# Reliability Analysis of IEEE 802.16 Mesh Networks

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**Abstract.** The problem of terminal-pair reliability has been extensively studied for wired computer networks, however comparatively less research has been done on the reliability of wireless networks. With mesh networks becoming a more attractive option than the traditional point-to-multipoint network design due to extended coverage and increased overall link quality, a method is needed to evaluate the added reliability obtained through the available path diversity in fixed wireless mesh networks. Methods are available to evaluate the reliability of wired networks, however they do not take into account the outage conditions found in a wireless environment such as multipath fading and interference. This paper combines the currently available reliability algorithms for wired networks along with methods to calculate the link and node outage in a fixed station IEEE 802.16 environment in order the simulate the reliability of a mesh subscriber station's connection to the base station. An example calculation is also provided.

## 1 Introduction

Fixed Broadband Wireless Access (FBWA) networks provide an attractive alternative to replace traditional last mile wired solutions such as cable or DSL as well as to provide Internet access in areas lacking a wired infrastructure [1]. Wireless networks tend to be easier and more cost-effective to deploy than wired networks. In the past network providers would have to rely on proprietary solutions in deploying a wireless broadband network, however the recent approval of the IEEE 802.16 (WiMAX) standard allows network providers to deploy a network at lower costs due to economies of scale and lower component costs.

The IEEE 802.16 standard for FBWA was first approved in 2001, however only frequency ranges between 10 and 66 GHz were supported. Due to the inability of high frequency signals to propagate around obstacles, Line of Sight (LOS) links were required. The more recent IEEE 802.16-2004 [2] standard specifies operating frequencies between 2 and 11 GHz allowing Non-Line of Sight (NLOS) operation. The physical layer described in IEEE 802.16-2004 uses either a 256-carrier Orthogonal Frequency Division Multiplexing (OFDM) with a Time Division Multiple Access (TDMA) scheme or a 2,048-carrier OFDM with an Orthogonal Frequency Division Multiple Access (OFDMA) scheme. The standard also supports power control and adaptive modulation schemes. Seven different modulation schemes are supported ranging from BPSK to 64-QAM.

Two different network topologies are currently supported in IEEE 802.16: Point-to-Multipoint (PMP) and mesh. A third topology, relay is being developed by the IEEE 802.16j Working Group (WG). In PMP, a Base Station (BS) distributes traffic directly to roof-mounted Subscriber Stations (SSs). In the mesh topology SSs can form multihop connections to the BS through other SSs as well as allowing SSs to connect directly to each other. The advantage of the mesh topology is to provide extended coverage range and an improved overall link quality. Rather than connecting directly to the BS through a low quality link, a SS can choose to connect through another SS where both links are of a higher quality. Intra-cell frequency reuse is another advantage of mesh networks which leads to a higher capacity in the network, however this could lead to increased interference and this interference must be considered in reliability calculations. Intra-cell frequency re-use is implemented in an IEEE 802.16 network by allowing multiple users within a cell to transmit within the same time slot as long as they are not within the interference range of each other dictated either by a centralized or distributed resource reservation scheme [2].

Reliability is an important component in the design and deployment of any communications network. Much work has been done in the area of reliability of wired networks [3–7], however comparatively less work has been done for wireless networks [8, 9]. An important measure of wireless network reliability is the terminal-pair reliability. Terminal-pair reliability is defined as the probability of successful communication between any two terminals in a network. For FBWA applications the terminals will be the SS and the BS. In the mesh topology, multiple routes to the BS may be available for every SS and the SS is free to choose any of these routes based on the routing protocol. The network is represented as a probabilistic graph $G = (V, E)$ where $V$ is the set of vertices (nodes) and $E$ is the set of edges (links). Each node and each edge will have a corresponding operational probability. In [10] it was shown that the analysis of terminal-pair reliability is NP-hard and has exponential time complexity, although efficient methods for computing bounds are available. This paper seeks to combine wired network reliability algorithms with wireless network propagation models in order to present an efficient way to calculate the terminal-pair reliability in IEEE 802.16 mesh networks. To the best of our knowledge no previous work has been done on the reliability of FBWA mesh networks.

The remaining sections are organized as follows. Sect. 2 describes the propagation models used in the calculation of link reliability. Sect. 3 describes a method to calculate the reliability of a single communication link. In Sect. 4, the terminal-pair reliability calculation will be presented. An example is shown in Sect. 5 and concluding remarks are given in Sect. 6.

