SECTION A

Question 1

A uniform, thin-walled beam has the open, singly symmetrical cross section shown in Figure Q1. The cross section has an axial slit in it at point 1 and the uniform thickness of the walls is \( t \). The slit is of negligible length such that the walls 1-2 and 2-3 are of equal length. The beam is subjected to a vertical shear force \( S_y \) acting through the shear centre of the cross section.

(a) Calculate the second moment of area \( I_{xx} \) of the thin-wall section about its centroidal \( x \)-axis.

[5 marks]

(b) Sketch the shear flow distribution in the walls and determine the position of the shear centre \( \xi_s \) from point 3 in the cross section.

[20 marks]
Question 2

A uniform, singly symmetrical, thin-walled closed section of constant wall thickness $t$ has cross section and dimensions shown in Figure Q2. BAF and CDE are straight vertical walls and BC and EF are horizontal walls, together forming the shape of a rectangle.

(a) Assuming that the direct stresses are distributed according to the theory of bending and the shear modulus $G$ is constant, calculate the shear flow distribution for a shear force $S_y$ applied vertically through its vertical wall FAB. [20 marks]

(b) Sketch the shear flow distribution along the walls. [5 marks]

![Figure Q2 (Not to scale)](image-url)
Question 3

A beam has the thin-walled, four-cell cross-section, formed by an outer circular cell of radius 100 mm and an inner equilateral triangular cell which are rigidly connected at points ABC as shown in Figure Q3. The thickness of the outer circular cell wall is 2 mm throughout and the thickness of the straight walls AO and COB partitioning the inner cells is 1 mm throughout. The three-cell cross-section beam is subjected to a torque of 50,000 Nm and has a shear modulus of materials $G = 27$ GPa.

Assuming that the cross-section shape is maintained by closely spaced rigid diaphragms, determine:

(a) the shear stresses in the walls,  
(b) the rate of twist for this cross-section.

[22 marks]

[3 marks]

Figure Q3 (Not to scale)
SECTION B

Question 4

(a) Use your knowledge of metals processing to explain how a metallic body panel for use as an aircraft skin is made. Your answer should explain why casting is not a suitable manufacturing route. [11 marks]

(b) Explain why it is desirable to use small-diameter rolls for the production of sheet metals? [6 marks]

(c) If plate steel were to be rolled and reduced in thickness from 0.40m to 0.35m in a single pass, what would be the maximum roll radius if the forming pressure was to be kept below 400 MPa? Assume the yield stress of the steel sheet is 200 MPa and the following equation applies.

\[ p_{\text{max}} = \sigma_y \left( 1 + \frac{w}{2d} \right) \]

where:
- \( p_{\text{max}} = \) maximum forming pressure,
- \( \sigma_y = \) yield stress of the steel,
- \( w = \) width of section under forming pressure and
- \( d = \) final thickness. [5 marks]

(d) A design requirement for the roll mill will be to ensure that the rollers exhibit a minimum deflection. How might this requirement be satisfied in practice if small diameter and hence flexible rolls are to be used? [3 marks]

Question 5

(a) Sketch typical stress-strain curves for each of the following materials:
- (i) metals,
- (ii) ceramics,
- (iii) polymers, and
- (iv) elastomers. [4 \times 2 marks = 8 marks]

(b) Using key features of these stress-strain curves, contrast the relative mechanical properties of each type of material, making specific reference to their elastic modulus, strength, processing requirements and the operating environments that they can withstand. [9 marks]

(c) Discuss with examples the importance of these properties in determining end use for two engineering applications of each material. [8 marks]
Question 6

Thrust deflectors and reversers on aircraft engines are controlled by hydraulic actuators. The forces required to reverse thrust can be large, so the actuators (of which each engine may have four) are heavy. One of the heavier parts is the piston rod, identified in Figure Q6. It carries axial compressive loads only, and is designed as a hollow tube, to save weight. The cross section is chosen to carry the loads without compressive failure or buckling.

![Figure Q6. A piston rod](image)

The main objective is to minimise the weight. The length \( L \) and the outer radius \( R \) are specified. The piston rod must transmit a maximum axial force \( F \), it must not fail by yielding in compression or by elastic buckling and it should operate at a temperature up to 300°C. The piston rod as a thin-walled tube of wall thickness \( t \).

