

Mathematics for Actuarial Science: Answer sheet 3

Sheet 5

1. (a) $\frac{29}{4}$, (b) 0.
2. $y = 2x - 2$.
3. $-2xe^{-x^2}$, $2(2x^2 - 1)e^{-x^2}$, $\pm\frac{1}{\sqrt{2}}$.
4. Max: 9, Min: 8.
5. (a) $8x + \frac{1}{x^2}$, $8 - \frac{2}{x^3}$,
 (b) $-\frac{1}{2}$, minimum at $(-\frac{1}{2}, 0)$,
 (c) $-\frac{1}{2}$, 1.
6. $27\frac{1}{2}$.
7. $\frac{2x-2}{x^2-2x+2}$, $\frac{1}{2}\ln 2$.
8. $\frac{1}{4}\ln 2$.
9. $\frac{5}{18}$.
10. $\frac{1}{1+2x} + \frac{5}{(1+2x)^2} - \frac{2}{1+x}$.
11. $\frac{1}{3}\sin^3 x + C$.
12. $\frac{e^{2x}}{2} - e^x + C$.
13. $\frac{(4x-3)^{5/2}}{40} + \frac{(4x-3)^{3/2}}{8} + C$.
14. $\frac{1}{2}\tan^2 x + C$.
15. $-2\cos\sqrt{x} + C$.
16. $\frac{(x-5)^7}{7} + \frac{8(x-5)^6}{3} + \frac{64(x-5)^5}{5} + C$.
17. $-\frac{1}{3}\ln|1 + \cos 3\theta| + C$.
18. $-\frac{1}{10}\cos 10t - \frac{1}{4}\cos 4t + C$.
19. $\frac{3}{8}t + \frac{1}{4}\sin 2t + \frac{1}{32}\sin 4t + C$.

Sheet 6

1. (a) $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$.
 (b) $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$.
2. (a) $\frac{2}{9}(3x+8)^{3/2} + C$.
 (b) $(3x+8)e^x - 3e^x + C$.
 (c) $\frac{2}{15}(3x+8)^{5/2} - \frac{16}{9}(3x+8)^{3/2} + C$.
3. $\frac{1}{4}x^2 - \frac{1}{2}\sin x \cos x + \frac{1}{4}\sin^2 x + C$.
4. $-\frac{2}{9}\pi$.
5. $I_n = \frac{1}{2}x^n e^{2x} - \frac{n}{2}I_{n-1}$,
 $\frac{1}{4}e^{2x}(2x^4 - 4x^3 + 6x^2 - 6x + 3) + C$.
6. (a) $\frac{1}{3}\ln 7$.
7. $2\sqrt{2}$.
8. (a) 8, (b) -4, (c) $\frac{1}{3} + 2\ln 2$.
9. $\frac{7}{11}$.
10. 0.
11. $\frac{3}{\sqrt{10}}\tan^{-1}\left(x\sqrt{\frac{2}{5}}\right) + C$.