## 2 Channel Models

An appropriate channel model is needed in order to assess the reliability of a link in a wireless network. The accuracy of a channel model depends upon the radio architecture used in the network. The IEEE 802.16 WG has developed a path loss model which includes measures for mean path loss, shadowing and fast fading [11]. This model supports microcell networks with rooftop antenna placements in the desired frequency range and was therefore chosen as the channel model for this paper.

### 2.1 Path Loss Model

The path loss model adopted by the 802.16 WG is based on measurements obtained by AT&T wireless services across the United States at 1.9 GHz with a receiver antenna height of 2 m [12]. The model provides three different terrain types A, B and C. Terrain A is a hilly terrain with moderate to heavy foliage density. Terrain C is a flat terrain with light foliage density. Terrain B is either a flat terrain with moderate to heavy foliage density or a hilly terrain with light foliage density. The path loss in dB is given by the following:

$$PL = A + 10\gamma log_{10}(d/d_{\mathrm{o}}) + X_{\mathrm{f}} + X_{\mathrm{h}} + s \text{ for } d > d_{\mathrm{o}} \tag{1}$$

where $\gamma$ is the path loss exponent, $d$ is the distance between transmitter and receiver in meters, $d_{\mathrm{o}} = 100\,\mathrm{m}$, $s$ is a log normal shadow fading factor due to hills, buildings and other large obstacles, $A$ is the free space path loss at 100 m, $X_{\mathrm{f}}$ is the frequency correction factor and $X_{\mathrm{h}}$ is the receiver antenna height correction factor. $\gamma$, $A$, $X_{\mathrm{f}}$ and $X_{\mathrm{h}}$ are defined as:

$$\gamma = a - bh_{\mathrm{b}} + c/h_{\mathrm{b}} \tag{2}$$

$$A = 20log_{10}(4\pi d_{\mathrm{o}}/\lambda) \tag{3}$$

$$X_{\mathrm{f}} = 6log_{10}(f/2000) \tag{4}$$

$$X_{\mathrm{h}} = \begin{cases} -10.8log_{10}(h_{\mathrm{r}}/2) \text{ for Terrain A and B} \ , \\ -10.0log_{10}(h_{\mathrm{r}}/3) \text{ for Terrain C } (h_{\mathrm{r}} \leq 3 \text{ m}) \ , \\ -20.0log_{10}(h_{\mathrm{r}}/3) \text{ for Terrain C } (h_{\mathrm{r}} > 3 \text{ m}) \ . \end{cases} \tag{5}$$

Constants $a$, $b$, $c$ and the standard deviation for $s$ ($\sigma$) can be found in Table 1.

### 2.2 Multipath Fading Model

Normally in wireless communications links, time variations of the channel are due to mobility of the receiver, however in fixed wireless systems time variations are due to the movement of scatterers between the transmitter and receiver. It has been previously shown that the temporal characteristics of the fixed wireless channel follow a Rice distribution with the strongest impact being caused

**Table 1.** Constant values for the path loss model

| Parameter | Terrain A | Terrain B | Terrain C |
|---|---|---|---|
| a | 4.6 | 4.0 | 3.6 |
| b | 0.0075 | 0.0065 | 0.005 |
| c | 12.6 | 17.1 | 20 |
| $\sigma$ | 10.6 | 9.6 | 8.2 |

by wind and foliage [13, 14]. The Rice distribution is characterized by a direct path, LOS component as well as multipath, scattered components existing in the received signal. The probability density function (pdf) is given by

$$f(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + r_{\mathrm{d}}^2}{2\sigma^2}\right) I_{\mathrm{o}}\left(\frac{r r_{\mathrm{d}}}{\sigma^2}\right) \tag{6}$$

where $r_{\mathrm{d}}$ is the amplitude of the direct path component, $\sigma$ is the amplitude of the multipath components and $I_o$ is the zero-order modified Bessel function. The Rice distribution is completely defined by the Rice Factor (R.F.), $K$ where $K[\mathrm{dB}] = 10log_{10}(\frac{r_{\mathrm{d}}^2}{2\sigma^2})$. Similarly, the cumulative distribution function (cdf) has been shown to be

$$F(r) = 1 - Q\left(\sqrt{2K}, \frac{r}{\sigma}\right) \tag{7}$$

where $Q(\cdot)$ is the Marcum $Q$ function. The 802.16 WG has adopted the model developed in [14] for modeling the $K$ factor which is given by

