Diploma in Actuarial Management
Stochastic Modelling Coursework
Coursework to be submitted by Wednesday 30 June 2004

Question 1 (Markov models)

A global climate model has five states: hot, warm, temperate, cool and cold. The climate in one century is dependent on the climate in the previous century, with transition probabilities given by the transition matrix

$$P = \begin{pmatrix} .88 & .11 & .01 & 0 & 0 \\ .05 & .92 & .03 & 0 & 0 \\ 0 & .05 & .86 & .08 & .01 \\ 0 & 0 & .05 & .90 & .05 \\ 0 & 0 & 0 & .04 & .96 \end{pmatrix}$$

a) Run four simulations of the climate process using Excel, each running for about 200 centuries, each using the **same** sequence of pseudo-random numbers, but each starting from a **different** state. Plot all four sequences on a single chart. Comment.

b) Run four simulations as above, but with the differences that all should start from the **same** state (state 2: temperate) and that they should use four **different** sequences of pseudo-random numbers. Plot and comment as before.

c) Solve the equation $\pi^T P = \pi^T$ with constraint $\pi_1+\pi_2+\pi_3+\pi_4+\pi_5 = 1$ to find the theoretical long-run proportion of time the climate spends in each of the four states.

d) Calculate the proportion of time each of your simulated processes in (b) spent in each of the four states and comment on this in the light of your answer to (c).

Question 2 (Brownian motion and related processes)

A share price S_t evolves according to a Geometric Brownian model:

$$S_t = S_0 e^{mt+s B_t}$$

The closing price of the share is recorded on each of 21 successive days, giving data $S_0, \ldots, S_{20}$. The data are to be used to estimate the parameters μ and σ.

a) Write down the distribution of $\ln(S_{t+1}/S_t)$ for each day t, in terms of μ and σ.

b) If the observed values are

1206, 1184, 1167, 1181, 1153, 1160, 1169, 1152, 1181, 1209, 1213,

1199, 1191, 1177, 1182, 1204, 1191, 1262, 1287, 1254, 1255

obtain suitable estimates for μ and σ.

c) Using your estimated values and the data provided, calculate the probability that $S_{30} > 1350$. (You will need Normal tables for this, or use the NORMDIST function in Excel.)

d) Carry out 20 simulations of the share price from time 20 to time 30 and write down the number of times that $S_{30} > 1350$. Comment.

e) Is there any feature of the data which might lead you to believe that a Lévy process model might be more suitable than a Geometric Brownian motion?