On gearing of helical screw compressor rotors

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Abstract: Twin-screw compressor rotors are effectively helical gears. When these are formed from a hobbing cutter, the hobbing tool and the rotor together constitute a pair of crossed helical gears. In the present paper, the envelope gearing method is used to derive a meshing condition for crossed helical gears which is then used to create the profile of a hobbing tool. A reverse transformation enables the rotor profile thereby manufactured to be calculated. Simplification of the main gearing condition leads to the meshing expression for helical gears which may be used for the design of screw compressor rotors.

Keywords: helical screw compressors, screw rotor profiles, crossed helical gears, meshing condition

NOTATION

C	rotor centre distance
i	gear ratio
k	ratio $= 1 - 1/i$
p	lead per unit angle
r	radius
$\mathbf{r}$	position vector
t	profile parameter
x	coordinate
y	coordinate
z	coordinate
θ	rotation angle, main rotor
Σ	shaft angle
τ	rotation angle, gate rotor, tool
ϕ	pressure angle
φ	rotor coordinate angle
ψ	helix angle

Subscripts

r	rack
0	transversal
1	main rotor
2	gate rotor, tool

1 INTRODUCTION

Twin-screw compressors or expanders are positive displacement rotary machines comprising a meshing pair of helicoid lobed rotors on parallel axes, contained in a casing, which together form a working chamber whose volume depends on the angle of rotation. A typical pair of rotors is shown in Fig. 1. In this case the driving rotor, situated on the right, rotates clockwise when operating as a compressor. Admission of the gas occurs through the low-pressure surface portion on the upper face of the rotors while the gas is discharged mainly through the high-pressure portion on the bottom-rear end face of the rotors. The compression process therefore leads to axial and bending forces on the rotors and also contact forces between the rotor lobes.

During the suction process, exposure of the rotors across the suction port on the top of the rotors allows the gas to fill the passages between the rotor lobes and the casing. Further rotation then leads to cut-off of the port and progressive reduction in the trapped volume. This continues until the passages are exposed to the discharge port through which the gas flows out at approximately constant pressure.

The adiabatic and volumetric efficiencies of screw compressors are dependent on both the profile and number of the lobes on each rotor. For such machines to perform effectively, the rotors must meet all the requirements of helical gears and, in addition, they should maintain a seal along the entire high-pressure side. This can be done by keeping the area of the triangular-shaped surface formed between the rotor tips and the housing cusp, known as the blowhole area, as small as possible. The blowhole can be kept small by making the curves generated by the mutual gearing action of the small flank portions close to the tips of both rotor lobes as trochoids. Trochoids, as well as the other curves usually generated for different sections of screw compressor rotor tooth profiles, can be manufactured by the use of

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Fig. 1 Screw rotors in mesh

formed tools. This means that tools suitable for screw rotor production must, in turn, be calculated by some appropriate gearing procedure.

The envelope method which states that two surfaces are in mesh if each generates or envelops the other under a specified relative motion is suitable for this purpose. Following Litvin [**1**], the procedure starts with a given generating surface $\boldsymbol{r}_1(t, \theta)$ for which a meshing or generated surface is to be determined. A family of such surfaces is given in parametric form by $\boldsymbol{r}_2(t, \theta, \tau)$, where t is a profile parameter while θ and τ are motion parameters. The envelope equation

$$f(t, \theta, \tau) = \frac{\partial \boldsymbol{r}_2}{\partial \tau} \cdot \left(\frac{\partial \boldsymbol{r}_2}{\partial t} \times \frac{\partial \boldsymbol{r}_2}{\partial \theta} \right) = 0 \tag{1}$$

