Mortality Profit Example

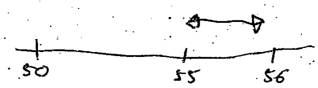
5 years ago a life office sold the following combination of policies to 100 lives aged 50. None of them died during the 5 years.

(a) A non-profit endowment assurance with a term of 10 years, a sum assured of £10,000 payable at the end of the year of death or on survival to the end of the term, and a premium of £750 payable annually in advance.

(b) A temporary annuity with a term of 9 years, paying £750 annually in arrears.

The office values both the endowment and the annuity contracts using the net premium method, assuming that mortality follows the A1967-70 ultimate table and interest is 6% p.a. During the next year (that is, between durations 5 and 6 years), 2 policyholders died. What is the total mortality profit arising at the end of the 6th year?

Solution



The endowment reserve at the end of the year is for each policy,

$$10{,}000.\,{}_6V_{50:\overline{10|}} = 10000\left(1 - \frac{\ddot{a}_{56:\overline{4|}}}{\ddot{a}_{50:\overline{10|}}}\right) = 5236.2$$

$S - {}_{t+1}V$

$= 10000 - 5236.2$

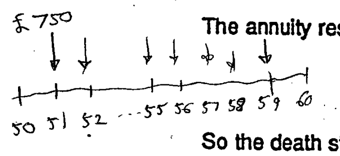
So the death strain at risk under the endowment assurance contract is, for each policy, 4,763.8. The number of expected deaths is $100.q_{55} = 0.84413$. The number of actual deaths is 2.

temporary annuity:-

So the mortality profit under the endowment contracts is -5,506.33

$(0.84413 - 2) \times 4763.8 = -5506.33$

The annuity reserve at the end of the year is



$$750.\ddot{a}_{56:\overline{4|}} = 2{,}715.0$$

So the death strain at risk is -2,715.

The mortality profit under the annuity contracts is

$$(0.84413 - 2).(-2{,}715) = 3{,}138.19$$

Therefore the total mortality profit is

$$3{,}138.19 - 5{,}506.33 = -2{,}368.14$$

0. (a) $_tV_x = A_{x+t} - P_x \ddot{a}_{x+t}$

$({}_tV_x + P_x) = A_{x+t} - P_x a_{x+t}$

Substitute $A_{x+t} = vq_{x+t} + vp_{x+t} A_{x+t+1}$

$a_{x+t} = vp_{x+t} \ddot{a}_{x+t+1}$

Then $A_{x+t} - P_x a_{x+t} = vq_{x+t} + vp_{x+t}(A_{x+t+1} - P_x \ddot{a}_{x+t+1})$

Hence $({}_tV_x + P_x)(1+i) = q_{x+t} + p_{x+t}\, {}_{t+1}V_x$

$= {}_{t+1}V_x + q_{x+t}(1 - {}_{t+1}V_x)$.

Net premium policy value plus net premium and interest at the assumed rate provides, at the end of the year, next yr's net premium policy value plus for expected deaths a sum sufficient to increase this policy value to the death benefit

(b) Mortality Profit/Loss = EOS − ADS

$= \sum_{\substack{\text{in force} \\ \text{on 1/1}}} q_{x+t}(S - {}_{t+1}V) - \sum_{\substack{\text{actual} \\ \text{deaths}}} (S - {}_{t+1}V)$

Per unit benefit, DSAR at end of 1993 are:

$1 - {}_7V_{40} = \frac{\ddot{a}_{47}}{\ddot{a}_{40}} = 0.89663$

$1 - {}_{11}V_{45} = \frac{\ddot{a}_{56}}{\ddot{a}_{45}} = 0.79719$

$1 - {}_{20}V_{35:\overline{25}|} = \frac{\ddot{a}_{55:\overline{5}|}}{\ddot{a}_{35:\overline{25}|}} = 0.28643$

$1 - {}_3V_{40:\overline{25}|} = \frac{\ddot{a}_{43:\overline{22}|}}{\ddot{a}_{40:\overline{25}|}} = 0.92153$

Then EOS = $250{,}000 \times 0.89663 \times q_{46} + 250{,}000 \times 0.79719 \times q_{55} + 301{,}000 \times 0.28643 \times q_{54} + 250{,}000 \times 0.92153 \times q_{42}$

= 343643

ADS = $10{,}000 \times 0.89663 + 2{,}000 \times 0.79719 + 1{,}000 \times 0.28643$

= 10847.11

MORTALITY LOSS = 7410.68