Diploma 104
Solutions to Assignment 2

1

Let * denote smoker functions

$$\mu_x^* = 2\mu_x \qquad {}_tp_x^* = e^{-\int_0^t \mu_{x+r}^* dr} = e^{-\int_0^t 2\mu_{x+r} dr} = ({}_tp_x)^2$$

The median future lifetime of a non-smoker is s s.t.

$${}_sp_{50} = \tfrac{1}{2} \Rightarrow l_{50+s} = 16334.93 \Rightarrow s = 76.86 - 50 = 26.86$$

The median future lifetime of a smoker is r s.t.

$${}_rp_{50}^* = \tfrac{1}{2} \Rightarrow ({}_rp_{50})^2 = \tfrac{1}{2} \Rightarrow {}_rp_{50} = \left(\tfrac{1}{2}\right)^{\frac{1}{2}} = 0.707107$$

$$\Rightarrow \quad r = 20.56$$

$$\Rightarrow \quad s - r = 6.30 \text{ years}$$

2

For full marks, it is necessary to describe the payments completely, including the amount of the annuity payments, when the first payment is made, what happens if the policyholder does not die during the term, etc.

This benefit has no value if the life survives n years.

If the life dies within n years s/he receives an annuity of 1 p.a. at the end the year of death and of each subsequent year up to the end of the nth year.

3

$$\mathring{e}_{40} = \int_0^{60} {}_tp_{40}\,dt = \int_0^{60}\left(1 - \frac{t}{60}\right)^{0.5} dt$$

$$= \left[-60\left(1 - \frac{t}{60}\right)^{1\frac{1}{2}} \times \frac{2}{3}\right]_0^{60}$$

$$= 60 \times \frac{2}{3} = 40$$

4

(a) Sv^{T_x}

(b) $X\ddot{a}_{\overline{\min(n,K_x+1)}|}$ OR $X\ddot{a}_{\overline{K_x+1}|}$ if $K_x < n$, $X\ddot{a}_{\overline{n}|}$ if $K_x \geq n$ OR $X\ddot{a}_{\overline{K_x+1}|}$ if $K_x < n-1$, $X\ddot{a}_{\overline{n}|}$ if $K_x \geq n-1$

(c) Sv^n if $T_x \geq n$, 0 if $T_x < n$ OR Sv^n if $K_x \geq n$, 0 if $K_x < n$

(d) Sv^{K_x+1} if $K_x < n-1$, Sv^n if $K_x \geq n-1$ OR Sv^{K_x+1} if $K_x < n$, Sv^n if $K_x \geq n$ OR $Sv^{\min(n,K_x+1)}$

(e) $Xv^n a_{\overline{K_x-n}|}$ if $K_x > n$, 0 if $K_x \leq n$

(f) $X\bar{a}_{\overline{5}|}$ if $T_x \leq 5$, $X\bar{a}_{\overline{T_x}|}$ if $T_x > 5$ OR $X\bar{a}_{\overline{\max(5,T_x)}|}$

5

(i) $\ddot{a}_{x:\overline{n+1}|}$ (ii) $_{m|}A^1_{x:\overline{n}|}$ (iii) $_{s|}\bar{A}^1_{x:\overline{n-s}|}$ or $\left(\bar{A}^1_{x:\overline{n}|} - \bar{A}^1_{x:\overline{s}|}\right)$

(8) (a) $2000\, A^{\;\;1}_{65:\overline{12}|}$

$$= 2000 \cdot \frac{M_{65} - M_{77}}{D_{65}}$$

$$= 2000 \times \frac{363.82 - 214.40}{689.23} = 433.59$$

(b) $1000 \cdot \ddot{a}_{65:\overline{9}|} + 1500 \frac{D_{74}}{D_{65}} \cdot \ddot{a}_{74:\overline{7}|}$

$$= 1000\left(\ddot{a}_{65} - \frac{l_{74}}{l_{65}} \cdot v^9 \cdot \ddot{a}_{74}\right)$$

$$+ 1500\, \frac{l_{74}}{l_{65}} \cdot v^9 \cdot \left(\ddot{a}_{74} - \frac{l_{81}}{l_{74}} \cdot v^7 \cdot \ddot{a}_{81}\right)$$

$$= 1000\left(13.666 - \frac{8616.170}{9647.797} \times (1.04)^{-9} \times 9.870\right)$$

$$+ 1500\left(\frac{8616.170}{9647.797} \times (1.04)^{-9} \times \left(9.870 - \frac{6582.891}{8616.170} \times (1.04)^{-7} \times 7.148\right)\right)$$

$$= 7472.97 + 5383.56 = 12856.53$$

(9) Let PV of this annuity be denoted by G

$$G = \begin{cases} 2500\, a_{\overline{K_{67}}|} & \text{if } K_{67} < 3 \\ 2500\, a_{\overline{3}|} & \text{if } K_{67} \geq 3 \end{cases}$$

Then

$$E(G) = 2500 \times \left(a_{\overline{1}|} \cdot {}_{1|}q_{67} + a_{\overline{2}|} \cdot {}_{2|}q_{67} + a_{\overline{3}|} \cdot {}_{3}p_{67}\right)$$

$$= 2500 \times \left(0.9132 \times \frac{70.580}{9605.483} + 1.7472 \times \frac{81.124}{9605.483} + 2.5089 \times \frac{9392.621}{9605.483}\right)$$

$$= 6186.92$$

$$E(G^2) = 2500^2 \times \left(a_{\overline{1}|}^2 \cdot {}_{1|}q_{67} + a_{\overline{2}|}^2 \cdot {}_{2|}q_{67} + a_{\overline{3}|}^2 \cdot {}_{3}p_{67}\right)$$

$$= 2500^2 \times \left(0.9132^2 \times \frac{70.580}{9605.483} + 1.7472^2 \times \frac{81.124}{9605.483} + 2.5089^2 \times \frac{9392.621}{9605.483}\right)$$

$= 38668737.31$

$\Rightarrow Var(G) = E(G^2) - (E(G))^2$

$= 38668737.31 - (6186.92)^2$

$= 390758.224$

$\Rightarrow SD(G) = \sqrt{390758.224} = 625.11$

8

(i) $\ddot{a}_{35:\overline{15}|} = \dfrac{N_{35} - N_{50}}{D_{35}}$

$= \dfrac{52663.13 - 23839.41}{2507.40} = 11.495$

(ii) $\bar{A}_{35:\overline{15}|} \simeq (1+i)^{\frac{1}{2}} A^{1}_{35:\overline{15}|} + A_{35:\overline{15}|}^{\ \ 1}$

$= (1.04)^{\frac{1}{2}} \left[\dfrac{M_{35} - M_{50}}{D_{35}}\right] + \dfrac{D_{50}}{D_{35}}$

$= (1.04)^{\frac{1}{2}} \left(\dfrac{481.90 - 449.71}{2507.40}\right) + \dfrac{1366.61}{2507.40}$

$= 0.55812$

(iii) ${}_{5|}a_{35:\overline{15}|} = \dfrac{N_{41} - N_{56}}{D_{35}}$

$= \dfrac{39017.36 - 16440.95}{2507.40}$

$= 9.004$