together with equations for these surfaces completes a system of equations. If a generating surface 1 is defined by the parameter t, the envelope may be used to calculate another parameter θ, now a function of t, as a meshing condition to define a generated surface 2, now the function of both t and θ. The cross-product in the envelope equation represents the normal to the surface 2 and $\partial \boldsymbol{r}_2/\partial \tau$ is the relative sliding velocity of two points on the surfaces 1 and 2 which coincide in the common tangential point of contact of these two surfaces. Since the equality to zero of a scalar triple product is an invariant property under the applied coordinate system and since the relative velocity may be concurrently represented in both coordinate systems, a convenient form of the meshing condition can be defined:

$$f(t, \theta, \tau) = \frac{\partial \boldsymbol{r}_1}{\partial \tau} \cdot \left(\frac{\partial \boldsymbol{r}_1}{\partial t} \times \frac{\partial \boldsymbol{r}_1}{\partial \theta} \right) = -\frac{\partial \boldsymbol{r}_2}{\partial \tau} \cdot \left(\frac{\partial \boldsymbol{r}_1}{\partial t} \times \frac{\partial \boldsymbol{r}_1}{\partial \theta} \right) = 0 \tag{2}$$

This is applied here to derive the condition of meshing action for crossed helical gears of uniform lead with non-parallel and non-intersecting axes. The method constitutes a gear generation procedure which is generally applicable. It can be used for the synthesis of screw compressor rotors, which are effectively helical gears with parallel axes. Formed tools for rotor manufacturing are crossed helical gears on non-parallel and non-intersecting axes with a uniform lead as in the case of hobbing, or with no lead as in formed milling and grinding. Templates for rotor inspection are the same as planar rotor hobs. In all these cases the tool axes do not intersect the rotor axes.

Accordingly, this paper presents an application of the envelope method for production of a meshing condition for crossed helical gears. Twin-screw rotor gearing is then given as an elementary example of its use while a procedure for forming a hobbing tool is given as a complex case.

2 MESHING OF CROSSED HELICAL GEARS

The coordinate system for crossed helical gears with non-parallel and non-intersecting axes is given in Fig. 2. In this case both leads are uniform and of values p_1 and p_2. The shortest distance between the two axes is C and the angle between them is Σ.

The coordinate system X_1, Y_1, Z_1 by which gear 1 curved surface $\boldsymbol{r}_1 = [x_1, y_1, z_1]$ is defined is fixed, but the coordinate system X_{01}, Y_{01} rotates around Z_1 with gear 1. The transverse or end cross-section coordinates of gear 1 are $x_{01}(t)$ and $y_{01}(t)$. The transformation between these two coordinate systems is given as

$$x_1 = x_1(t, \theta) = x_{01} \cos\theta - y_{01} \sin\theta \tag{3}$$

$$y_1 = y_1(t, \theta) = x_{01} \sin\theta + y_{01} \cos\theta \tag{4}$$

$$z_1 = z_1(\theta) = p_1 \theta \tag{5}$$

where p_1 is the lead per unit angle of gear 1.

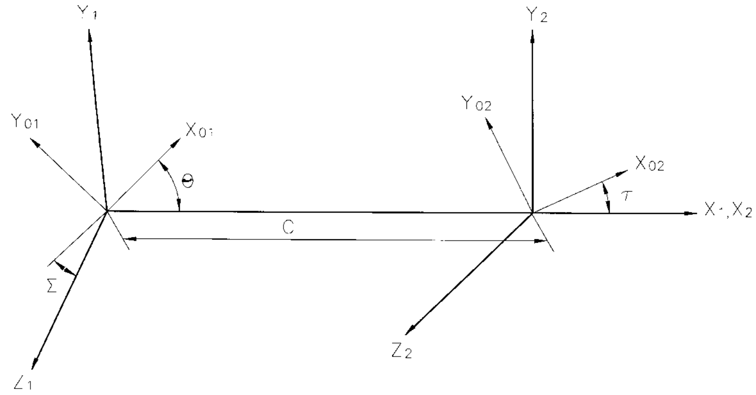

Fig. 2 Coordinate systems of crossed helical gears

Derivatives needed for meshing condition (2) are

$$\frac{\partial \boldsymbol{r}_1}{\partial t} = \left[\frac{\partial x_1}{\partial t}, \frac{\partial y_1}{\partial t}, 0\right] \quad \text{and} \quad \frac{\partial \boldsymbol{r}_1}{\partial \theta} = [-y_1, x_1, p_1] \tag{6}$$

