SECTION A

Question 1

The horizontal beam ABCD shown in Figure Q1(a) is simply supported at A and is continuous over the support at C. The beam cross-section is shown in Figure Q1(b). It is symmetrical about the YY axis and has an overall depth of 300 mm. It has unequal flanges so that the centroidal axis, XX, is 130 mm from the upper face. The second moment of area, I_{xx}, is $120 \times 10^6$ mm$^4$.

(a) Explain why the section has two values of elastic section modulus and find their values.  

(b) For the loading shown:

(i) sketch the deflected form, indicating points where the deflection, slope or curvature are zero,  

(ii) draw the shear force and bending moment diagrams, showing all principal values,  

(iii) calculate the maximum tensile and compressive stresses in the beam due to bending moment and state where they occur.  

(c) Explain why the beam in Figure Q1(a) would be statically indeterminate if the support at A had been fixed. Without calculation, sketch the general form of the bending moment diagram.
Question 2

![Figure Q2](image)

The uniform beam, ABCD, shown in Figure Q2 is simply supported at its ends, A and D. The beam carries a uniformly distributed load of 10 kN/m along the entire length and concentrated loads of 80 kN at B and 40 kN at C.

Young’s modulus for the material of the beam is $205 \times 10^3$ N/mm$^2$ and the second moment of area of the beam about the axis of bending is $335 \times 10^6$ mm$^4$.

(a) Find the value of the deflection at mid-span.

(15 marks)

(b) Explain, without calculation, how the position and value of the maximum deflection in the beam could be found.

(5 marks)
Question 3

![Figure Q3](image)

(a) The pin-jointed truss shown in Figure Q3 lies in a vertical plane and is simply supported at A and E. Each member of the truss is 2.2 m in length.

Find the force in each member of the truss due to the loads shown. State clearly whether each force is tension or compression.

(16 marks)

(b) Using the method of sections, confirm the value of the force in member GF due to the loading shown.

(4 marks)

Question 4

A solid, circular steel shaft, 400 mm in length, is required to transmit a torque of 1 kNm. The specification requires that

(i) the maximum shear stress should not exceed $85 \text{ N/mm}^2$,

and (ii) the angle of twist in the shaft should not exceed 0.02 radians.

Find the minimum radius required for the shaft.

The shear modulus, $G$, for steel is $78 \times 10^4 \text{ N/mm}^2$.

(20 marks)
SECTION B

Question 5

(a) A transport aircraft A is flying north-east at an angle 20 degree to north with a velocity of 720 km/h when a fighter jet B passes underneath A in a direction 50 degrees to the north. To passengers of aircraft A, the jet B appears to be flying sideways and moving east. Determine:

(i) The actual velocity of fighter B. (5 marks)

(ii) The velocity of B relative to A. (5 marks)

(b) A transport aircraft, see Figure Q5(b), is flying at a constant speed v at an altitude h=11 km is being tracked by radar located at O. If the angle θ is decreasing at a rate of 0.015 rad/s when θ=50º:

(i) Draw the velocity components in polar-coordinate system. (2 marks)

(ii) Determine the value of \( \dot{r} \) at this instant. (4 marks)

(iii) Determine the speed v of the aircraft. (4 marks)

Figure Q5(b)
Question 6

(a) An aircraft shown in Figure Q6 has a mass of 25000 kg with mass centre at G and is flying horizontally with a constant speed under the engine thrust, T. If the total drag, D, is 320 KN, then

(i) draw a free body diagram of the aircraft.   \( 2 \text{ marks} \)

(ii) calculate the thrust force. \( 3 \text{ marks} \)

(iii) calculate the vertical aerodynamic forces acting on the wings at A and B. \( 7 \text{ marks} \)

(b) If the thrust is suddenly increased to 380 kN, assuming that all other forces remain unchanged, calculate the instantaneous angular acceleration of the aircraft and indicate whether the aircraft is rising or falling. The radius of gyration is 3.9 m. \( 8 \text{ marks} \)