$$K[\mathrm{dB}] = 10log_{10}(F_{\mathrm{s}}) + 10log_{10}(F_{\mathrm{h}}) + 10log_{10}(F_{\mathrm{b}}) + log_{10}(k_{\mathrm{o}}d^{\gamma}) + u \tag{8}$$

where $k_{\mathrm{o}} = 10$, $\gamma = -0.5$ , $u$ is a lognormal R.V. with zero mean and standard deviation $\sigma_{\mathrm{K}} = 8$, $d$ is the distance in km and

$$F_{\mathrm{s}} = \begin{cases} 1.0 \text{ , winter (no leaves on trees)} \\ 2.5 \text{ , summer (leaves on trees)} \end{cases}$$

$$F_{\mathrm{h}} = \left(\frac{h_{\mathrm{r}}}{3}\right)^{0.46} \text{ , } h_{\mathrm{r}} \text{ receiver antenna height}$$

$$F_{\mathrm{b}} = \left(\frac{b}{17}\right)^{-0.62} \text{ , } b \text{ antenna beamwidth (degrees)}$$

## 3 Link Reliability

In order to calculate the terminal pair reliability the reliability of all individual links in the network must be known. In IEEE 802.16 mesh networks the uplink and downlink connection will be formed along the same path and therefore a bi-directional link is needed between all nodes along the path. A single wireless link between two nodes can fail for two reasons: either of the end nodes fail due to equipment failure or the wireless link fails due to fading. The wireless link can fail because either the Signal to Noise Plus Interference Ratio (SINR) falls below the required protection ratio, $\Psi$ for a given Bit Error Rate (BER) due to multipath fading of Co-Channel Interference (CCI) or the received signal power falls below the minimum received power requirement of the receiver, $\Gamma$ due to multipath fading of the signal. Fading can also occur due to rain, although rain fading can be neglected in the 802.16 frequency range.

### 3.1 Link Outage Probability

As previously mentioned, the multipath fading of all SSs in the network can be modeled using the Rice distribution. If we denote the instantaneous power of the received signal as $p_{\mathrm{o}}$, the instantaneous power of interferer $i$ as $p_i$, the outage probability due to multipath fading of the signal by $P^1_{\mathrm{out}}$ and the outage probability due to the multipath fading of $L$ statistically independent Rice interferers by $P^2_{\mathrm{out}}$, then the overall outage probability is given by

$$\begin{aligned} P_{\mathrm{out}} &= \Pr\left\{ (p_{\mathrm{o}} < \Gamma) \cup \left( p_{\mathrm{o}} < \Psi \sum_{i=1}^{L} p_i \right) \right\} \\ &= P^1_{\mathrm{out}} \cup P^2_{\mathrm{out}} \\ &= P^1_{\mathrm{out}} + P^2_{\mathrm{out}} - P^1_{\mathrm{out}} P^2_{\mathrm{out}} \end{aligned} \tag{9}$$

because $P^1_{\mathrm{out}}$ and $P^2_{\mathrm{out}}$ are assumed to be independent.

$P^1_{\mathrm{out}}$ is easily calculated using (7), but a simple closed expression for the sum of $L$ independent Rice interferers is not available because the solution involves an $L$-fold convolutional integral [15]. However, in [16] a technique for computing $P^2_{\mathrm{out}}$ using the Moment Generating Function (MGF) of the Rice distribution was developed, where

$$P^2_{\mathrm{out}} = \frac{1}{2\pi} \int_0^{\pi} \tilde{\phi}(\theta)\, d\theta \tag{10}$$

if

$$\tilde{\phi}(\theta) = \mathrm{Real}[(1 - j\tan(\theta/2))\phi_\gamma(c + jc\tan(\theta/2))] \tag{11}$$

where $0 < c < a_{\min}$ if $a_{\min} = \min\{a_i\}$ for $i = 1 \ldots L$ with $a_i$ being the $i^{\mathrm{th}}$ pole of the MGF, $\phi_\gamma(s)$. Although $c$ should be chosen to ensure the MGF in (11) decays as rapidly as possible, in [16] it was shown that choosing $c = a_{\min}/2$ is sufficient. The MGF is given by

$$\phi_\gamma(s) = \phi_o(s/\Psi) \prod_{k=1}^{L} \phi_k(-s) \tag{12}$$

where $\Psi$ is the SINR protection ratio, $\phi_o$ is the MGF of the desired signal and $\phi_k$ is the MGF of the $k^{\mathrm{th}}$ interferer. For the Rice distribution the MGF is given as

$$\phi(s) = \frac{1+K}{1+K+s\bar{p}} \exp\left( \frac{-sK\bar{p}}{1+K+s\bar{p}} \right) \tag{13}$$

where $K$ is the rice factor and $\bar{p}$ is the mean received signal power. The integral in (10) can be calculated using the Gauss-Chebychev integration method presented in [16] or any other numerical integration method.

