_r\right) = 0 \qquad (2)$$

As may be seen, this equation is explicit in angle θ, which can be derived directly from it. Once solved, θ serves to generate the rotors profile and also to calculate the rotor sealing lines. The rotor profile coordinates x_{01} and y_{01} of the main rotor and x_{02} and y_{02} of the gate rotor are:

$$\begin{aligned} x_{01} &= x_r \cos\theta - \left(y_r - r_{1w}\right)\sin\theta \\ y_{01} &= x_r \sin\theta + \left(y_r - r_{1w}\right)\cos\theta \end{aligned} \qquad (3)$$

$$\begin{aligned} x_{02} &= C - x_r \cos\theta - \left(y_r - r_{2w}\right)\sin\theta \\ y_{02} &= x_r \sin\theta + \left(y_r - r_{2w}\right)\cos\theta \end{aligned} \qquad (4)$$

Table 1 'N' profile rack gradients and pressure angles

Profile part	Curve	Gradient	Limits
D-C	Circle	$dx_r/dy_r=-\tan\varphi$	$0 < \varphi < \alpha_1$
C-B	Straight line	$dx_r/dy_r=const=\tan(\pi-\alpha_1)$	$\varphi = \alpha_1$
B-A	Parabola	$dx_r/dy_r=0.5y^{-0.5}$	$\tan\alpha_1<dx/dy<0$
A-H	Trochoid on the rack from the circle on the main rotor	$dx_{01}/dy_{01}=-\tan\varphi$ on the main rotor	$0 < \varphi < \alpha_1$
H-G	Trochoid on the rack from the circle on the gate rotor	$dx_{02}/dy_{02}=-\tan\varphi$ on the gate rotor	$0 < \varphi < \alpha_1$
G-F	Straight line	$dx_r/dy_r=const=\tan(\pi-\alpha_2)$	$\varphi = \alpha_2$
F-E	Circle	$dx_r/dy_r=-\tan\varphi$	$0 < \varphi < \alpha_2$
E-D	Straight line	$dx_r/dy_r=const=0$	$\varphi = \pi$

3 REVIEW OF PRESSURE ANGLES IN THE ‘N’ PROFILE

Any rotor profile may be used to demonstrate pressure angle profile generation. However, taking into account that rack gradient generation is superior to rotor gradient generation, an “N” profile rack generated rotor has been used as an example in this paper without any loss in generality. Stosic and Hanjalic, 1997, who included a full specification of the curves from which the rack is constructed, described this profile and included analyses of several aspects of its generation. The ‘N’ rotor profile is presented in Fig. 2 while its curves and their gradients are defined in Table 1.