(a) Derive the equation for the mass of the rod. [2 marks]

(b) What is the failure criterion for a yield failure? [2 marks]

(c) By eliminating the wall thickness from your equations, calculate the mass of the piston rod and derive the material performance index \( M_1 \) for this minimum mass constraint. [4 marks]

(d) Buckling occurs when an axially-loaded strut is compressed by force \( F = \pi^2EI/L^2 \) where \( I \) is given by \( I = \pi R^4 t \) for a cylinder. Calculate an alternative material performance index \( M_2 \) for a minimum mass strut that will not buckle under load. [4 marks]

(e) Draw lines on the appropriate graphs that are attached to the examination paper using both derived performance indices \( M_1 \) and \( M_2 \) and use the lines to identify the most suitable materials. (Ensure that you attach the annotated graphs you have used to your answer booklet.) [6 marks]

(f) Create a table for the four selected materials; include the values of the performance indices and comments on their suitability. [7 marks]
(A) The stiffness matrix of a bar element in global co-ordinates is given by

\[
\overline{k} = \begin{bmatrix}
\bar{k}_{11} & \bar{k}_{12} \\
\bar{k}_{21} & \bar{k}_{22}
\end{bmatrix}
\]

where

\[
\bar{k}_{11} = \bar{k}_{22} = -\bar{k}_{12} = -\bar{k}_{21} = \frac{EA}{L} \begin{bmatrix}
l^2 & lm \\
lm & m^2
\end{bmatrix}
\]

where \(EA\) and \(L\) are respectively the axial rigidity and the length of the bar; \(l\) and \(m\) are the direction cosines of the local co-ordinate axes with respect to the global co-ordinate axes.

(B) The stiffness matrix of a beam element in global co-ordinates is given by

\[
\overline{k} = \begin{bmatrix}
\bar{k}_{11} & \bar{k}_{12} \\
\bar{k}_{21} & \bar{k}_{22}
\end{bmatrix}
\]

where

\[
\bar{k}_{11} = \begin{bmatrix}
F & G & H \\
G & P & Q \\
H & Q & R
\end{bmatrix}, \quad \bar{k}_{22} = \begin{bmatrix}
F & G & -H \\
G & P & -Q \\
-H & -Q & R
\end{bmatrix}
\]

\[
\bar{k}_{12} = \begin{bmatrix}
-F & -G & H \\
-G & -P & Q \\
-H & -Q & B
\end{bmatrix}
\]

and

\[
\bar{k}_{21} = \begin{bmatrix}
-F & -G & -H \\
-G & -P & -Q \\
H & Q & B
\end{bmatrix}
\]

where

\[
F = \frac{EA}{L} l^2 + \frac{12EI}{L^3} m^2, \quad G = \frac{EA}{L} lm - \frac{12EI}{L^3} lm, \quad H = -\frac{6EI}{L^2} m
\]

\[
P = \frac{EA}{L} m^2 + \frac{12EI}{L^3} l^2, \quad Q = \frac{6EI}{L^2} l, \quad R = \frac{4EI}{L}, \quad B = \frac{2EI}{L}
\]

\(EA\) and \(EI\) are respectively the axial and bending rigidity of the beam; \(l\) and \(m\) are the direction cosines of the local co-ordinate axes with respect to the global co-ordinate axes and \(L\) is the length of the beam.
Information Sheet 2

The following relationships are given in the usual notation.

(i) Shear flow in a singly symmetric closed section

\[ q_z = q_o - \frac{S_y}{I_{yy}} \int y \, ds \]

(ii) Rate of twist and torque are related as

\[ \frac{d \theta}{d z} = \frac{T}{GJ} \]

(iii) Torsion constant \((J)\) for closed and open sections are respectively given by

\[ J_c = 4A^2 \int \frac{d s}{t} \quad \text{and} \quad J_o = \int \frac{t^3}{3} \, ds \]

(iv) Bredt-Batho formula

\[ T = 2Aq \]

(v) Bending stress due to asymmetric bending

\[ \sigma_z = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x \quad \text{or} \]

\[ \sigma_z = \left[ \frac{I_{xx}}{I_{yy}} \frac{I_{xx} - I_{xy}^2}{I_{yy} - I_{xy}^2} \right] y + \left[ \frac{I_{yy}}{I_{xx}} \frac{I_{xx} - I_{xy}^2}{I_{yy} - I_{xy}^2} \right] x \]

(vi) Differential equations governing vertical and horizontal deflections due to asymmetric bending are respectively given by

\[ \frac{d^2 v}{dz^2} = -\frac{M_x}{E I_{xx}} \quad \text{and} \quad \frac{d^2 u}{dz^2} = -\frac{M_y}{E I_{yy}} \]

where

\[ M_x = \frac{M_x - M_y I_{xy} / I_{yy}}{1 - I_{xy}^2 / I_{xx} I_{yy}} \quad \text{and} \quad M_y = \frac{M_y - M_x I_{xy} / I_{xx}}{1 - I_{xy}^2 / I_{xx} I_{yy}}. \]
Information Sheet 3 (To be handed in with the exam script)
Information Sheet 4 (To be handed in with the exam script)