since

$$\frac{\partial x_1}{\partial \theta} = -x_{01} \sin\theta - y_{01}\cos\theta = -y_1 \tag{7}$$

$$\frac{\partial y_1}{\partial \theta} = x_{01}\cos\theta - y_{01}\sin\theta = x_1 \tag{8}$$

$$\frac{\partial z_1}{\partial \theta} = p_1 \tag{9}$$

Similarly, the coordinate system X_2, Y_2, Z_2, by which gear 2 curved surface $\boldsymbol{r}_2 = [x_2, y_2, z_2]$ is defined is fixed, but the coordinate system X_{02}, Y_{02} rotates with gear 2. The transverse coordinates of gear 2 are x_{02} and y_{02}. The relation between these two coordinate systems is

$$x_2 = x_{02}\cos\tau - y_{02}\sin\tau \tag{10}$$

$$y_2 = x_{02}\sin\tau + y_{02}\cos\tau \tag{11}$$

$$z_2 = p_2\tau \tag{12}$$

where p_2 is the lead per unit angle of gear 2. Since

$$\frac{\partial x_2}{\partial \tau} = -x_{02}\sin\tau - y_{02}\cos\tau = -y_2 \tag{13}$$

$$\frac{\partial y_2}{\partial \tau} = x_{02}\cos\tau - y_{02}\sin\tau = x_2 \tag{14}$$

$$\frac{\partial z_2}{\partial \tau} = p_2 \tag{15}$$

a derivative of $\boldsymbol{r}_2$ in system 2 is

$$\frac{\partial \boldsymbol{r}_2}{\partial \tau} = [-y_2, x_2, p_2] \tag{16}$$

The transformation between coordinate systems X_1, Y_1, Z_1 and X_2, Y_2, Z_2, as defined in Fig. 2 is as follows:

$$x_2 = x_1 - C \tag{17}$$

$$y_2 = y_1\cos\Sigma - z_1\sin\Sigma \tag{18}$$

$$z_2 = y_1\sin\Sigma + z_1\cos\Sigma \tag{19}$$

where C is the distance between non-parallel axes Z_1 and Z_2 and Σ is the angle between them. Hence, the derivative of $\boldsymbol{r}_2$ in system 1 is

$$\frac{\partial \boldsymbol{r}_2}{\partial \tau} = [p_1\theta\sin\Sigma - y_1\cos\Sigma,\; p_2\sin\Sigma + (x_1 - C)\cos\Sigma,\; p_2\cos\Sigma - (x_1 - C)\sin\Sigma] \tag{20}$$

Insertion of (6) and (13) into (2) gives a convenient form of the envelope equation for crossed helical gears which can now be used to evaluate the required meshing condition $\theta = \theta(t)$:

$$[C - x_1 + (p_1 - p_2)\cot\Sigma]\left(x_1\frac{\partial x_1}{\partial t} + y_1\frac{\partial y_1}{\partial t}\right) + p_1\left[p_1\theta\frac{\partial y_1}{\partial t} + (p_2 - C\cot\Sigma)\frac{\partial x_1}{\partial t}\right] = 0 \tag{21}$$

The shaft angle Σ, centre distance C, and unit leads of two crossed helical gears, p_1 and p_2, are not interdependent. A meshing of crossed helical gears is still preserved: both gear racks have the same normal cross-section profile and the rack helix angles are related to the shaft angle as $\Sigma = \psi_{r1} + \psi_{r2}$. This is achieved by implicit shift of the gear racks in the x direction, forcing them to adjust accordingly to the appropriate rack helix angles. This certainly includes special cases, like that of gears which may be oriented so that the shaft angle is equal to the sum of gear helix angles: $\Sigma = \psi_1 + \psi_2$. Furthermore, the centre distance may be equal to the sum of the gear pitch radii: $C = r_1 + r_2$.