Since the uplink and downlink are assumed to be independent, the overall outage probability for a link between two nodes is then

$$P_{\text{out}}^{\text{link}} = 1.0 - \left[\left(1.0 - P_{\text{out}}^{\text{up}}\right)\left(1.0 - P_{\text{out}}^{\text{down}}\right)\right] \tag{14}$$

where $P_{\text{out}}^{\text{up}}$ and $P_{\text{out}}^{\text{down}}$ are the outage probability for the uplink and downlink respectively and can be calculated using (9).

### 3.2 Node Outage Probability

Equipment failure can be due to many different factors such as weather damage, component failure, vandalism or power outage. The equipment reliability is a function of the Mean Time Between Failures (MTBF) and the Mean Time to Repair (MTTR) and the node outage probability is given by

$$P_{\text{out}}^{\text{node}} = 1.0 - \left[\frac{MTBF}{MTBF + MTTR}\right] \ . \tag{15}$$

## 4 Terminal-Pair Reliability

Much research has been performed in the area of network reliability and many different algorithms are available for calculating the terminal-pair reliability. The terminal-pair reliability algorithm calculates the probability of a connection existing between a source and sink node by decomposing the network into a disjoint event tree where an event is the set of all links in the network. Events can be further divided into success events and failure events. A success event is a set of links which leads to a path between the source and sink and a failure event is an event with no path between source and sink. Since all of the events in the tree are disjoint the reliability can then be calculated as the product of the probabilities of all the success events. The identification of the disjoint events is a well-known NP-hard problem [10] and much of the recent work in this area has focused on speeding up this process by reducing the number of required computations. A simple and efficient algorithm for a network with perfect vertices was presented in [5, 6] and was later extended to include imperfect nodes in [4]. The efficiency of this algorithm has been compared to other algorithms in [4, 5].

The network consists of an undirected graph $G = (V, E)$ with imperfect vertices with success probability, $\alpha$, and imperfect edges with success probability, $\beta$, and all vertices and edges are considered to fail independently. All undirected edges of $G$ can be transformed into two anti parallel directed edges by the transformation shown in Fig. 1. These two anti parallel edges can still be treated as independent random variables (for proof see [17]).

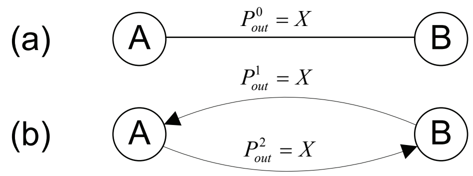


**Fig. 1.** (a) Undirected link (b) Transformation to two anti parallel directed links

An event is represented by a vector $E$ of size $l$, where $l$ is the number of edges in the network. The elements of $E$ are defined as follows:

$$e_i = \begin{cases} 1 & \text{if edge } i \text{ is operational} \ , \\ -1 & \text{if edge } i \text{ has failed} \ , \\ 0 & \text{if edge } i \text{ is not specified} \ . \end{cases}$$

The algorithm works by finding an exhaustive set of all disjoint success events, $S_i$, and is described in Fig. 2.

*Input:* Directed graph $G = (V, E)$
Create a new graph $G'$ corresponding to event $E$
Find the minimum path between the two terminals for network $G'$
If path found:
- Event $E$ is a success event, $S$
- Partition the network further by taking the complement of event $E$, where the complement of $E$ is defined as $\overline{E} = \{\overline{e_1}\} \cup \{e_1\overline{e_2}\} \cup \{e_1e_2\overline{e_3}\} \cup \ldots \cup \{e_1e_2\ldots\overline{e_l}\}$ where $e_i$ is the $i^{\text{th}}$ element of $E$ [6]

Else: Event $E$ is a failure event, $F$
Repeat until all events in $G$ have been processed or the bounds in (19) are reached

