Diploma 104: Solutions to Assignment 3

① (i) We consider whole life policies, with annual premiums limited to m years.

${}_mP_x$ is the symbol for the net premium:

$${}_mP_x = \frac{A_x}{\ddot{a}_{x:\overline{m}|}} = \frac{M_x}{N_x - N_{x+m}}$$

The prospective reserve (for unit sum assured)

$$= A_{x+t} - {}_mP_x \cdot \ddot{a}_{x+t:\overline{m-t}|}$$

$$= \frac{M_{x+t}}{D_{x+t}} - {}_mP_x \cdot \frac{N_{x+t} - N_{x+m}}{D_{x+t}} \quad \cdots (A)$$

The Retrospective reserve (for unit sum assured)

$$= {}_mP_x \cdot \ddot{s}_{x:\overline{t}|} - \frac{M_x - M_{x+t}}{D_{x+t}}$$

$$= {}_mP_x \cdot \frac{N_x - N_{x+t}}{D_{x+t}} - \left(\frac{M_x - M_{x+t}}{D_{x+t}}\right) \quad \cdots (B)$$

(ii) From the equation of value (for the premium)

$${}_mP_x \cdot \frac{N_x - N_{x+m}}{D_x} - \frac{M_x}{D_x} = 0$$

$$\Rightarrow {}_mP_x \cdot \frac{N_x - N_{x+m}}{D_x} \cdot \frac{D_x}{D_{x+t}} - \frac{M_x}{D_x} \cdot \frac{D_x}{D_{x+t}} = C$$

Hence $${}_mP_x \cdot \frac{N_x - N_{x+m}}{D_{x+t}} - \frac{M_x}{D_{x+t}} = 0 \quad \cdots (C$$

Now Equations (A) + (C) = Equation (B)

= Retrospective reserv

as required

(2) The age at 1/1/94 of the 3 deaths are 67, 66 and 67.

The reserves at 31/12/94 in respect of the 3 policies are:

$$2000\left(1 - \frac{\ddot{a}_{68}}{\ddot{a}_{55}}\right) = 2000\left(1 - \frac{11.135}{15.873}\right)$$

$$= 596.99$$

$$2000\left(1 - \frac{\ddot{a}_{67}}{\ddot{a}_{55}}\right) = 2000\left(1 - \frac{11.515}{15.873}\right)$$

$$= 549.11$$

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and $1000\left(1-\frac{\ddot{a}_{68}}{\ddot{a}_{57}}\right) = 1000\left(1-\frac{11.135}{15.195}\right)$

$= 267.19$

Since the age at 1/1/94 of the 3 deaths were, respectively, 67, 66 and 67, we know that the reserves of 596.99 and 267.19 relate to the data given in the 2nd row of the first table in the question

[ie: Age
67 80 000 39000];

and the reserve of 549.11 relates to the data given in the 1st row

[ie: Age
66 100,000 52000]

and there were no deaths related to the 3rd row.

Hence, DSAR in total is

$(100000 - (52000 + 549.11)) +$

$(80000 - (39000 + 596.99 + 267.19))$

$+ (60000 - 32000)$

$= 47450.89 + 40135.82 + 28000$

$EDS = q_{66} \times 47450.89 + q_{67} \times 40135.82$

$+ q_{68} \times 28000$

$= 0.015940 \times 47450.89 + 0.017824 \times 40135.82$

$+ 0.019913 \times 28000$

$= 2029.31$

$ADS = (2000 - 596.99) + (2000 - 549.11)$

$+ (1000 - 267.19) = 3586.71$

$\Rightarrow$ Mort loss of 1557.40

(ii)(a) Expected death claims =

$q_{66} \times 100\,000 + q_{67} \times 80\,000 + q_{68} \times 60000$

$= 4214.70$

Actual death claims = 5000

(b) Although the expected death claims were relatively close to the actual death claims a significant loss due to mortality was made during the year.

This is because the DSAR varies for the different policies, depending on age and duration. The actual deaths were disproportionately concentrated on the higher DSAR lives $\Rightarrow$ significant loss made.