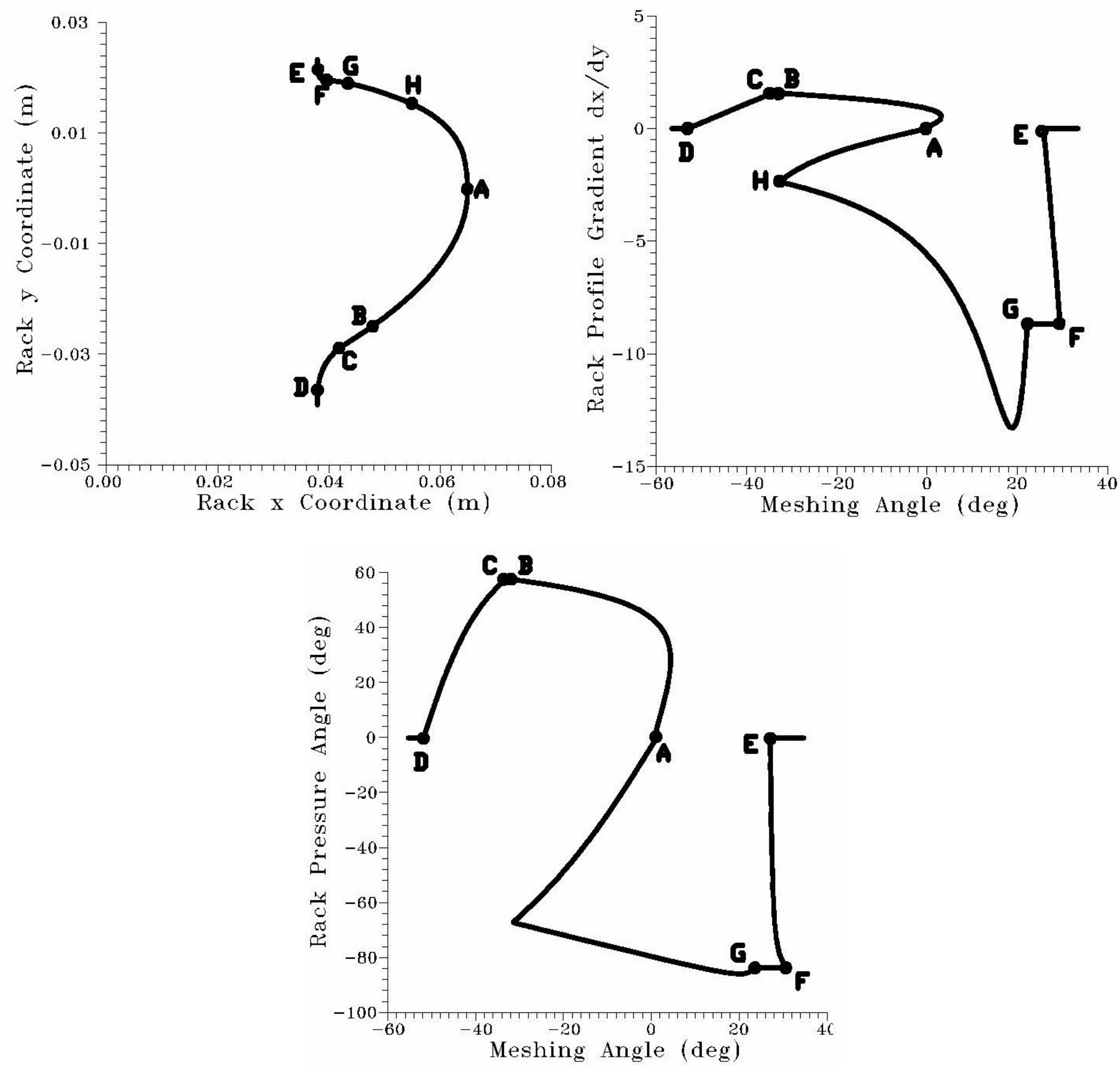

Figure 2 ‘N’ Rotor rack, its profile gradient and pressure angle

4 OPTIMISATION OF THE ROTOR PROFILE

The criteria for screw profile optimisation are valid irrespective of the machine type and duty. Thus, an efficient screw machine must admit the highest possible fluid flow rates for a given machine rotor size and speed. This requires that the fluid flow cross-sectional area through the rotors must be as large as possible. In addition, the maximum delivery per unit size or weight of the machine must be accompanied by minimum power utilization. This implies that the efficiency of the energy interchange between the fluid and the machine is a maximum. Accordingly unavoidable losses such as fluid leakage and energy losses must be kept to a minimum. However, increased leakage may be acceptable if it is associated with greater bulk fluid flow rates. Overall, the required compressor delivery rate must be obtained by simultaneous optimisation of the rotor size and speed to minimise the compressor weight while maximising its efficiency. It follows that a multivariable minimisation procedure is needed for screw compressor design with the optimum function criterion comprising a weighted balance between compressor size and efficiency or specific power.

The algorithm of the thermodynamic and flow processes used in optimization calculations is based on a mathematical model comprising a set of differential equations of conservation of mass and energy and algebraic equations of state and instantaneous compressor volume which fully describe the physics of all the processes within the screw compressor.