Pairs of crossed helical gears may be with either both helix angles of the same sign or each of opposite sign (left- or right-handed), depending on the combination of their lead and shaft angle Σ. The crossed helical gear meshing condition in the form of equation (21) can be solved only by numerical methods. For the given parameter t, the coordinates x_{01} and y_{01} and their derivatives $\partial x_{01}/\partial t$ and $\partial y_{01}/\partial t$ are known. A guessed value of parameter θ is then used to calculate $x_1, y_1, \partial x_1/\partial t$ and $\partial y_1/\partial t$ from equations (3) to (5). A revised value of θ is then derived from equation (21) and the procedure repeated until the difference between two consecutive values becomes sufficiently small.

For given transverse coordinates and derivatives of gear 1 profile, θ determined from (21) may be used to calculate the x_1, y_1, z_1 coordinates of its helicoid surface from equations (3) to (5). The gear 2 helicoid surface may then be calculated from equations (17) to (19). Coordinate z_2 may then be used to calculate τ from (12) and, finally, its transverse profile point coordinates may be obtained from (10) and (11).

Reference books on gears consider this application too complex to be derived in general form. Only a few cases, such as straight-sided tools for involute generation, are considered there. However, Andreev [**2**] and recently Tang [**3**] give a singular case of non-lead form tools for screw compressor rotor milling, valid for $p_2 = 0$. The same is occasionally described in specialist engineering literature on screw compressor manufacturing.

A number of cases can be identified from this equation:

1. When $\Sigma = 0$ the equation meets the meshing condition of screw machine rotors and also helical gears with parallel axes. For such a case the gear helix angles have the same value but the opposite sign and the gear ratio $i = p_2/p_1$ is negative. The same equation may also be applied for the generation of a rack formed from gears. In addition it describes the formed planar hob, front milling tool and the control instrument.
2. If a disc formed milling or grinding tool is considered, it is sufficient to place $p_2 = 0$. This is a singular case where tool free rotation does not affect the meshing process. Therefore, a reverse transformation cannot be obtained directly.
3. The full scope of the meshing condition is required for the generation of the profile of a formed hobbing tool. This is therefore the most complicated type of gear which can be generated from it.

All three cases will be demonstrated in more detail in the next section.

3 SCREW COMPRESSOR ROTOR MESHING

Screw machine rotors have parallel axes and a uniform lead and they are therefore a form of helical gears. Axis distance for this particular case is $C = r_1 + r_2$, where r_1 and r_2 are the rotor pitch circle radii. Rotors make line contact and the meshing criterion in the transverse plane perpendicular to their axes is the same as that of spur gears. Although spur gear meshing fully defines helical screw rotors, it may be more convenient to use the envelope condition for crossed helical gears which was given in equation (21) and simplify it by setting $\Sigma = 0$ in order to obtain the required meshing condition.

To start the procedure of rotor profiling, the profile point coordinates in the transverse plane of one rotor, x_{01} and y_{01}, and their first derivative, say $\mathrm{d}y_{01}/\mathrm{d}x_{01}$, must be known. This primary profile may be specified on either the main or gate rotors or in sequence on both. Also the primary profile may be defined as a rack. In Fig. 3 the main rotor profile is created from its rack, transferred to the main rotor and used in that form for the later analysis. The trochoid on the rack high-pressure side is formed by the gearing action of the main and gate rotor tip circles. More details of this profile are given by Stošić and Hanjalić [**4**].

