Question 1

A critical isentropic flow exists within a converging-diverging duct with the following parameters: A-in/A-throat = 3.00, intake diameter D-in = 90cm, p-in = 0.15MPa, T-in = 300K, p-out = 35,000 Pa. Determine the following quantities.

(a) The flow stagnation pressure and temperature. Would the stagnation density change if a normal shock were present in the duct? Would the stagnation temperature change in this case? Please explain your answer. [5 marks]

(b) The intake speed and outlet Mach number. [5 marks]

(c) The duct intake mass flow rate. [5 marks]

(d) The duct outlet diameter. [5 marks]

(e) Calculate the thrust this duct would generate if it were a propulsion device. [5 marks]

Question 2

An air flow does 250kJ/kg of work as it expands in an ideal turbine, with M-in = 0.3, p-in= 275.75kPa, T-in = 1,500K. Determine:

(a) the critical pressure and temperature; [5 marks]

(b) the outlet Mach number; [5 marks]

(c) the outlet pressure; [5 marks]

(d) the outlet temperature; [5 marks]

(e) the outlet air speed. [5 marks]
Question 3

An aircraft is propelled by a single-shaft turbojet engine at a speed of 280 m/s at an altitude where the ambient pressure is 25 kPa and the ambient temperature is 220 K. The performance data for the engine, when operating at this flight condition, are as follows:

- Air mass flow rate entering the intake: 30 kg/s
- Fuel mass flow rate to combustor: 0.77 kg/s
- Intake pressure recovery factor: 0.96
- Compressor stagnation pressure ratio: 24
- Compressor isentropic efficiency: 0.90
- Mechanical efficiency: 99%
- Combustion chamber stagnation pressure loss: 4%
- Combustion chamber exit stagnation temperature: 1600 K
- Turbine isentropic efficiency: 0.94
- Isentropic efficiency of the convergent-divergent nozzle: 0.95

Allowing for the increase in mass flow rate due to the addition of the fuel, and neglecting all other losses, determine:

(a) the power required to drive the compressor, [8 marks]
(b) the stagnation temperature and pressure at entry to the nozzle, [8 marks]
(c) the nozzle exit velocity, [6 marks]
(d) the net thrust produced by the engine. [3 marks]


For the products of combustion $c_p = 1155$ J/kg.K, $R = 287$ J/kg.K, $k = 4/3 = 1.333$
Question 4

A straight radial vaned centrifugal compressor is required to supply 10 kg/s of air to a test rig. An electric motor with an output shaft power of 1600 kW running at a rotational speed of 3000 rpm is available to drive the compressor through a 1:5 step-up gearbox (running the compressor at 15000 rpm). The compressor is designed to operate at ambient temperature of 288 K, and an ambient pressure of 101.3 kPa. In the inducer section, prewhirl guide vanes make an angle of 20° to the axial direction and the absolute velocity of the air leaving the prewhirl guide vanes is 140 m/s. The inducer inner diameter (hub) is 80 mm. If the isentropic efficiency is expected to be 0.88, the slip factor 0.93, and the power input factor 1.05, calculate:

(a) the inducer outer diameter, [10 marks]
(b) the outer diameter of the impeller, [8 marks]
(c) the compressor pressure ratio. [7 marks]

Neglect the stagnation pressure loss in the inducer.

For air $c_p = 1005$ J/kg.K, $k = 1.4$, $R = 287$ J/kg.K.
Question 5

(a) For a particular axial flow compressor stage, designed using free vortex principles, the rotational speed of the rotor is 170 rev/s, the axial velocity of the air is 180 m/s, and the mean blade speed is 220 m/s. If the degree of reaction at the mean blade height is 0.60, and the stage temperature rise is 27K, calculate the rotor blade inlet and exit angles at this section. Assume a work done factor of 0.87. [7 marks]

(b) Determine the degree of reaction at the blade root section using the relationships for free vortex flow if the blade hub/tip ratio is 0.50. [18 marks]

For air: \( c_p = 1.005 \text{ kJ/kg.K}, \ k = 1.4 \)
Question 6

(a) Atmospheric pollution from aircraft engines is a major environmental issue.
   (i) Explain how the main pollutants are formed in the power plant and how they react with the atmosphere. [7 marks]
   (ii) Discuss the main combustion chamber design solutions that are being employed to alleviate the problem. [6 marks]