Figure Q6
Question 7

A slender rod with a mass \((m_R)\) of 3.6 kg and a length \((L)\) of 600 mm long is welded to a disk with a mass \((m_D)\) of 2 kg, and a radius \((r)\) of 120 mm that rotates about the pivot point A. A spring is attached to the disk and is un-stretched when the rod AB is horizontal. The spring stiffness constant is 85 N/m. If the system is released from rest in the position shown in Figure Q7, determine the following when the rod has rotated \(70^\circ\):

(a) The extension in the spring. \(\text{ (4 marks) }\)

(b) The rod angular velocity by using work-energy equations \(\text{ (16 marks) }\)

The mass moment of inertias of the rod and disk about their centre of mass are \((I_{G})_R=m_RL^2/12\) and \((I_{G})_D=m_Dr^2/2\), respectively.

![Figure Q7](image-url)

Question 8

Train A with a mass of 17500 kg is travelling at 75.6 km/h when it strikes a second train B, which has a mass of 18200 kg and is moving in the same direction at a speed of 28.8 km/h. If the coefficient of restitution for the collision is 0.7 and all resistances to motion are negligible, then

(a) calculate the speed of the trains A and B immediately after the impact. \(\text{ (8 marks) }\)

(b) as a result of the damage done during the collision in case (a) the trains A and B have a constant friction resistance to the motion of 15 kN. Calculate the distance that the train B will move before coming to rest. \(\text{ (8 marks) }\)

(c) if the train B was stationary before the impact, what would be the coefficient of restitution if the train A comes to rest after the impact? \(\text{ (4 marks) }\)
DATA SHEET

PART I MECHANICS OF SOLID

One-dimensional motion

\[ v = \frac{ds}{dt} \quad a = \frac{dv}{ds} = \frac{d^2s}{dt^2} = \frac{dv}{ds} \]

Constant velocity, \( a = 0 \)

\[ s = s_o + vt \]

Constant Acceleration

\[ v = v_o + at \]

\[ s = s_o + v_o t + \frac{1}{2} at^2 \]

\[ v^2 = v_o^2 + 2as \]

Curvilinear motion

Cartesian (rectangular) co-ordinates

Position vector \( r = xi + yj + zk \)

Velocity vector \( v = v_i \dot{i} + v_j \dot{j} + v_k \dot{k} \)

Acceleration \( a = a_i \dot{i} + a_j \dot{j} + a_k \dot{k} \)

Normal (n) and tangential (t) coordinates

\[ v = v_n \dot{n} + v_t \dot{t} \]

\[ a_n = \frac{v^2}{r} \quad a_t = \dot{v} = \frac{dv}{dt} \]

Radial-Transverse (Polar) coordinates

Position vector \( r = r \dot{r} \)

Velocity vector \( v = \dot{r} \dot{r} + r \dot{\theta} \dot{\theta} \)

Acceleration \( a = (\ddot{r} - r \dot{\theta}^2) \dot{r} + (2 \dot{r} \dot{\theta} + r \ddot{\theta}) \dot{\theta} \)

\[ a_r = (\ddot{r} - r \dot{\theta}^2) \quad a_\theta = (2 \dot{r} \dot{\theta} + r \ddot{\theta}) \]

Rigid body plane motion

\[ \sum M_G = I_G \alpha \]

\[ \sum M_o = I_G \alpha + ma_G d \]

Energy equation \( \Delta U = \Delta KE + \Delta PE + \Delta SE \) where \( \Delta KE = 0.5mv^2, \Delta PE = mgh \) and

\[ SE = V_e = \frac{1}{2} k x^2 \]

Kinetic energy of rigid body in plane motion

\[ KE = T = \frac{1}{2} I_G \omega^2 + \frac{1}{2} I_G \dot{\omega}^2 \]

Coefficient of Restitution

\[ e = \frac{V_A - V_B}{U_A - U_B} = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} \]

Moment of a Force

\[ M = r \times F = (rF \sin \theta) \phi \]