③ (i) Let P = annual premium

$$P\ddot{a}_{35:\overline{30|}} = 25000\,\bar{A}^{1}_{35:\overline{30|}} + 50000\,A_{35:\overline{30|}}^{\;\;1}$$

Now $\bar{A}^{1}_{35:\overline{30|}} \simeq (1.04)^{\frac{1}{2}} A^{1}_{35:\overline{30|}}$

and $A^{1}_{35:\overline{30|}} = \dfrac{M_{35} - M_{65}}{D_{35}} = \dfrac{481.90 - 363.82}{2507.40}$

$= 0.047093$

$\Rightarrow \bar{A}^{1}_{35:\overline{30|}} = (1.04)^{\frac{1}{2}} \times 0.047093 = 0.048025$

$\therefore P \times 17.629 = 25000 \times 0.048025$

$+ 50000 \times \dfrac{D_{65}}{D_{35}}$ $D_{65} \leftarrow 689.23$, $D_{35} \leftarrow 2507.40$

$\Rightarrow P = \dfrac{14944.54}{17.629} = 847.72$ pa

(ii) Prosp. pol. Val after 10 yrs

= EPV of future benefits − EPV of future premiums

$= 25000\,\bar{A}^{1}_{45:\overline{20|}} + 50000\,A_{45:\overline{20|}}^{\;\;1}$

$- 847.72 \times \ddot{a}_{45:\overline{20|}}$

$$\bar{A}^{1}_{45:\overline{20}|} \simeq (1.04)^{\frac{1}{2}} \times \frac{M_{45} - M_{65}}{D_{45}}$$

$$= (1.04)^{\frac{1}{2}} \times \frac{463.20 - 363.82}{1677.97} = 0.060399$$

$$A_{45:\overline{20}|}^{\ \ \ 1} = \frac{D_{65}}{D_{45}} = \frac{689.23}{1677.97} = 0.41075$$

Hence, prosp pol val

$$= 25000 \times 0.060399 + 50000 \times 0.41075$$

$$- 847.72 \times 13.780 = 10,365.89$$

(iii) Retro pol val = [Accum of Prems − Accul of Bfts] over first 10 years

$$= \left[847.72\ \ddot{a}_{35:\overline{10}|} - 25000\ \bar{A}^{1}_{35:\overline{10}|}\right] \times \frac{D_{35}}{D_{45}}$$

where $\bar{A}^{1}_{35:\overline{10}|} \simeq (1.04)^{\frac{1}{2}} \times \frac{481.90 - 463.20}{2507.40}$

$$= 0.007606$$

⇒ Retro Reserve

$$= \left[847.72 \times \left(\frac{52663.13 - 31583.93}{2507.40}\right) - 25000 \times 0.007606\right]$$

$$\times \frac{2507.40}{1677.97}$$

$$= 10365.19$$

(iv) The Retro Res = Prosp Res since:

Both types of reserve have been calculated on the same basis and this basis is the same as the premium basis

(4) (i) Mortality profit = EDS − ADS

$$= \sum_{\substack{\text{P/holders in} \\ \text{force at} \\ \text{start of year}}} q_{x+t} \cdot (S - {}_{t+1}V) - \sum_{\substack{\text{claims} \\ \text{during} \\ \text{year}}} (S - {}_{t+1}V)$$

Reserves per unit sum assured:-

Endowment: $${}_{11}V_{35:\overline{25}|} = 1 - \frac{\ddot{a}_{46:\overline{14}|}}{\ddot{a}_{35:\overline{25}|}} \quad \frac{10.818}{16.027}$$

$$= 0.32501$$

Whole life: $${}_{4}V_{40} = 1 - \frac{\ddot{a}_{44}}{\ddot{a}_{40}} \quad \frac{19.075}{20.005}$$

$$= 0.046488$$

Pure endowment:

Premium:- $$P \underbrace{\ddot{a}_{40:\overline{20}|}}_{13.927} = \frac{D_{60}}{D_{40}} \quad \frac{882.85}{2052.96}$$

$\Rightarrow P = 0.030878$

Reserve: $${}_{18}V = \frac{\overset{882.85}{D_{60}}}{\underset{967.90}{D_{58}}} - P\, \overset{1.937}{\ddot{a}_{58:\overline{2}|}}$$

$$= 0.85232$$

[Note: for pure endowments (and term assurance, there is no easy formula for the reserve of the form

$1 - \ddot{a} / \ddot{a}$

$\Rightarrow$ must calculate reserve from first principles as above.