Since $\Sigma = 0$, the meshing condition for a screw

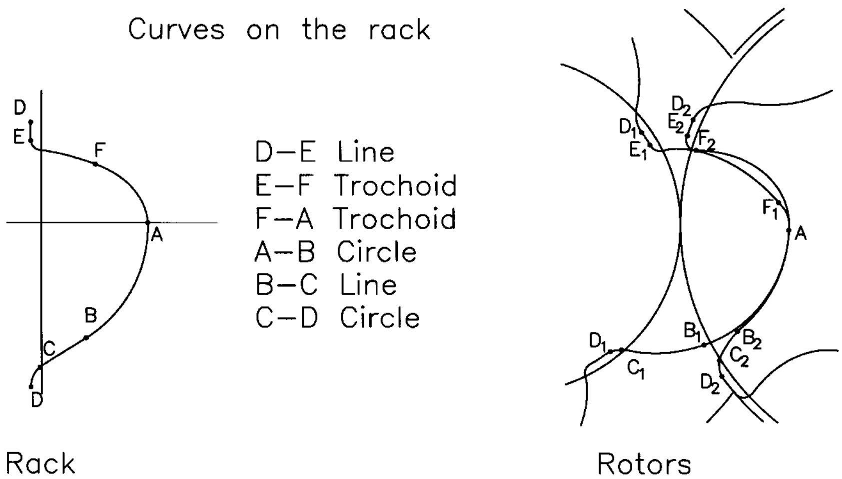

Fig. 3 ‘N’ rack and rotor profile points

machine rotor simplified from (21) is

$$\frac{dy_{01}}{dx_{01}}\left(ky_{01}-\frac{C}{i}\sin\theta\right)+kx_{01}+\frac{C}{i}\cos\theta=0 \tag{22}$$

where $i = p_2/p_1$ and $k = 1 - 1/i$. Once obtained, the distribution of θ along the profile may be used to calculate the meshing rotor profile point coordinate, as well as to determine the sealing lines and paths of proximity between the two rotors. Rotor rack coordinates may also be calculated from the same θ distribution.

Since $\tau = \theta/i$ for parallel axes, the meshing profile equations of the gate rotor in the transverse plane are obtained from equations (3), (4), (10), (11), (17) and (18) as

$$x_{02} = x_{01}\cos k\theta - y_{01}\sin k\theta - C\cos\frac{\theta}{i} \tag{23}$$

$$y_{02} = x_{01}\sin k\theta + y_{01}\cos k\theta + C\sin\frac{\theta}{i} \tag{24}$$

Rack coordinates may be obtained uniquely from equations (23) and (24) if the rack/rotor gear ratio i tends to infinity:

$$x_{0r} = x_{01}\cos\theta - y_{01}\sin\theta \tag{25}$$

$$y_{0r} = x_{01}\sin\theta + y_{01}\cos\theta - r_1\theta \tag{26}$$

The sealing line of screw compressor rotors is somewhat similar to the gear contact line. Since there exists a clearance gap between rotors, the sealing line is a line consisting of points of the most proximate rotor position. Its coordinates are x_1, y_1, z_1 and they are calculated from equations (3) to (5) for the same θ distribution. The most convenient practice to obtain an interlobe clearance gap is to consider the gap as the shortest distance between two rotor racks of the main and gate rotor sealing points in the cross-section normal to the rotor helicoid. The rotor racks, obtained from the rotors by the reverse procedure, may include all manufacturing and positioning imperfections. Therefore the resulting clearance distribution may represent real-life compressor clearances. From normal clearances, a transverse clearance gap may be obtained by the appropriate transposition.

A configuration of five to six lobes on the main and gate rotor is presented as an example here. The rotor helix angles are 40° and have opposite signs. Rotor centre distance is 90 mm. The meshing rotors (bold line) and their sealing lines (dashed line), with a front view in the centre and a side view on the right, as well as the uniform interlobe clearance distribution between two rotor racks in the transverse cross-section, with a scale factor of 100, are presented in Fig. 4.