**Fig. 2.** Terminal-pair reliability algorithm

Using this method a symbolic expression for the reliability of the network can be obtained as

$$R = \sum_{i=1}^{|S|} \Pr\{S_i\} \tag{16}$$

where $\Pr\{S_i\}$ is the probability of the success event $S_i$ and $|S|$ is the number of successful events. Finally the algorithm is extended to include imperfect vertices giving the network reliability as

$$R = \alpha_s \sum_{i=1}^{|S|} \prod_{j=1, j \neq s}^{N} \Pr\{S_{ij}\} \tag{17}$$

where $\alpha_s$ is the success probability of the source node, $N$ is the number of nodes in the network and $\Pr\{S_{ij}\}$ can be computed as follows:

$$\Pr\{S_{ij}\} = \begin{cases} 1 & N = 0, K = 0, \\ \alpha_j \prod_{i=1}^{N} (1 - \beta_i) \prod_{i=1}^{K} \beta_i & N \geq 1, K \geq 1, \\ (1 - \alpha_j) + \alpha_j \prod_{i=1}^{N} (1 - \beta_i) & N \geq 1, K = 0, \\ \alpha_j \prod_{i=1}^{K} \beta_i & N = 0, K \geq 1. \end{cases} \tag{18}$$

In (18), $K$ is the number of operational links directed into node $j$, $N$ is the number of failed links directed into node $j$ and $\alpha_j$ and $\beta_i$ are the success probabilities of node $j$ and link $i$, respectively. Using (17), the terminal pair reliability may be calculated and if the algorithm is stopped before it is completed, then the bounds of the calculation will be

$$\alpha_s \sum_{i=1}^{|S|} \prod_{j=1, j \neq s}^{N} \Pr\{S_{ij}\} \leq R \leq \alpha_s - \alpha_s \sum_{i=1}^{|F|} \prod_{j=1, j \neq s}^{N} \Pr\{F_{ij}\} \tag{19}$$

where $\Pr\{F_{ij}\}$ is also given by (18), except $\beta_i$ refer to the links in the set of failure events $F$.

## 5 Example Calculation

In order to demonstrate the techniques shown in this paper, the terminal-pair reliability between node 1 and node 5 in the example mesh network shown in Fig. 3 will now be evaluated. Node 1 represents the source SS and node 5 represents the BS or sink. The assumed network parameters are given in Table 2. It is also assumed that the network has a frequency reuse factor of 2 so that there is always one interferer present for every node.

The first step will be to calculate the outage probabilities for all links in the network. An example will now be provided for the link between node 1 and node 2. Assuming a distance of 600 m between node 1 and node 2 and the distance of the interferer from node 1 to be 1500 m and from node 2 to be 2500 m, the following values are calculated for the link between node 1 and node 2 (Note:

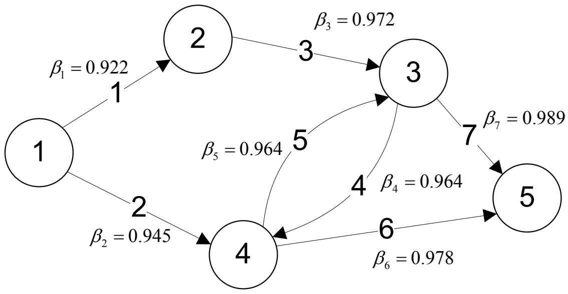


**Fig. 3.** Example network

**Table 2.** Network parameters

| | |
|---|---|
| Environment | Type C (Summer) |
| SS Antenna Height | 7 m |
| SS Transmitter Power | 35 dBm |
| Antenna | Omni, Gain = 8 dBi |
| Frequency | 3.5 GHz |
| Channel BW | 5.5 MHz |

The signal R.F. has been calculated using (8) assuming $u = 0$):

$$\begin{array}{lll}
\text{Path Loss}_{1\leftrightarrow 2} & = +\ 123.3 & \text{dB} \\
\text{Shadow Factor } (s_{2\to 1}) & = -\ 4.25 & \text{dB} \\
\text{Shadow Factor } (s_{1\to 2}) & = +\ 1.32 & \text{dB} \\
\text{Power Received}_{\text{node1}} & = -\ 76.1 & \text{dBm} \\
\text{Power Received}_{\text{node2}} & = -\ 81.6 & \text{dBm} \\
\text{SINR}_{\text{node1}} & = +\ 22.4 & \text{dB} \\
\text{SINR}_{\text{node2}} & = +\ 24.7 & \text{dB} \\
\text{Signal R.F., } K_{\text{o}} & = +\ 4.58 & \text{dB} \\
\text{Node 1 Interferer R.F., } K_i & = +\ 2.59 & \text{dB} \\
\text{Node 2 Interferer R.F., } K_i & = +\ 1.48 & \text{dB}
\end{array}$$