Profit

Endowment :-

$$\overset{0.001465}{q_{45}} \times 350\,000\,(1 - 0.32501)$$

$$- 50000 \times (1 - 0.32501) = -33{,}403$$ (i.e. a loss)

Whole life :-

$$\overset{0.001208}{q_{43}} \times 425\,000\ (1 - 0.046488)$$

$$- 25000\ (1 - 0.046488) = -23348$$ (i.e. a loss)

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Pure Endowment [Note Sum assured = 0]

$$q'_{57} \times 200000 \times (0 - 0.85232)$$ with $q'_{57} = 0.005650$

$$-25000\ (0 - 0.85232) = 20,345$$ (i.e. a profit)

$\Rightarrow$ overall loss = 36,406

(ii) For endowment assurance,

Expected number of deaths = $\frac{350000}{25000} \times q_{45} = 0.0205$

cf Actual number of deaths = 2 people

For whole life,

Expected number of deaths = $\frac{425000}{25000} \times q_{43} \simeq 0.0205$

cf Actual number = 1 person

For Pure Endowment,

Expected number of deaths = $\frac{200000}{25000} \times q_{57} = 0.045$

cf Actual number = 1 person

Hence, many more deaths than expected has created a substantial loss for Endowment Assurance and Whole Life policies, and a profit for Pure Endowment. The losses, o course, exceed that profit so overall a loss is made.

[Note: For pure endowments, some deaths are expected
$\Rightarrow$ deaths amongst pure endowment policy holders don't necessarily lead to a profit for such policies. You must still compare the actual with the expected number of deaths

— many students think that all deaths amongst ~~pu~~ pure endowment policyholders (and annuitants) will give rise to profit — but that's not the case]

(5) (i) (a) Possibilities are :-

1) die in year 1 $\Rightarrow L = 100000\,v - 2867\,\ddot{a}_{\overline{1}|}$

$= 94339.62 - 2867 = 91472.62$

2) die in yr 2 $\Rightarrow L = 100000\,v^2 - 2867\,\ddot{a}_{\overline{2}|}$

$= 88999.64 - 2867 \times 1.943396$

$= 83472.92$

3) die in yr 3 $\Rightarrow L = 100000\,v^3 - 2867\,\ddot{a}_{\overline{3}|}$

$= 83961.93 - 2867 \times 2.833393$

$= 75838.59$

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4) No death $\Rightarrow L = -2867 \times \ddot{a}_{\overline{3}|} = -8,123.34$

(b) Probabilities are: i) $\frac{d_{62}}{l_{62}} = \frac{1472}{84173} = 0.017488$

ii) $d_{63}/l_{62} = \frac{1625}{84173} = 0.019305$

(iii) $d_{64}/l_{62} = \frac{1783}{84173} = 0.021,183$

(iv) $l_{65}/l_{62} = \frac{79293}{84173} = 0.942024$

$\Sigma = 1.0$
(as we require)

$\Rightarrow E(L) = 0.017488 \times 91472.62 + 0.019305 \times 83472.9$

$+ 0.021183 \times 75838.59 + 0.942024 \times -8123.34$

$= -2,834.77$ ($\Rightarrow$ Premium of 2867 contains a margin since EPV $\neq 0$)

$E(L^2) = 0.017488 \times (91472.62)^2 + 0.019305 \times (83472.92)$

$+ 0.021183 \times (75838.59)^2 \times 0.942024 \times (-8123.34)^2$

$= 464835037 = (21560.03)^2$

$\Rightarrow SD = \sqrt{E(L^2) - (E(L))^2} = 21373$

(iii) With $E(L) = -2,834.77$ (i.e. same as for the term assurance just considered), we expect a smaller standard deviation for the endowment assurance.

This is because for the endowment assurance the payment is certain to be made – it is only the timing of the payment which is uncertain.

– Whereas for the term assurance, both the payment and the timing are uncertain.