Question 1

A pin-jointed space frame, shown in Figure Q1, consists of five bar members connected together at point 1 with fully fixed supports at points 2, 3, 4, 5 and 6. Point 2 lies on the vertical XY plane and points 4, 5 and 6 lie on the vertical YZ plane, while point 3 lies on the horizontal XZ plane. Points 1, 2, 4, 5 and 6 all located at a vertical height of 4 m from the horizontal plane XZ plane. The frame is loaded at point 1 with forces $F_x = 800$ kN, $F_y = 500$ kN and $F_z = 300$ kN as shown.

The cross-sectional areas of the members are:

$$A_{14} = A_{15} = A_{16} = 5,000 \text{ mm}^2, \quad A_{12} = A_{13} = 10,000 \text{ mm}^2$$

Take the Young’s modulus $E = 200,000 \text{ MN/m}^2$.

(a) Calculate the displacements at point 1. [17 marks]

(b) Calculate the member force in each member, stating whether the member is in tension or compression. [8 marks]

Figure Q1 (Not to scale)
Question 2

The displacement distribution within a triangular plate element subject to in-plane loading (see Fig. Q2) is expressed in terms of the shape functions $N_1, N_2, N_3, \ldots \ldots \text{ etc.}$ as

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4 + N_5 u_5 + N_6 u_6$$

$$v = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4 + N_5 v_5 + N_6 v_6$$

where points 4, 5 and 6 are at the side mid-points.

For the triangular element having the corner coordinates given below determine the shape function $N_4$ for node 4. [5 marks]

The side 1-2 of the triangular element is subjected to in-plane distributed loading which varies linearly from 1 kN/m at node 1 to 2 kN/m at node 2. Establish the nodal force in the x-direction $P_{x4}$ at node 4 in the nodal force representation of the distributed loading. [15 Marks]

Evaluate the strains $\varepsilon_x$ and $\varepsilon_y$ in the plate at node 4 of the triangular element corresponding to the nodal displacements: $u_4 = 1 \text{ mm}, \quad v_4 = -2 \text{ mm}$

and all other nodal displacements zero. [5 marks]

![Figure Q2](image)

<table>
<thead>
<tr>
<th>Node</th>
<th>Coordinates (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Question 3

(a) Explain the effect of cyclic stress on fatigue behaviour of structures. What are the important cyclic stress parameters and their relative influence on fatigue boundary?

[5 marks]

(b) Explain the phenomenon of stress concentration and how this affects the fatigue strength of structures.

[5 marks]

(c) The stepped cylindrical shaft shown in Fig. Q3 is subjected to a steady tensile load of 50 kN and an alternating bending moment $M$. The yield strength of the rotor material is 300 MN/m² and the fatigue limit in un-notched pure reversed bending is 200 MN/m². Assuming that the fatigue boundary is given by Soderberg relationship

$$S_u = S_D \left(1 - \frac{S_m}{S_Y}\right),$$

where $S_D$ is semi-range stress at zero mean stress, $S_m$ is the mean stress and $S_Y$ is the yield stress, calculate the maximum value of $M$ to avoid fatigue failure in the rotor. The stress concentration factor $K_t$ for the fillet radius is 1.55 and the notch sensitivity factor $q = 0.9$.

[15 marks]
Question 4

Form the stiffness matrix $\mathbf{K}$ and the applied load vector $\mathbf{F}$ for a finite element analysis of the structure shown in Fig. Q4.

AB and BC are uniform beams, built-in to rigid support at A and C. All other members are pinned at their ends.

The bending rigidity $EI$ of both AB and BC is $10^7 \text{Nm}^2$ and the extensional rigidity $EA$ of both is $5 \times 10^7 \text{N}$. The extensional rigidity of all other members is $2 \times 10^7 \text{N}$.

Note that for a uniform beam of length $L$, built-in at each end, and subjected to a uniformly distributed loading $w$ per unit length, the fixing moment at each end is $wL^2/12$.

[25 marks]

Fig. Q4
Question 5

The continuous-beam members ABCDE (see Fig. Q5) consists of four straight uniform beam elements, rigidly jointed together at B, C, D, and to a rigid support structure at A and E. The structure is braced underneath by three bar elements, which can sustain axial loads only. The bars are pin jointed to the beam and to the rigid support at their ends as shown. The structure is symmetric about the line CF and is subjected to the anti-symmetric loading case as shown. The node numbering for one symmetric half of the structure corresponding to the left-hand side is shown in parenthesis alongside the letters A, B, C and F.

Establish the overall stiffness matrix $K$ of the structure, and the loading matrix $P$, necessary for a finite element solution of the problem for the given loading. Take full account of the symmetry of the structure to reduce the size of the $K$ matrix.

[25 marks]

Members AB, BC, CD and DE have bending rigidity $(EI) = 10^{10}$ Nm$^2$ and extensional rigidity $(EA) = 10^{10}$ N

Members BF, CF, FD have extensional rigidity $(EA) = 2 \times 10^{10}$ N

Fig. Q5
Question 6

The configuration of a carbon-epoxy laminate is (0/90), with a ply thickness of 2 mm.

(a) Form the A, B and D matrices. [15 marks]

(b) Ascertain if the laminate possesses the following properties:

(i) in-plane isotropy
(ii) in-plane orthotropy
(iii) bending isotropy
(iv) bending orthotropy [4 marks]

(c) Given the stress system in the reference axes (X – Y) as $\sigma_x = 150 \text{ N/mm}^2$, $\sigma_y = 50 \text{ N/mm}^2$ and $\tau_{xy} = 75 \text{ N/mm}^2$, determine the stress system in the material axes (1-2) when $\theta$, the angle between the reference X-axis with the fibre axis is 60°. [6 marks]

The reduced stiffness matrix of a unidirectional carbon-epoxy is given by

$$
Q = \begin{bmatrix}
200 & 3 & 0 \\
3 & 10 & 0 \\
0 & 0 & 5
\end{bmatrix} \text{ kN/mm}^2
$$
**Question 7**

A two-layered unsymmetric isotropic beam made of materials “a” and “b” is shown in Fig. Q7. The beam is loaded in its own plane by an axial load and a bending moment relative to the mid-plane.

(a) If \(N\) is the resultant axial force per unit width and \(M\) is the corresponding resultant bending moment per unit width, show how \(N\) and \(M\) are related to axial strain and curvature, in terms of the material properties.

[20 marks]

Assume that the bond between the two materials is rigid and there is no slippage between them.

Use appropriate notation wherever necessary.

(b) Comment briefly on the results.

[5 marks]

Examiners: Dr. C.W. Cheung (Q1-Q3)  
Professor J.R. Banerjee (Q4-Q7)  
External examiner: Professor D.I.A. Poll