(b) A single stage axial flow turbine has a mean diameter of 0.6 m and rotates at 200 rev/s. The stator blade exit angle and the gas exit angle for the stage are 55° and 15° respectively. The axial velocity is 300 m/s and is constant through the stage. If the gas mass flow rate is 45 kg/s, calculate
   (i) the shaft power produced by the stage, [6 marks]
   (ii) the blade loading coefficient, [4 marks]
   (iii) the flow coefficient. [2 marks]
Steady Flow Energy Equation

\[
\frac{dQ}{dt} - \frac{dW_{\text{ext}}}{dt} = \frac{dm}{dt} \left[ (h_2 - h_1) + \left( \frac{v_2^2 - v_1^2}{2} \right) \right]
\]

\[
q - w_{\text{ext}} = \left( h_2 - h_1 \right) + \left( \frac{v_2^2 - v_1^2}{2} \right)
\]

Stagnation Enthalpy

\[
h_t = h + \frac{v^2}{2}
\]

Steady Flow Energy Equation

\[
\frac{dQ}{dt} - \frac{dW_{\text{ext}}}{dt} = \frac{dm}{dt} (h_{2t} - h_{1t})
\]

\[
q - w_{\text{ext}} = (h_{2t} - h_{1t})
\]

For a perfect gas \( h_t = c_p T_t \)

\[
\frac{dQ}{dt} - \frac{dW_{\text{ext}}}{dt} = \frac{dm}{dt} c_p \left( T_{2t} - T_{1t} \right)
\]

\[
q - w_{\text{ext}} = c_p \left( T_{2t} - T_{1t} \right)
\]

For adiabatic processes

\[
\frac{dQ}{dt} = 0
\]

\[- \frac{dW_{\text{ext}}}{dt} = mc_p \left( T_{2t} - T_{1t} \right)\]

Stagnation Temperature

\[
T_{1t} = T_t + \frac{v_{1t}^2}{2c_p}
\]

Mach Number \( M = v/a \)
Stagnation/static properties

\[ \frac{p_{st}}{p_1} = \left( \frac{T_{st}}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \]

Velocity of sound for a gas

\[ a = \sqrt{\gamma RT} = \sqrt{\frac{\gamma R T}{M}} \]

\[ \frac{T_{st}}{T_1} = 1 + \frac{\gamma - 1}{2} M_i^2 \]

\[ \frac{p_{st}}{p_1} = \left[ 1 + \frac{\gamma - 1}{2} M_i^2 \right]^{\frac{\gamma}{\gamma-1}} \]

Isentropic Processes for a perfect gas

\[ \frac{p_{2t}}{p_{st}} = \left( \frac{T_{2t}}{T_{st}} \right)^{\frac{\gamma}{\gamma-1}} \]

\[ \frac{p_{3t}}{p_{4t}} = \left( \frac{T_{3t}}{T_{4t}} \right)^{\frac{\gamma}{\gamma-1}} \]

The isentropic efficiency for a compressor is defined to be

\[ \eta_c = \frac{h'_{2t} - h_{2t}}{h_{2t} - h_{1t}} = \frac{T'_{2t} - T_{1t}}{T_{2t} - T_{1t}} \]

The isentropic efficiency for a turbine is defined to be

\[ \eta_t = \frac{h_{3t} - h'_{4t}}{h_{3t} - h_{4t}} = \frac{T_{3t} - T'_{4t}}{T_{3t} - T_{4t}} \]

The isentropic efficiency for an intake is defined to be

\[ \eta_i = \frac{T_{0t} - T_{0i}}{T_{0t} - T_0} \]

Intake pressure recovery factor = \( \frac{p_{st}}{p_{0t}} \)

Combustion chamber fuel-air ratio \( f \)

\[ f = \frac{c_p (T_{3t} - T_{2t})}{1 + f} = \frac{c_p (T_{3t} - T_{2t})}{\eta_{c/e} C.V.} \]
The isentropic efficiency for a nozzle is defined to be

\[ \eta_n = \frac{T_{4t} - T_5}{T_{4t} - T_3} \]