The energy equation is in form of internal energy rather than enthalpy as the derived variable. This was found to be computationally more convenient, especially when evaluating the properties of real fluids because their temperature and pressure calculation is not explicit. However, since the internal energy can be expressed as a function of the temperature and specific volume only, pressure can be calculated subsequently directly. All the remaining thermodynamic and fluid properties within the machine cycle are derived from the internal energy and the volume and the computation is carried out through several cycles until the solution converges.

Leakage in a screw machine forms a substantial part of the total flow rate and plays an important role because it affects the delivered mass flow rate and compressor work and hence both the compressor volumetric and adiabatic efficiencies.

Injection of oil or other liquids for lubrication, cooling or sealing purposes, modifies the thermodynamic process in a screw compressor substantially. Special effects, such as gas or its condensate mixing and dissolving in or flashing out of the injected fluid must be accounted for separately if they are expected to affect the process. In addition to lubrication, the major purpose for injecting oil into a compressor is to seal the gaps and cool the gas.

The solution of the set of differential equations is performed numerically by means of the Runge-Kutta 4th order method, with appropriate initial and boundary conditions. As the initial conditions were arbitrary selected, the convergence of the solution is achieved after the difference between two consecutive compressor cycles becomes sufficiently small. Once solved, internal energy and mass in the compressor working chamber serve to calculate the fluid pressure and temperature.

The solution of the mathematical model of the physical process in the compressor provides a basis for a more exact computation of all desired integral characteristics with a satisfactory degree of accuracy. The most important of these properties are the compressor mass flow rate, the indicated work and power and specific power, volumetric and adiabatic efficiencies. A full and detailed description of the presented model of the compressor thermodynamics is given in Hanjalic and Stosic, 1997.

When attempting to optimise a compressor design, the criterion for the desired result, such as the minimum power consumption or operational cost must first be resolved. However, the power consumption is coupled to other requirements, which should also be included, such as low compressor price, or investment cost. The problem becomes obvious if the requirement for low power consumption conflicts with the requirement for low compressor price, which is overcome by weighting the various elements of the target function.

The Box complex method was used here to find the minimum of the target function, which, in the case described in this paper, was the specific power. The constrained simplex method emerged from the simplex method, which was introduced by G. Box, 1957 and developed 1969 by M. Box. It is also suitable for the constrained cases because that only a few starting trials were needed, and the simplex immediately moves away from unsuitable trial conditions.

This is a multivariable constrained optimisation process. The task is to maximise a target function $f(x_1, x_2, ..., x_n)$, subjected simultaneously to the effects of explicit and implicit constraints and limits, $g_i \le x_i \le h_i,\ i = 1,n$ and $g_i \le y_i \le h_i,\ i = n+1,m$ respectively, where the implicit variables $y_{n+1},\dots,y_m$ are dependent functions of x_i. The constraints g_i and h_i are either constants or functions of the variables x_i.

Since the nonlinear problem is to be solved, it is necessary to use k points in a simplex, where k=2n. These starting points are randomly generated so that both the implicit and explicit conditions in are satisfied. Let the points x^h and x^g be defined by

$$f(x^h) = \max\ f(x^1), f(x^2), ..., f(x^k)$$
$$f(x^g) = \min\ f(x^1), f(x^2), ..., f(x^k) \tag{5}$$

calculate the centroid $\bar{x}$ of those points other than x^l by

$$\bar{x} = \frac{1}{k-1}\sum_{i=1}^{k} x_j^i, \quad x^i \ne x^l \tag{6}$$

The main idea of the algorithm is to replace the worst point x^l by a new and better point. The new point x^r is calculated as a reflection of the worst point through the centroid. This is done as

$$x^r = \bar{x} + \alpha(\bar{x} - x^l) \tag{7}$$

where the reflection coefficient α is chosen according to Box as α=1.3.

