Question 1

(a) Explain the importance of wing dihedral angle $\Gamma$ in determining the lateral stability of an aircraft. [5 marks]

(b) Derive an expression for the “rolling moment due to sideslip” derivative $L_v$ for an aircraft with unswept wings of arbitrary shape, semispan $s$, dihedral angle $\Gamma$ and lift curve slope $a (= dC_L/d\alpha)$, flying at true airspeed $U_0$ in air of density $\rho$. Ignore any effects other than from the wing. [9 marks]

(c) Use the result of part (b) to derive a value for the dimensionless derivative $C_{l\beta}(= l_v)$ for an unswept wing of taper ratio $\lambda (= c_l/c_r)$ of 0.5 and gross wing area $S_w$. [8 marks]

(d) Calculate the value of $C_{l\beta}$ for a light aircraft having the following data and flying in air of density 1.226 kg/m$^3$:

$\lambda = 0.5 \quad \Gamma = 5$ deg. \quad $a = 0.085$ per deg.

$S_w = 15 \text{ m}^2 \quad \text{Span } b = 8.8 \text{ m} \quad U_0 = 120$ knots [3 marks]

Note: 1 knot = 0.5144 m/s
**Question 2**

A transport aircraft has a mass of 45000 kg, a gross wing area of 160 m$^2$ and is cruising at an altitude of 30000 ft ($\sigma = 0.374$) at 450 kts TAS. Its stick-fixed lateral stability quartic at this altitude is

$$\lambda^4 + 5\lambda^3 + 10\lambda^2 + 25\lambda + 0.5 = 0$$

(a) Describe the three lateral stability modes this aircraft is likely to exhibit, and relate each to one or more of the four roots of this equation.

[6 marks]

(b) Making suitable stated assumptions where necessary, determine the approximate value of each of the four roots of the quartic equation.

[13 marks]

Note: Do not perform more than one iteration in finding any root – approximate answers are acceptable. You may use, without derivation, the following:

$$\left(\mu^2 + \omega^2\right) = \frac{E}{\lambda_1 \lambda_2} \quad \text{and} \quad C = \left(\mu^2 + \omega^2\right) + \lambda_1 \lambda_2 + 2\mu(\lambda_1 + \lambda_2)$$

(c) Assess whether the above aircraft satisfies the following criteria for Category B flight:

- Roll damping: maximum time to half amplitude 1.0 s
- Spiral mode: minimum time to double amplitude 20 s
- Dutch roll: minimum damping ratio $\zeta$ 0.08
- Minimum undamped natural frequency $\omega_n$ 0.4 rad/s

[6 marks]

**Note:** 1 knot = 0.5144 m/s  At sea level, $\rho_0 = 1.225$kg/m$^3$
Question 3

(a) On the simplifying assumption that the motion of an aircraft in pitch is governed only by the derivatives $M_w$ and $M_q$, the equation of motion can be written as

$$I_{yy} \ddot{\theta} = M_w w + M_q q$$

Using the non-dimensionalising rules set out on the attached sheet and noting that the characteristic length in this case will be the wing mean chord derive the dimensionless version of the above equation.

[10 marks]

(b) The following data refer to a large transport aircraft:

- Mass = 280 tonnes
- Gross wing area = 501 m$^2$
- Wing mean chord = 8.3 m
- $I_{yy} = 4.45 \times 10^7$ kg m$^2$
- True air speed = 248 m/s (ie $M = 0.84 \ @ \ 36000$ ft)
- Air density = 0.327 kg/m$^3$

In addition, wind tunnel testing has shown that $m_q$ ($= C_{mq}/2$) = −13.3 and $m_w$ ($= C_{ma}$) = −0.49. Using the equation you derived in part (a), determine the periodic time and the time to half amplitude of the oscillating motion involved.

[15 marks]
**Question 4**

(a) An aircraft in straight and level flight is suddenly subjected to an aileron deflection of $\delta$ radians. If it is constrained to move only in roll, show that the rate of roll $p (= \dot{\phi})$ in terms of its rolling inertia $I_x$, and the derivatives $L_p$ and $L_{\delta a}$ is given by the equation

$$p = -\left[ \frac{L_{\delta a}}{L_p} \right] [1 - e^{(L_p/I_x)t}] \delta a$$

[10 marks]

(b) A light aircraft of mass 1100 kg, gross wing area 15 $m^2$ and wing span 9.1 m is flying at 120 knots at sea level. The aircraft has a dimensionless rolling inertia $i_x$ of 0.064 and wind tunnel tests have shown that $\ell_p = -0.27$ and $\ell_{\delta a} = -0.15$ (angles in radians).

