

Mathematics for Actuarial Science: Answer sheet 1

Sheet 1

1. $x = \sqrt{\frac{b}{a-c^2d}}, \quad 2.$
2. $(p^2 + q) + 2p\sqrt{q},$
 $(p^2 + 3pq) + (3p^2 + q)\sqrt{q}.$
3. $32 - 240x + 720x^2 - 1080x^3 + 810x^4 - 243x^5.$
4. $A = 44, B = 924, C = 12320.$
5. 1215.
6. $\frac{1}{3}, \quad 1.$
7. $3x^2 - 3x - 2 = 0.$
8. $k \leq -6 \text{ or } k \geq 2, \text{ (a) } -155,$
 $\text{ (b) } x^2 - 14x - 479 = 0.$
9. $x^4 + 3x^3 - 2x + 1.$
10. $(x - 4)(x^2 - 4x + 13), \quad 4.$
11. $(x - 2)(x - 3)(2x + 1).$
12. $a = -1, b = 4.$
13. $k = 9.$
14. $-\frac{2}{(1+x)^2} + \frac{3}{1+3x}.$
15. $\frac{1}{1-2x} - \frac{2x}{1+x^2}.$
16. $(x^2 + 2\sqrt{3}x + 4)(x^2 - 2\sqrt{3}x + 4),$
 $\frac{x+2\sqrt{3}}{16\sqrt{3}(x^2+2\sqrt{3}x+4)} - \frac{x-2\sqrt{3}}{16\sqrt{3}(x^2-2\sqrt{3}x+4)}.$
17. $1 - \frac{1}{x^2} - \frac{2}{3(x+1)} - \frac{1-2x}{3(x^2-x+1)}.$

Sheet 2

1. $a = 25, d = -3, \text{ (b) } S = -3810.$
2. $n = 33, d = \frac{7}{2}, S = 1450.$
3. $\frac{1}{3}n(4n^2 - 1).$
4. (a) $f^{-1}(x) = \frac{2x+1}{x-3}, x \neq 3.$
(b) $f(f(x)) = \frac{10x+1}{x+5}, x \neq -5, x \neq 2.$
5. (a) $x \geq 0, \text{ (b) } 0, 8,$
(c) i. 2, 6, ii. 1, $-\frac{4}{3}.$
6. (a) $\mathbb{R}, y \geq -\frac{1}{4}, \text{ (b) } x \leq 1 \text{ or } x \geq 2,$
 $y \geq 0, \text{ (c) } \mathbb{R}, y \geq 0.$
7. (a) $-\frac{2}{3} < x < 2, \text{ (b) } x < \frac{1}{2}.$
8. 0.524, 2.618, 3.386, 6.034.
9. 26.6, 90, 206.6, 270.
10. $(n \pm \frac{1}{6})\pi.$
11. $\cos 4x = 2\cos^2 2x - 1$
 $= 8\sin^4 x - 8\sin^2 x + 1,$
 $\sin x = \pm\frac{1}{2}\sqrt{2 \pm \sqrt{2(1+a)}}.$
12. 1.231, 3.142, 5.052.
13. $R = 5, \alpha = 0.6435. \text{ (a) i. } 1.80, 5.77,$
ii. 0.21, 0.64, 2.31, 4.40.
(b) max 1 when $\theta = 3.785,$
min $\frac{1}{11}$ when $\theta = 0.644.$
14. $y = x^4.$
15. $\ln 2, \ln 3.$
16. $p - \frac{1}{2}q, \text{ (b) } a^2b^{-3}.$
17. (a) 4, 16 $\sqrt{2}.$
18. $\frac{1}{3}, \frac{2}{3}, \text{ (b) } 4, 16\sqrt{2}.$
19. $x < -6, x > -1.$
20. $2 < x < \frac{5}{2}.$
21. (a) $-1 < x < \frac{1}{2} \text{ or } x > 3,$
(b) $u < -\ln 2 \text{ or } u > \ln 3.$