Further rearrangement of rotor meshing condition (22) gives a form which is frequently used for profiling of spur and helical gears. Let ϕ be a pressure angle or the angle of the normals of the rotors and rack at the contact point. For the given rotor $\tan\phi_1 = -dx_{01}/dy_{01}$. Let φ be a profile angle at the contact point, $\tan\varphi_1 = y_{01}/x_{01}$, which implies $x_{01} = r_{01}\cos\varphi_1$ and $y_{01} = r_{01}\sin\varphi_1$, where r_{01} is the point radius. Let θ be a meshing condition or the rotation angle of the main rotor for which the rotors and rack are in contact at local points (x_{01}, y_{01}), (x_{02}, y_{02}) and (x_r, y_r). Insertion into (22) produces a relation between these angles required for a meshing condition θ:

$$\frac{\sin(\phi_1+\theta)}{r_{01}} = \frac{\sin(\phi_1-\varphi_1)}{r_1} \tag{27}$$

A graphical presentation of this meshing condition

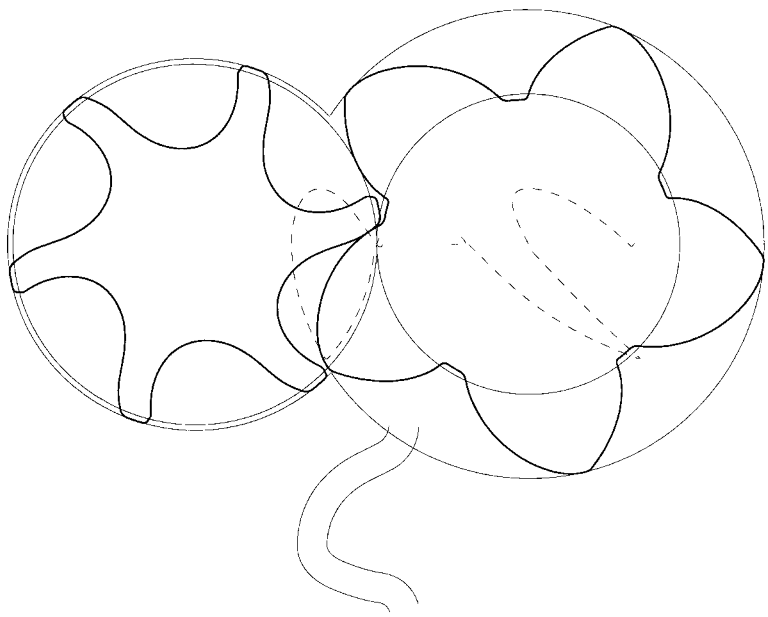

Fig. 4 'N' rotors in mesh, sealing line and clearance distribution

(Fig. 5) confirms that the situation where normals of the two gears and rack at their contact point pass through the pitch point is nothing but a special case of the envelope meshing condition.

4 GENERATION OF FORMED TOOLS FOR SCREW ROTOR MANUFACTURE

A screw compressor rotor and its formed hobbing tool are meshing crossed helical gears with non-parallel and non-intersecting axes. Apart from the gashes forming the cutter faces, the hob is simply a helical gear in which each tooth is referred to as a thread [**5**]. Owing to their axes not being parallel, there is only point contact between them, whereas there is line contact between the screw machine rotors. The need to satisfy equation (21) leads to the rotor–hob meshing requirement for the given rotor transverse coordinate points x_{01} and y_{01} with dy_{01}/dx_{01}. The hob transverse coordinate points x_{02} and y_{02} can then be calculated. These are sufficient to obtain the coordinate $R_2=\sqrt{(x_{02}^2+y_{02}^2)}$. The axial coordinate z_2, calculated directly, and R_2 are hob axial plane coordinates which define the hob geometry.

