Fundamentals of Insurance Maths
Diploma — Assignment no. 1
Guidelines to Solutions

Q(1) $f(x) = c\gamma x^{\gamma-1} \cdot \exp[-cx^{\gamma}]$

$$\Rightarrow F(x) = \int_0^x c\gamma y^{\gamma-1} e^{-cy^{\gamma}} dy$$

$(= P[X \le x])$

Let $z = y^{\gamma} \Rightarrow dz = \gamma y^{\gamma-1} dy$

$$F(x) = \int_0^{x^{\gamma}} c e^{-cz} dz = \left[-e^{-cz}\right]_0^{x^{\gamma}}$$

$$= 1 - e^{-cx^{\gamma}}$$

The survival function; $S(x) = P[X > x]$

$$= 1 - F(x)$$

$$= e^{-cx^{\gamma}}$$

The probability that the animal dies between age 5 and 10, given survival to age 2 :

$$\frac{S(5) - S(10)}{S(2)} = \frac{e^{-1.1275} - e^{-1.3337}}{e^{-0.89191}}$$

$$= 0.1520$$

12 (a) $\mu^*_{80+t} = \mu_{80+t} - [.05 + .01t] \qquad 0 \le t \le 10$

μ^*_{80+t} as experienced; μ_{80+t} according to a(55) males ultimate.

$${}_{10}p^*_{80} = e^{-\int_0^{10} \mu^*_{80+t}\,dt} = e^{-\int_0^{10} (\mu_{80+t} - [.05+.01t])\,dt}$$

$$= e^{-\int_0^{10} \mu_{80+t}\,dt} * e^{+\int_0^{10} .05\,dt} * e^{+\int_0^{10} .01t\,dt}$$

$$= {}_{10}p_{80} * e^{.5} * e^{[.01t^2/2]_0^{10}} = {}_{10}p_{80} * e^{.5} * e^{.5} = {}_{10}p_{80} * e = \frac{l_{90}}{l_{80}} * e$$

${}_{10}p_{80}$: a(55) males ultimate

$$= .5353$$

$$\therefore {}_{10}q^*_{80} = 1 - .5353 = .4647.$$

b) The life becomes ultimate at age 62

Then $l_{62} = l_{60} * {}_2p^*_{60}$ where ${}_2p^*_{60} = e^{-\int_0^2 0.04879\,dt}$

And then $l_{65} = l_{62} * {}_3p^*_{62}$

${}_3p^*_{62}$ is to be evaluated using the same mortality as the a(55) males ultimate table.

i.e. ${}_3p^*_{62} = \frac{l'_{65}}{l'_{62}}$ $\quad l'_{62}, l'_{65}$ read from the a(55) males ult. table

$$= \frac{788316}{834752}$$

Thus $l_{60} * {}_2p^*_{60} * {}_3p^*_{62} = l_{65} = 100,000$ (given)

$$\therefore l_{60} = \frac{100000}{{}_2p^*_{60} * \frac{788316}{834752}}$$

Finally to evaluate ${}_2p^*_{60} = e^{-\int_0^2 0.04879\,dt} = e^{-0.04879 \times 2}$

An astute reader may notice that $\delta = 0.04879$ is the force of interest corresponding to $i = 5\%$ so that $e^{-0.04879} = \frac{1}{1.05}$

giving ${}_2p^*_{60} = \left(\frac{1}{1.05}\right)^2$

Thus $l_{60} = \frac{105891}{(\frac{1}{1.05})^2} = 105891 * (1.05)^2 = 116744$

③ Use ' to denote special mortality functions
Other functions come from ELT 12 (males)