Since 802.16 receivers employ adaptive modulation, the link will only fail once the SINR or received power falls below the required thresholds for the most robust modulation scheme. The required received power for the most robust modulation scheme (BPSK 1/2) is $\Gamma = -89\,\text{dBm}$ and the required SINR protection ratio is $\Psi = 3.0\,\text{dB}$ for a BER of $10^{-6}$ [2, 18]. In order to calculate the outage probability for the link between node 1 and node 2, the outage probability of the downlink and uplink must be calculated. The received power fade margin for the downlink is equal to: $FM = -76.1\,\text{dBm} - (-89\,\text{dBm}) = 12.9\,\text{dB}$. Using this power requirement ratio and the signal R.F. given in Table 2, the outage probability due to fading of the signal received by node 1 from node 2 can be found using (7):

$$P_{\mathrm{out}}^{1(2\to 1)} = 1 - Q\left(\sqrt{2\cdot K_{\mathrm{o}}}, \sqrt{\frac{2\cdot(1+K_{\mathrm{o}})}{FM}}\right) = 0.0133$$

where $K_{\mathrm{o}}$ and $FM$ must be linear values. In [15] a simplified expression of (10) was presented assuming only one interferer is present:

$$\begin{aligned} P_{\mathrm{out}}^{2} =& Q\left[\sqrt{\frac{2K_i\Psi}{b+\Psi}}, \sqrt{\frac{2K_{\mathrm{o}}b}{b+\Psi}}\right] \\ & - \frac{b}{b+\Psi}\exp\left[-\frac{K_i\Psi + K_{\mathrm{o}}b}{b+\Psi}\right] \\ & \times I_{\mathrm{o}}\left(\frac{\sqrt{4K_iK_{\mathrm{o}}\Psi b}}{b+\Psi}\right) \end{aligned} \tag{20}$$

where $K_{\mathrm{o}}$ and $K_i$ are the Rice factors of the signal and the interferer, respectively and $b = \mathrm{SINR}\cdot(K_i+1)/(K_{\mathrm{o}}+1)$. Using (20), the outage probability due to multipath fading of the interferer for the downlink between node 1 and node 2 is $P_{\mathrm{out}}^{2(2\to 1)} = 1.25\times 10^{-3}$. The overall outage probability for the downlink $2\to 1$ can then be calculated as

$$\begin{aligned} P_{out}^{2\to 1} &= P_{\mathrm{out}}^{1(2\to 1)} + P_{\mathrm{out}}^{2(2\to 1)} - P_{\mathrm{out}}^{1(2\to 1)}\cdot P_{\mathrm{out}}^{2(2\to 1)} \\ &= 0.0145 \end{aligned}$$

Similarly, for the uplink, the received power fade margin is 7.38 dB which gives $P_{\mathrm{out}}^{1(1\to 2)} = 0.0642$ using (7). Then with $P_{\mathrm{out}}^{2(1\to 1)} = 6.85\times 10^{-4}$ from (20), the overall outage probability for the uplink $1\to 2$ is $P_{out}^{1\to 2} = 0.0649$. Finally, the overall outage probability for the link between node 1 and node 2 is calculated using (14):

$$\begin{aligned} P_{\mathrm{out}}^{1\leftrightarrow 2} &= 1 - (1 - P_{\mathrm{out}}^{2\to 1})(1 - P_{\mathrm{out}}^{1\to 2}) \\ &= 0.0785 \end{aligned}$$

The outage probability could then be similarly calculated for all the other links in the network and the assumed values for the link availabilties (link availability is the complement of the outage probability) are shown in Fig. 3. The outage probabilities for all nodes must then be calculated. Assuming that for all nodes the $MTTF$ is 720 hours and the $MTBF$ is 8 hours, the outage probability for all nodes can be found using (15) and will be $P_{\mathrm{out}}^{\mathrm{node}} = 0.011$ or, in terms of availability probability, $\alpha_i = 0.989$ for $i = 1\ldots 5$.