The transverse coordinates of the screw machine rotors described in the previous section are used as an example here to produce hob coordinates. Rotor unit leads p_1 are 48.754 mm for the main rotor and -58.504 mm for the gate rotor. Single-lobe hobs are generated for unit leads p_2: 6.291 mm for the main rotor and -6.291 mm for the gate rotor. The corresponding hob helix angles ψ_2 are 85° and 95°. The same rotor–hob centre distance $C=110$ mm and shaft angle $\Sigma=50°$ are given for both rotors. The hob axial plane coordinates are shown in Fig. 6.

Reverse calculation of the hob–screw rotor transformation permits determination of the transverse rotor profile coordinates that will be obtained as a result of the manufacturing process. These may be compared with those originally specified to determine the effect of manufacturing errors such as imperfect tool setting or tool and rotor deformation upon the final rotor profile.

For the purpose of reverse transformation, the hob longitudinal plane coordinates R_2 and z_2 and dz_2/dR_2 should be given. An axial coordinate z_2 is used to calculate $\tau=z_2/p_2$ which is then used to calculate the hob transverse coordinates:

$$x_{02}=R_2\cos\tau, \qquad y_{02}=R_2\sin\tau \tag{28}$$

These are then used as given coordinates in equation (21) to produce a meshing criterion and then the transverse plane rotor coordinates.

A comparison of original rotors and manufactured rotors is given in Fig. 7 with the difference between them scaled 100 times. Two types of error are considered. The gate rotor (left) is produced with a 30 μm offset in centre distance between the rotor and tool, and the main rotor with a 0.04° offset in the tool shaft angle Σ.

As already stated, milling and grinding tools may also be generated by placing $p_2=0$ in equation (21) and then following the procedure of this section. However, if screw rotors are expected to be calculated from the tool coordinates, the singularity imposed does not permit calculation of the tool transverse plane coordinates. The main meshing condition cannot therefore be applied. For this purpose another condition is derived for the reverse milling tool–rotor transformation, from which a meshing angle τ is calculated:

$$\left(R_2+z_2\frac{dz_2}{dR_2}\right)\cos\tau+(p_1+C\cot\Sigma)\frac{dz_2}{dR_2}\sin\tau + p_1\cot\Sigma-C=0 \tag{29}$$