The point x^r is examined with regard to explicit and implicit constraints and, if it is feasible, x^l is replaced with x^r unless $f\left(x^r\right) \le f\left(x^l\right)$. In that case, it is moved halfway towards the centroid of the remaining points. This is repeated until it stops repeating as the lowest value. However, this cannot handle the situation where there is a local minimum located at the centroid. The method used here is to gradually move the point towards the maximum value if it continues to be the lowest value. This will, however, mean that two points can come very close to each other compared to other points, with a risk of collapsing the complex. Therefore, a random value is also added to the new point. In this way, the algorithm will take some extra effort to search for a point with a better value, but in the neighbourhood of the point of the maximum value. It is consequently guaranteed that a point better than the worst of the remaining points will be found. Expressed as an equation

$$x^{r(new)} = 0.5\left[x^{r(old)} + c\bar{x} + (1-c)x^h\right] + (\bar{x} - x^h)(1-c)(2R-1) \qquad (8)$$

where

$$c = \left(\frac{n_r}{n_r + k_r - 1}\right)^{\frac{n_r + k_r - 1}{n_r}} \qquad (9)$$

and k_r is the number of times the point has repeated itself as lowest value and n_r is a constant. Here n_r=4 has been used. R is a random number in the interval [0,1].

If a point violates the implicit constraints, it is moved halfway towards the centroid. In order to handle the case of the centroid violating the implicit constraint, the point is gradually moved towards the maximum value. If the maximum value is located very close to the implicit constraint, this will take many iterations and the new point will be located very close to the maximum value and will not really represent any new information. Therefore a random value is also added in this case.

All effects are present and often they exert an opposing influence. The geometry of screw machines is dependent on a number of parameters whose best values to meet specified criteria can, in principle, be determined by a general multi-variable optimization procedure. In practice it is preferable to restrict the number of parameters to a few, which are known to be the most significant, and restrict the optimization to them only. More information on screw compressor optimisation is published by Stosic, Smith and Kovacevic, 2003.

5 OPTIMISATION CONSTRAINTS

The optimsation constraints, which are explicit, are presented in Table 1. Those, which are implicit, will be discussed further.

Since y is an independent variable for this type of generation, its constraints are considered explicit too. The y coordinate on the rack is a uniform and rising function for any rotor, therefore, this must be satisfied as a constraint. y must never decrease. Moreover, it must not be constant, because this will result in an unbounded or discontinuous gradient for dx/dy.

Implicit constraints are imposed upon the optimsed variable, which is x, since y is used as an independent variable.

Specifying the outer and root diameters of the rotors sets limits to the permitted values of x since the values of these for both rotors are defined by the lowest and highest values of x on the rack. Thus, the points at the lowest value of x on the rack are at the rotor root of the main rotor and at the outer diameter of the gate rotor, while, the highest value of x corresponds to the main outer and gate root diameters. Moreover, the rack must start and end at the same value of x.

Constraints are usually imposed upon the curvature radii of the profile to satisfy the requirements of the applied tools. In such a case, all small circles on the rack and on the rotor profiles are subjected to restrictions, which are defined by the tool manufacturers. Therefore the smallest radii at certain areas of the profile are given additional length constraints.

