Question 1

(a) (i) Explain what is meant by absolute probability and what is meant by conditional probability.
[2 marks]

(ii) A given population (e.g., the population of the UK or a collection of similar engineering components) will have a failure probability density, $f(t)$, and a hazard rate, $h(t)$. Explain the terms "failure probability density" and "hazard rate" and say how they are related.
[4 marks]

(iii) Table Q1.1 below applies to a starting cohort of 100,000 live births and gives the number of people surviving at age 60 and at age 61. Estimate the probability density for failure, $f(t)$, and the hazard rate, $h(t)$, at age, $t = 60$.

<table>
<thead>
<tr>
<th>Age ($t$)</th>
<th>Survivors to age, $t$, $L(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100,000</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>60</td>
<td>88,799</td>
</tr>
<tr>
<td>61</td>
<td>87,994</td>
</tr>
</tbody>
</table>

Table Q1.1
[4 marks]

(b) Two dice are thrown.

(i) Given no advance information about the fall of the dice, what is the probability that the total score is 4?
[2 marks]

(ii) Use the formulae of conditional probability to calculate the probability of the total score being 4 when it is known that the first die showed an odd face after landing.
[6 marks]

(iii) Illustrate your answer to (ii) with a tree diagram and explain how this diagram has been influenced by the information that the first die has landed odd.
[2 marks]

Question 2

2-out-of-3 voting is a common structure for trip systems, as shown in the logic tree in the diagram below, where $M_1, M_2$ and $M_3$ are monitor signals derived from independent measurements of the same process variable.
(a) Referring to the diagram, write down the minimal cut sets, $C_i$, that together define the top event, $T$. Use these to produce an equation for the upper and lower limits for the probability of the top event, $T$.

[4 marks]

(b) Assuming each monitor has the same fault detection probability, $P_m$, derive the upper and lower limits for the probability to detect a fault and trip. Hence find the numerical values of these limits for $P_m = 0.8$.

[5 marks]

(c) The probability of the top event, $T$, $P(T)$, may be derived as:

$$P(T) = P(Y) + P(Z) - P(Y.Z)$$

where

$$Z = M_1.M_2$$
$$Y = M_2.W$$

in which

$$W = M_1 + M_3$$

where the operator "+" stands for logical "OR", while the operator "." stands for logical "AND".

Using this equation and the definitions of $W$, $Y$ and $Z$ given, apply the laws of logic and the theorems of elementary probability to show that

$$P(T) = P(M_1)P(M_2) + P(M_2)P(M_3) + P(M_1)P(M_3) - 2P(M_1)P(M_2)P(M_3)$$

[7 marks]
(d) From the result of (c), write down the exact expression for the probability that the system as a whole will detect correctly a fault and trip for the case where each monitor has the same fault detection probability, $P_m$. Evaluate this for $P_m = 0.8$, as in (b) above.

[2 marks]

(e) Compare the results of (b) with those of (d) and comment on how close or otherwise the approximate method came to the exact answer.

[2 marks]

Question 3.

Consider a repairable system as shown in the Markov diagram Figure 3.1 below, where the failure rate and the repair rate are constant, with values $\lambda$ and $\mu$ respectively.

\[ P(\text{Normal} | \text{Normal}) = \lambda \delta t \]  \hspace{1cm} (3.1)
\[ P(\text{Failed} | \text{Normal}) = 1 - \lambda \delta t \]  \hspace{1cm} (3.2)
\[ P(\text{Normal} | \text{Failed}) = \mu \delta t \]  \hspace{1cm} (3.3)
\[ P(\text{Failed} | \text{Failed}) = 1 - \mu \delta t \]  \hspace{1cm} (3.4)

Explain what each of the equations (3.1) to (3.4) means.