Critical pressure ratio for a choked convergent nozzle

\[ \frac{p_5}{p_{4t}} = \left( \frac{2}{\gamma + 1} \right)^{\gamma \rho^{-1}} \]

Degree of reaction for a compressor

\[ \Lambda = \frac{h_3 - h_1}{h_3 - h_1} = \frac{T_3 - T_1}{h_3 - h_1} = \frac{v_a}{2U} (\tan \beta_1 + \tan \beta_2) \]

Degree of reaction for a turbine

\[ \Lambda = \frac{h_3 - h_1}{h_1 - h_3} = \frac{T_3 - T_1}{h_1 - h_3} = \frac{v_a}{2U} (\tan \beta_3 - \tan \beta_2) \]

Turbo-fan Bypass ratio \( B = \frac{\dot{m}_c}{\dot{m}_{st}} \)

Specific work in a centrifugal compressor

\[ w = c_r \Delta T_i = P(U_2 v_{w2} - U_1 v_{w1}) = P(\sigma U_2^2 - U_1 v_{w1}) \]

Specific work in an axial compressor

\[ w = c_r \Delta T_i = \Omega U v_a (\tan \beta_1 - \tan \beta_2) \]

Specific work in an axial turbine

\[ w = c_r \Delta T_i = U v_a (\tan \beta_2 + \tan \beta_3) \]

The Blade loading coefficient \( \psi \)

\[ \psi = \frac{2c_r \Delta T_i}{U^2} = \frac{2v_a}{U} (\tan \beta_2 + \tan \beta_3) \]

Flow coefficient

\[ \phi = \frac{v_a}{U} \]
\[ M = \frac{1 + 0.27 \left( \frac{A}{A_0} \right)^2}{1.728 \left( \frac{A}{A_0} \right)}, \quad T_o \approx \left( 1 + \frac{M^2}{5} \right), \quad p_o = \left( 1 + \frac{M^2}{5} \right)^{3.5}, \quad \frac{A}{A_0} = \frac{125}{216M} \left( 1 + \frac{M^2}{5} \right)^3 \]

\[
T = \left( 1 + \frac{1 + \gamma}{2 + (\gamma - 1)M^2} \right), \quad p = \frac{1 + \gamma}{M} \left( 1 + \frac{1 + \gamma}{2 + (\gamma - 1)M^2} \right)^{1/2} \]

\[
\frac{1 - M^2}{\gamma M^2} + \frac{1 + \gamma}{2\gamma} \ln \left( \frac{(1 + \gamma)M^2}{2 + (\gamma - 1)M^2} \right) = f \left( \frac{x_0 - x}{L} \right)
\]

\[
\frac{T}{T_o} = \left( \frac{(1 + \gamma)M^2}{(1 + \gamma M^2)^2} \right), \quad \frac{p}{p_o} = \frac{(1 + \gamma)}{1 + \gamma M^2} \]

\[
\frac{(1 + \gamma)^2 M^2}{(1 + \gamma M^2)^2} \cdot \frac{1 + \gamma - 1}{2 M^2} = \frac{1 + \gamma}{2} \left( 1 - \frac{1}{\gamma R T_o} \right) \left( \frac{x_0 - x}{L} \right)
\]

\[
\frac{T}{T_o} = (M)^{(2(1-\gamma)/(1+\gamma)} \quad \frac{p}{p_o} = (M)^{2\gamma/(1+\gamma)} \]

\[
\left( 1 + \frac{\gamma - 1}{2} M^2 \right) M^{2(1-\gamma)/(1+\gamma)} = \frac{1 + \gamma}{2} + \frac{(\gamma - 1)w}{\gamma R T_o} \left( \frac{x_0 - x}{L} \right)
\]
Reference Tables

\[ M = \frac{1 - M^2}{\gamma M^2} + \frac{1 + \gamma}{2\gamma} \ln \left( \frac{(1 + \gamma)M^2}{2 + (\gamma - 1)M^2} \right) = \frac{f}{D/L} \left( \frac{x_* - x}{L} \right) \]

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\[ M \left( 1 + \frac{\gamma - 1}{2} M^2 \right) M^{2(\gamma - 1)(1 + \gamma)} = \frac{1 + \gamma}{2} + \frac{(\gamma - 1)w}{\gamma RT} \left( \frac{x^* - x}{L} \right) \]

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