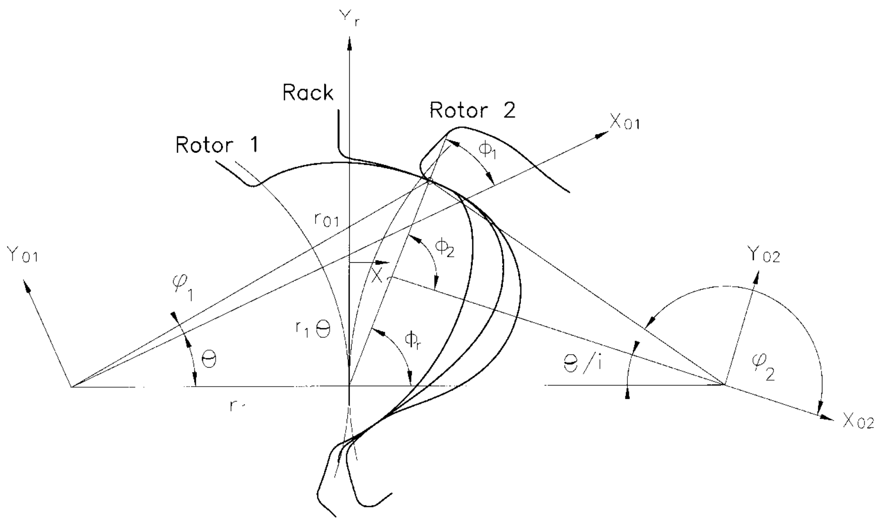

Fig. 5 Plane meshing of helical rotors

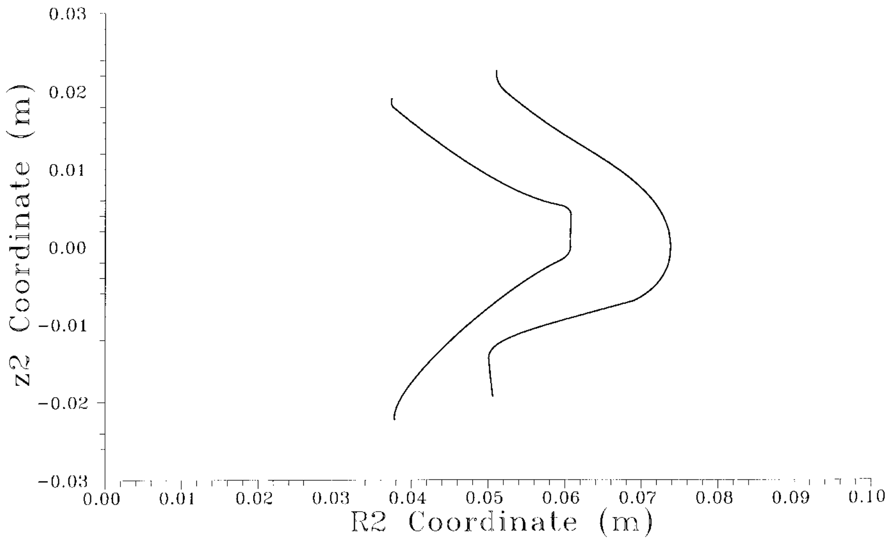

Fig. 6 Tool profiles generated from their given screw rotors

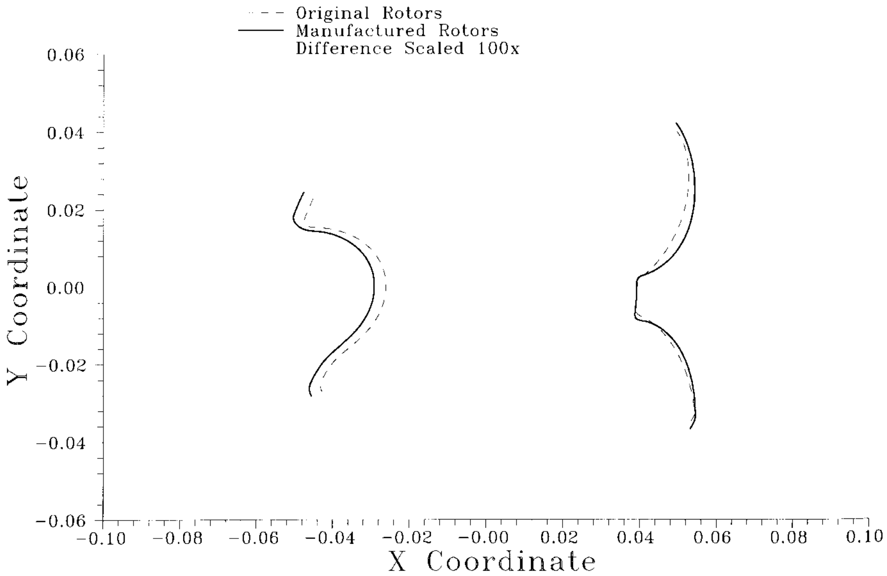

Fig. 7 Rotor profiles generated from a given tool

5 CONCLUSIONS

Screw compressor rotors can be generated from formed tools and these may all be formed as crossed helical gears with non-parallel and non-intersecting axes. For this purpose the envelope theory of gearing has been applied as the meshing requirement for crossed helical gears and illustrated by two examples.

The first of these is the profiling of a formed hobbing tool from known rotors and the reverse profiling of the screw machine rotors from a known hob. The second example is the profiling of another screw rotor and its rack from the known primary rotor. For this purpose, the full meshing condition of crossed helical gears is simplified to that of helical gears, i.e. gears with parallel axes. Although used here primarily in screw machine rotor gearing, the meshing condition shown may be conveniently employed in general gearing practice.

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