



## FAM Cswk no. 4

①

- (i) An irredeemable fixed interest stock pays coupons of  $c\%$  annually and the next coupon payment is due in one year's time.

If  $i$  is the rate of interest per annum effective, show that the discounted mean term of this stock is:

$$1 + \frac{1}{i}$$

[4 marks]

- (ii) An insurance company has liabilities of £8 million due in 6 years' time and £10 million due in 10 years' time. The company's assets consist of a zero-coupon bond redeemable in one year's time and an irredeemable stock paying annual coupons of  $4\%$  per annum, the next coupon being due in one year's time.

If the rate of interest is  $6\%$  per annum effective, calculate the nominal amount of each stock that the company should hold in order that Redington's first two conditions for immunization against small changes in the rate of interest are satisfied.

[7 marks]

- (iii) Without undertaking any further calculations, explain why it appears very likely that Redington's third condition for immunization will also be satisfied.

[2 marks]

[Total 13 marks]

②

- At time  $t = 0$ , the one-year zero coupon yield is  $4\%$  per annum effective and the one-year forward rate per annum effective at time  $t$  ( $t = 1, 2, \dots$ ) is given by:

$$f_{t,1} = (4 + t) \%$$

- (i) Calculate the issue price per £100 nominal of a three-year  $4\%$  coupon bond issued at time  $t = 0$ , paying coupons annually in arrears and redeemable at  $102\%$ .
- (ii) Calculate the 3-year par yield at time  $t = 0$ .

(It may be assumed that coupon and capital payments can be discounted using the same factors).

[Total 7 marks]

- ③ £1,000 is invested in a bank account which pays interest at the end of each year. The interest will be reinvested in the account.

The rate of interest is fixed randomly at the beginning of each year and remains unchanged until the beginning of the next year. The rate of interest applicable in any one year is independent of the rate applicable in any other year.

During the first year the rate of interest per annum effective will be one of 4%, 5% or 7%, with equal probability.

During the second year, the rate of interest per annum effective will be either 6% with probability 0.6, or 8% with probability 0.4.

- (i) Calculate the expected accumulated amount in the bank account at the end of two years, accurate to two decimal places.

[3 marks]

- (ii) Calculate the variance of the accumulated amount in the bank account at the end of two years, accurate to the nearest whole number.

[7 marks]

[Total 10 marks]

④

In any year, the rate of interest on funds invested with a given insurance company is independent of the rates of interest in all previous years. Each year the value of  $(1 + i_t)$ , where  $i_t$  is the rate of interest earned in the  $t^{\text{th}}$  year, is lognormally distributed. The mean and standard deviation of  $i_t$  are 0.06 and 0.17 respectively.

- (i) Determine the parameters  $\mu$  and  $\sigma^2$  of the lognormal distribution of  $1 + i_t$

[5 marks]

- (ii) (a) Determine the distribution of  $S_{20}$ , where  $S_{20}$  denotes the accumulation of one unit of money over 20 years.

- (b) Determine the probability that  $S_{20} > 4.5$ .

[4 marks]

[Total 9 marks]

⑤

- (i) If the one year zero coupon yield is 5% per annum effective and the one year forward rate at time  $t = 1$  is 6% per annum effective, calculate the issue price per £100 nominal of a two-year 4% coupon bond issued at time  $t = 0$ , paying coupons annually in arrears and redeemable at par. (It may be assumed that coupon and capital payments can be discounted using the same factors).

[4 marks]

- (ii) If in addition to the information given in (i) the three year par-yield is 5.87% per annum effective, calculate the three year zero coupon yield.

[3 marks]

[Total 7 marks]