Now, that all the link and node outage probabilities are known, the terminal-pair reliability may be found. The symbolic outage probability assuming perfect nodes can be calculated using the algorithm in Fig. 2 along with (16). The resulting decomposition event tree is shown in Fig. 4 and the symbolic expression for the reliability between node 1 and node 5 is given by

$$\begin{aligned} R_{1\to 5} =& \beta_2\beta_6 + \beta_1\overline{\beta_2}\beta_3\beta_7 + \beta_1\beta_2\beta_3\overline{\beta_6}\beta_7 + \\ & \beta_1\overline{\beta_2}\beta_3\beta_4\beta_6\overline{\beta_7} + \overline{\beta_1}\beta_2\beta_5\overline{\beta_6}\beta_7 + \beta_1\beta_2\overline{\beta_3}\beta_5\overline{\beta_6}\beta_7 \ . \end{aligned}$$

Then using (17), the outage probability is extended to include imperfect nodes

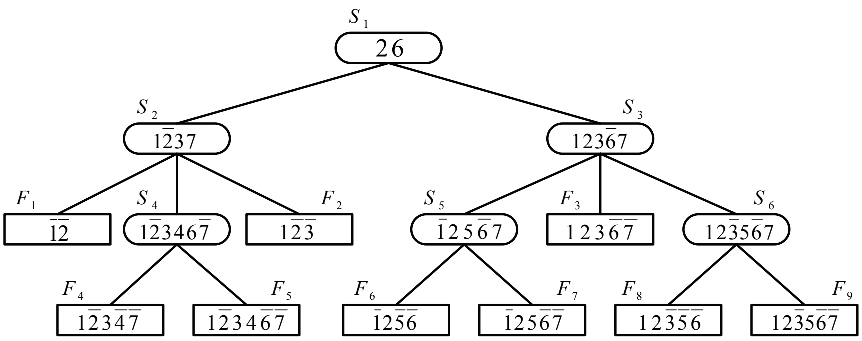

**Fig. 4.** Event tree: $i$ means link $i$ is operational, $\bar{i}$ means link $i$ has failed, $S_i$ are the success events and $F_i$ are the failure events

and the resulting symbolic terminal-pair reliability is

$$
\begin{aligned}
R_{1\to 5} = \alpha_1 \sum_{i=1}^{6} \prod_{j=2}^{5} \Pr\{S_{ij}\} = \alpha_1 \Big[ & (\alpha_4\beta_2)(\alpha_5\beta_6) + (\alpha_2\beta_1)(\alpha_3\beta_3)(\overline{\alpha_4} + \alpha_4\overline{\beta_2})(\alpha_5\beta_7) \\
& + (\alpha_2\beta_1)(\alpha_3\beta_3)(\alpha_4\beta_2)(\alpha_5\overline{\beta_6}\beta_7) + (\alpha_2\beta_1)(\alpha_3\beta_3)(\alpha_4\overline{\beta_2}\beta_4)(\alpha_5\beta_6\overline{\beta_7}) \\
& + (\overline{\alpha_2} + \alpha_2\overline{\beta_1})(\alpha_3\beta_5)(\alpha_4\beta_2)(\alpha_5\overline{\beta_6}\beta_7) + (\alpha_2\beta_1)(\alpha_3\overline{\beta_3}\beta_5)(\alpha_4\beta_2)(\alpha_5\overline{\beta_6}\beta_7) \Big] \ .
\end{aligned}
$$

Finally, inserting the link availability probabilities, $\beta_i$, shown in Fig. 3 and the node availability probabilities, $\alpha_i$, previously calculated into the symbolic terminal-pair reliability expression, the terminal-pair reliability of the connection between node 1 and node 5 is $R_{1\to 5} = 0.96957$.

## 6 Conclusion

An efficient method for calculating the terminal-pair reliability of IEEE 802.16 mesh networks was presented in this paper. Methods were shown for calculating the outage probabilities for all links and nodes in the network as well as for calculating the overall outage probability of the link between a SS and the BS based on the multihop/multipath link capabilities of a mesh network. The capability of a mesh node to choose the most reliable path to the BS and avoid poor links in the network is one of the advantages of using the mesh topology over PMP or relay. It allows mesh nodes to use higher modulation rates and achieve lower BERs with more reliable links.

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