(i) Calculate the maximum roll rate to be expected for a rapid aileron deflection of 10 deg (which is then held constant at 10 deg.)

[10 marks]

(ii) Calculate the time taken for the aircraft to reach 50% of this maximum rate of roll.

[5 marks]

**Note:** 1 knot = 0.5144 m/s and sea level air density = 1.225 kg/m$^3$

If you prefer to use Laplace transforms for this question, you may find the following table useful.

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$\mathcal{L}[f(t)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1/s$</td>
</tr>
<tr>
<td>$t$</td>
<td>$1/s^2$</td>
</tr>
<tr>
<td>$e^{at}$</td>
<td>$1/(s-a)$</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>s. $\mathcal{L}(x)$</td>
</tr>
</tbody>
</table>

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Question 5

(a) The pilot of a large twin-engined passenger jet in approach configuration is landing using the wing-down technique (i.e. heading along the runway centreline but sideslipping/banking into wind).

(i) Draw a diagram, looking along the aircraft’s centreline, showing the forces and moments acting.

(ii) From this diagram, write down the governing equations covering side force and rolling and yawing moments.

(b) The aircraft of part (a) is flying an approach at a steady 135 knots into a wind of 28 knots from a direction 20 deg. off the runway centreline. From the data below, determine:

(i) the bank angle required
(ii) the aileron angle
(iii) the rudder angle

(c) Comment on the sign of the answers to (b)(ii) and (b)(iii). Are they as expected?

Note: 1 knot = 0.5144 m/s  Reminder: $\beta = v/U_0$ so (for example) $Y_\beta = U_0 Y_v$

Data: All data are in S.I. units (kg, m, s, N, rad)

Mass = 212000

$Y_v = -16.2 \times 10^3$ $L_v = -3.8 \times 10^5$ $N_v = 1.1 \times 10^5$

$L_{\delta a} = -4.3 \times 10^6$ $L_{\delta r} = 1.3 \times 10^6$

$N_{\delta a} = -1.4 \times 10^6$ $N_{\delta r} = -9.1 \times 10^6$
Question 6

(a) Describe, using sketches, a commonly used mechanical arrangement for obtaining the collective and cyclic pitch changes on the rotor blades of a typical helicopter, identifying the main components using normal technology. Relate each type of pitch change to the appropriate mode of flight (i.e. axial and lateral), and describe briefly the normal method of yaw control. [7 marks]

(b) A helicopter has a main rotor with effectively zero-offset blades, such that the flapping hinges may be assumed to be at the rotor axis. The helicopter c.g. is positioned a distance \( h_R \) below the rotor at a point on the rotor shaft axis and in a datum plane that is normal to this axis. It is also given that the fuselage aerodynamic centre and the tail rotor axis are at normal distances \( h_A \) and \( h_T \) respectively from the datum plane. For the helicopter in steady trimmed flight, and making use of suitable force and geometric diagrams, show that the cyclic and flapping coefficients \( a_1 \) and \( b_1 \) are given by

\[
-a_1 = \frac{h_A}{h_R} \frac{D}{W} = -\frac{h_A}{h_R - h_A} \theta_F
\]

\[
-b_1 = \frac{h_T}{h_R} \frac{T_T}{W} = -\frac{h_T}{h_R - h_T} \phi_F
\]

for which the upwards flapping angle of a blade is given by

\[
\beta = \beta_0 - a_1 \cos \psi - b_1 \sin \psi
\]

where \( \psi \) is the azimuth angle of the blade measured from aft (advancing blade on the right hand side), \( W \) is the helicopter weight, \( D \) is the fuselage drag, \( T_T \) is the tail rotor thrust (thrust vector normal to the main rotor shaft axis), and \( \theta_F \) and \( \phi_F \) are the steady pitch and roll angles (in the conventional sense) of the fuselage. [10 marks]

(c) The helicopter has a mass of 5000 kg and fuselage drag \( D = C_D \frac{1}{2} \rho V^2 S_{FA} \) where the equivalent flat plate frontal area of the fuselage \( S_{FA} = 2.6 \text{ m}^2 \) and \( C_D = 1.0 \), \( \rho \) being the ambient air density, and \( h_R = 3.5 \text{ m} \), and \( h_A = 0.85 \text{ m} \). For the case of straight and level trimmed flight at a forward speed of \( V = 60 \text{ m/s} \) at sea level, determine the values of the coefficient \( a_1 \) and fuselage pitch angle \( \theta_F \). [4 marks]