(a) $${}_5q'_{55} = 1 - {}_5p'_{55} = 1 - \exp\left(-\int_0^5 \mu'_{55+t}\,dt\right)$$

$$= 1 - e^{-0.10} \quad \text{as } \mu'_{55+t} = 0.02$$
for $0 \leq t \leq 7$

$$= 0.09516$$

(b) $${}_{5|5}q'_{55} = {}_5p'_{55} \cdot {}_2q'_{60} + {}_7p'_{55} \cdot {}_3q_{62}$$

$$= {}_5p'_{55} - {}_{10}p'_{55}$$

$$= {}_5p'_{55} - {}_7p'_{55}\ {}_3p_{62}$$

$$= e^{-0.10} - e^{-0.14} \cdot \frac{l_{65}}{l_{62}}$$

$$= e^{-0.10} - e^{-0.14} \cdot \frac{68490}{75172}$$

$$= 0.11276$$

④ (a) ω is the lowest age for which $l_x = 0$

By inspection this is at age 100

$\Rightarrow \omega = 100$

(b) $$\mu_x = -\frac{1}{l_x} \cdot \frac{d}{dx}(l_x)$$

$$= \frac{-1}{l_0\left(1-\frac{x}{100}\right)^{\frac{1}{2}}} \cdot \frac{l_0 * \frac{1}{2}\left(-\frac{1}{100}\right)}{\left(1-\frac{x}{100}\right)^{\frac{1}{2}}}$$

$$= \frac{1}{2 * 100} * \left(\frac{1}{1-\frac{x}{100}}\right) = \frac{1}{2(100-x)}$$

(c) $$q_{80} = 1 - \frac{l_{81}}{l_{80}}$$

$$= 1 - \left(\frac{1-\frac{81}{100}}{1-\frac{80}{100}}\right)^{\frac{1}{2}}$$

$$= 1 - \left(\frac{19}{20}\right)^{\frac{1}{2}} = 0.02532057$$

(5) (i) $F_x(t) = 1 - S_x(t) = 1 - \left(\frac{\lambda}{(\lambda+t)}\right)^{\alpha}$

(ii) $${}_n p_x = S_x(n) = \left(\frac{\lambda}{\lambda+n}\right)^{\alpha}$$

(iii) $$f_x(t) = \frac{d}{dt} F_x(t) = \frac{d}{dt}\left\{1 - \left(\frac{\lambda}{\lambda+t}\right)^{\alpha}\right\}$$

$$= \frac{\alpha \lambda^{\alpha}}{(\lambda+t)^{\alpha+1}}$$

(iv) $$\mu_{x+n} = \frac{f_x(n)}{S_x(n)} = \frac{\alpha}{\lambda+n}$$

(6) Given that $\mu_x = A + BC^x$, we ca
show:

$${}_t p_x = s^t g^{c^x(c^t-1)} \quad \text{---(1}$$

where $$s = e^{-A} \quad \text{---(2}$$

$$g = \exp\left(\frac{-B}{\ln C}\right) \quad \text{---(3}$$

CW1A-5

Q6 ctd

Thus from (1)

$$ {}_5p_{70} = 0.73 = s^5 g^{c^{70}(c^5-1)} \qquad (4)$$

$$ {}_5p_{75} = 0.62 = s^5 g^{c^{75}(c^5-1)} \qquad (5)$$

$$ {}_5p_{80} = 0.50 = s^5 g^{c^{80}(c^5-1)} \qquad (6)$$

(4) ÷ (5) $\Rightarrow \dfrac{0.73}{0.62} = g^{c^{70}(c^5-1) - c^{75}(c^5-1)} = g^{-c^{70}(c^5-1)^2}$ (7)

(5) ÷ (6) $\Rightarrow \dfrac{0.62}{0.50} = g^{c^{75}(c^5-1) - c^{80}(c^5-1)} = g^{-c^{75}(c^5-1)^2}$ (8)

(7) $\Rightarrow \log_e\left(\dfrac{0.73}{0.62}\right) = -c^{70}(c^5-1)^2 \log g$ (9)

(8) $\Rightarrow \log_e\left(\dfrac{0.62}{0.50}\right) = -c^{75}(c^5-1)^2 \log g$ (10)

(10) ÷ (9) $\Rightarrow c^5 = \log\left(\dfrac{0.62}{0.50}\right) / \log\left(\dfrac{0.73}{0.62}\right) = 1.3170752$