[7 marks]
(c) Make use of the equations above to write down the unavailability a time $\delta t$ after
the current time, $t$. [3 marks]

(d) Hence show that the unavailability of the system obeys the equation of a first
order lag, namely

$$T \frac{dQ(t)}{dt} + Q(t) = a \quad (3.5)$$

What are the values of $T$ and $a$ ? [3 marks]

(e) Given values of 12 h for the mean time to failure (MTTF) and 2 h for the mean
time to repair (MTTR), calculate the probability that the system will be unavailable at
1 h and at 6 h ?

{Hint: the analytical solution of equation (3.1) for a constant value of $a$ and a zero
initial condition, $Q(0) = 0$, is:

$$Q(t) = a \left(1 - e^{-\frac{t}{T}}\right) \quad (3.6)$$

[3 marks]

(f) What is the steady state unavailability? [2 marks]

Question 4.

a) Explain how risk-management engineers define "risk". [2 marks]

(b) Engineers have developed structured approaches to analysing risk. Select four of
these and explain briefly how each contributes to an understanding of the reliability
and safety of engineered systems. [8 marks]

(c) Consider the opening passage below from the report "Get a life and take sensible

'Children are being prevented from taking part in activities by over-zealous
"health and safety pedants" terrified of taking risks, the chairman of the Health
and Safety Commission said yesterday.

Bill Callaghan said that misunderstandings and fear of the "compensation
culture" had led to organisations being over-cautious and preventing worthwhile
activities and trips. He cited examples of a university asking for a 69-page risk
assessment to be filled in before a field trip, a school banning egg boxes because of the risk of salmonella and children being prevented from playing conkers or required to wear goggles to do so.'

Do you agree that health and safety culture is being taken over by the over-zealous? Does this carry over to industrial health and safety? (You may choose to illustrate your answer with examples from the railway industry, the farming and food industries and the nuclear industry.)

**Question 5.**

(a) The two-parameter Weibull distribution may be described by the equation:

\[ F(t) = 1 - \exp \left( -\left( \frac{t}{\sigma} \right)^\beta \right) \]  

(1)

where \( t \) is elapsed time.

(i) Explain the meaning of \( F(t) \) and of the parameters \( \beta \) and \( \sigma \).  

[3 marks]

(ii) What is the condition for the two-parameter Weibull distribution to converge to the exponential distribution?  

[2 marks]

(iii) Demonstrate that the Weibull expression is equivalent to the equation:

\[ \ln \ln \frac{1}{1 - F(t)} = \beta \ln t - \beta \ln \sigma \]  

(2)  

[3 marks]

(b) The following failure data are available for 1000 components of the same design:

<table>
<thead>
<tr>
<th>Time interval from start of operation (h)</th>
<th>Number of components failed in the time interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 2</td>
<td>222</td>
</tr>
<tr>
<td>2 – 4</td>
<td>45</td>
</tr>
<tr>
<td>4 – 6</td>
<td>32</td>
</tr>
<tr>
<td>6 – 8</td>
<td>27</td>
</tr>
<tr>
<td>8 – 10</td>
<td>21</td>
</tr>
<tr>
<td>10 – 12</td>
<td>15</td>
</tr>
<tr>
<td>12 – 14</td>
<td>17</td>
</tr>
<tr>
<td>14 – 16</td>
<td>7</td>
</tr>
<tr>
<td>16 - 18</td>
<td>14</td>
</tr>
<tr>
<td>18 – 20</td>
<td>9</td>
</tr>
<tr>
<td>20 – 22</td>
<td>8</td>
</tr>
<tr>
<td>22 - 24</td>
<td>3</td>
</tr>
</tbody>
</table>
(i) Produce a table of the cumulative failure probability against time for a typical component for the first 24 hours of operation.  

[2 marks]

(ii) Assuming that the failure distribution is a two-parameter Weibull, determine graphically those two parameters.  

[6 marks]

(iii) From the results of (ii), comment on whether it might be reasonable to model the failure distribution as an exponential distribution.  

[2 marks]

(iv) Use the results of your curve fitting to produce a theoretical estimate of the number of failures up to 2 hours.  

[2 marks]