(d) If the power supplied to the rotor is 750 kW, the main rotor speed is 33 rad/s, the tail rotor arm is 8 m, and \( h_T = 2.2 \text{ m} \), determine also the values of the coefficient \( b_1 \) and the fuselage pitch angle \( \phi_F \). Also, indicate the direction of tilt of the rotor disc and of the fuselage, i.e. downwards to left or right, and downwards to forward or aft, respectively. [4 marks]
Question 7

(a) Show that the induced velocity in the fully contracted wake of a rotor in hover is twice the induced velocity in the rotor plane. Briefly outline any assumptions made

[7 marks]

(b) A four-bladed helicopter rotor has a rotor diameter of 10 m, a blade chord of 0.2 m, and rotates at a constant 33 rad/s. The total blade built-in twist is 8° reducing linearly towards the tip. Determine the induced velocity through the rotor disc and the total mass carried by the rotor out of ground effect (OGE) at sea-level ($\rho_0 = 1.225 \text{ kg/m}^3$) when the rate of climb is 7 m/s at a collective pitch setting of 24°, given the following equation for the downwash coefficient of a rotor in axial flight.

$$\lambda_i^2 + \lambda_i \left[ \left( \frac{sa}{8} \right) + \lambda_c \right] - \left( \frac{sa}{12} \right) \left[ \theta_0 \frac{3\kappa}{4} - \frac{3\lambda_c}{2} \right] = 0$$

In the equation, $s$ is the solidity, $a = 5.7$ per radian is the sectional lift curve slope, $\theta_0$ is the collective pitch, and $\kappa$ is the total blade built-in twist in radians. Also $\lambda_c = V_c/\Omega R$ where $V_c$ is the climb velocity, $\Omega$ is the rotor angular speed in rad/s and $R$ is the rotor radius. It may be assumed that $C_T = 4(\lambda_i^2 + \lambda_i \lambda_c)$, where $C_T$ is the thrust coefficient based on rotor tip speed and rotor disc area.

[9 marks]

(c) For the same collective pitch of 24°, and assuming that there is sufficient engine power and that the lift curve slope remains linear for all blade sections, determine the air density ratio $\sigma$ at which the rate of climb OGE becomes zero for a total mass carried aloft of 2300 kg. Determine the approximate altitude given the data in the table below.

$$\begin{array}{|c|c|c|c|c|}
\hline
\text{Altitude (ft)} & 0 & 5,000 & 10,000 & 15,000 \\
\sigma & 1.0 & 0.862 & 0.739 & 0.629 \\
\hline
\end{array}$$

[9 marks]
NON-DIMENSIONAL PARAMETERS and DERIVATIVES

Time
\[ \hat{t} = t / \tau \quad \tau = m / \rho US \]

Longitudinal

Velocity
\[ \hat{u} = u / U \quad \hat{w} = w / U = \alpha \]

Deriv. of force w.r.t. linear velocity
\[ x_u = X_u / \frac{1}{2} \rho US \]

Deriv. of force w.r.t. angular velocity
\[ x_q = X_q / \frac{1}{2} \rho US\bar{c} \]

Deriv. of moment w.r.t. linear velocity
\[ m_u = M_u / \frac{1}{2} \rho US\bar{c} \]

Deriv. of moment w.r.t. angular velocity
\[ m_q = M_q / \frac{1}{2} \rho US\bar{c}^2 \]

Moment of inertia
\[ i_y = I_y / m\bar{c}^2 \]

“Relative density”
\[ \mu_1 = m / \rho S\bar{c} \]

Lateral

Velocity
\[ \hat{v} = v / U = \beta \]

Deriv. of force w.r.t. linear velocity
\[ y_v = Y_v / \frac{1}{2} \rho US \]

Deriv. of force w.r.t. angular velocity
\[ y_p = Y_p / \frac{1}{2} \rho USb \]

Deriv. of moment w.r.t. linear velocity
\[ n_v = N_v / \frac{1}{2} \rho USb \]

Deriv. of moment w.r.t. angular velocity
\[ n_p = N_p / \frac{1}{2} \rho USb^2 \]

Moment of inertia
\[ i_x = I_x / m(b / 2)^2 \]

“Relative density”
\[ \mu_2 = m / \rho S(b / 2) \]
# \[ 2m / \rho S\bar{c} \] in U.S. \quad \frac{1}{4} \text{ in denominator in U.S.} \\

Notation
\[ b = \text{span} \quad \bar{c} = \text{mean chord} \quad U = \text{TAS} \quad S = \text{gross wing area} \]
\[ m = \text{mass} \quad \rho = \text{local air density} \quad t = \text{time} \]