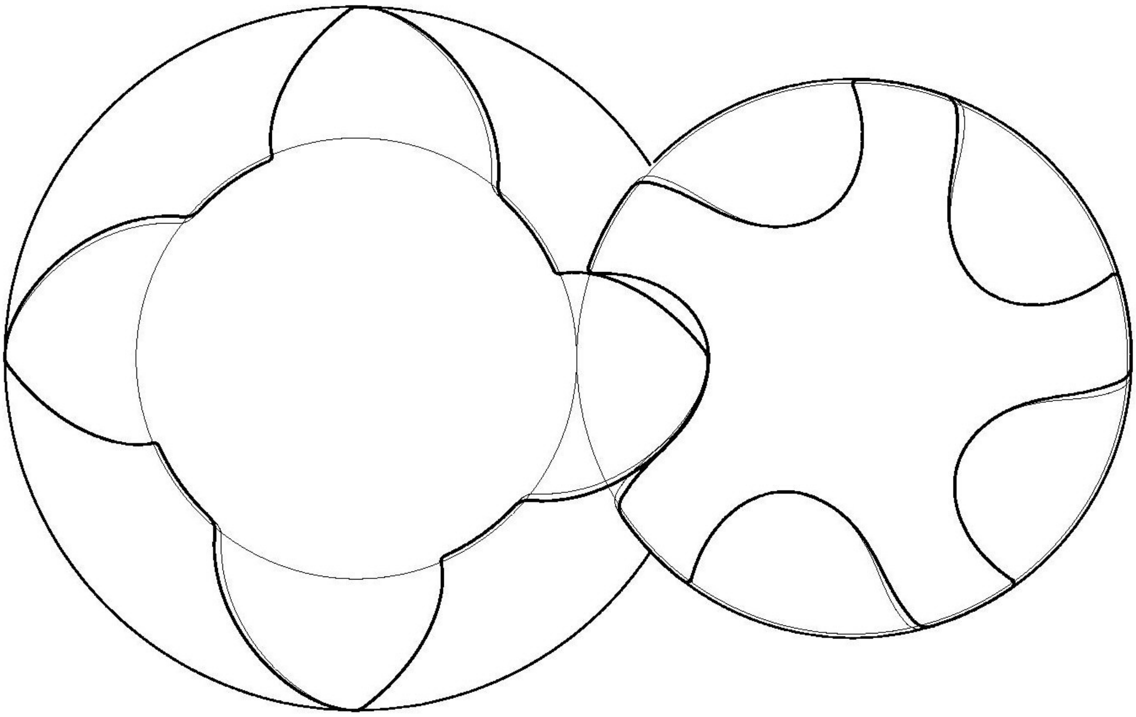

Figure 3 'N' Rotors generated by pressure angle method, fixed parameters - light line, optimised parameters - bold line

Additional constraints are requested by the manufacturing process. Namely, to mill or grind rotors with the minimum tool wear, the rotor pressure angle at the rotor pitch circles has to be as large as possible, therefore its minimum value should be defined in advance as an optimization constraint. This angular requirement is a demand upon the profile gradient, because the profile gradient dx/dy is a tangent of the pressure angle. This requirement is implicitly satisfied by rack generated rotors if a straight line defines the profile in the vicinity of the pitch circles, when the line slope angles α_1 and α_2 define the rack and rotor pressure angles.

6 EXAMPLE OF ROTORS GENERATED BY THE PRESSURE ANGLE METHOD

For the purpose of validation a fixed geometry generation of a 4/5-220 mm 'N' profile has been performed, once by using an ordinary and well proven procedure and the second time by use of the profile pressure angle generation method.

No difference was detected between the fixed profile coordinates generated by the standard method and by the pressure angle method up to the 8^{th} significant digit, which was 0.01 μm.

A rotor optimisation was conducted for the lowest specific power as a target function. As a result of the optimisation, very similar rotor geometry was obtained for both generation procedures, with a specific power difference between them of less than 0.5 %. This outcome was expected, because the optimsation procedure used, searches for the local minimum in a stochastic manner and, therefore some discrepancy in the optimised result was not a surprise.

The rotors calculated by the fixed procedure and their optimised counterparts are presented in Fig 3 in dashed and bold lines respectively.

7 CONCLUSION

The pressure angle procedure was incorporated into a rotor profile generation routine and used to generate ‘N’ rotors. New optimisation constraints were determined to suit the pressure angle method and applied to optimisation of the screw compressor rotors. The results presented in this paper confirm that the pressure angle method may be considered consistent with the standard ’N’ profile generation procedure and the constraints imposed upon the variables in the optimisation process were found to be adequate.

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