$\Rightarrow c = 1.056628$

(9) $\Rightarrow \log_e g = -0.0343701$

(3) $\Rightarrow B = 0.0018932$

(4) $\Rightarrow \log_e 0.73 = 5\log_e s + c^{70}(c^5-1)\log_e g$

$\Rightarrow \log_e s = 0.040079$

(2) $\Rightarrow A = -\log_e s = -0.040079$

Thus:

$A = -0.040079$
$B = 0.0018932$
$c = 1.056628$

CW1A-6

Q7. We are told that $\mu'_x = 3\mu_x$ for all ages x. What is the relationship between q'_y and q_y?

$$q'_y = 1 - p'_y = 1 - e^{-\int_0^1 \mu'_{x+t}\,dt}$$

$$= 1 - e^{-3\int_0^1 \mu_{x+t}\,dt}$$

$$= 1 - \left(e^{-\int_0^1 \mu_{x+t}\,dt}\right)^3$$

$$= 1 - (p_y)^3$$

$$= 1 - (1-q_y)^3$$

$$\Rightarrow q'_y = 3q_y - 3q_y^2 + q_y^3$$

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(8) $\mu_{x+t} = -\frac{1}{{}_tp_x}\frac{d}{dt}\left({}_tp_x\right)$

$= -\frac{1}{\left(1-\frac{t}{(70-x)}\right)^{\frac{1}{2}}} \cdot \frac{1}{2}\left(1-\frac{t}{(70-x)}\right)^{-\frac{1}{2}} \cdot -\frac{1}{(70-x)}$

$= \frac{\frac{1}{2}\cdot\frac{1}{(70-x)}}{1-\frac{t}{(70-x)}}$

Let $t \to 0$, $\mu_x = \frac{1}{2(70-x)}$

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(9) a) ${}_{0.4}q_{64} = 0.4 \times q_{64} = 0.4 \times 0.021$

$= 0.0084$

b) ${}_{0.4}q_{63.6} = 1 - {}_{0.4}p_{63.6}$

$= 1 - \frac{{}_1p_{63}}{{}_{0.6}p_{63}}$

$= \frac{0.4 q_{63}}{1 - 0.6 q_{63}}$

$= \frac{0.4 \times 0.019}{1 - 0.6 \times 0.019}$

$= 0.0076876$

c) ${}_{0.8}q_{63.6} = 1 - {}_{0.4}p_{63.6} \times {}_{0.4}p_{64}$

$= 1 - (1 - 0.0076876)(1 - 0.0084)$

$= 0.016023$

10 (a)

$$ {}_2p_{61} = e^{-\int_0^2 \mu_{61+t}\,dt} $$

$$ = e^{-\int_0^1 0.015\,dt} \cdot e^{-\int_1^2 \mu_{61+t}\,dt} $$

$$ = e^{-0.015} \cdot e^{-0.017} $$

$$ = 0.985112 \times 0.983144 $$

$$ = 0.96851 $$

(b)

$$ \mathring{e}_{61} = \int_0^\infty {}_tp_{61}\,dt $$

$$ = \int_0^2 {}_tp_{61}\,dt + \int_2^\infty {}_tp_{61}\,dt $$

$$ = \int_0^2 {}_tp_{61}\,dt + {}_2p_{61} \int_0^\infty {}_tp_{63}\,dt $$

$$ = \int_0^2 {}_tp_{61}\,dt + {}_2p_{61} \cdot \mathring{e}_{63} $$

$$ \Rightarrow \mathring{e}_{63} = \frac{\mathring{e}_{61} - \int_0^2 {}_tp_{61}\,dt}{{}_2p_{61}} $$

and

$$ \int_0^2 {}_tp_{61}\,dt = \int_0^1 {}_tp_{61}\,dt + \int_1^2 {}_tp_{61}\,dt $$

$$ = \int_0^1 e^{-0.015t}\,dt + p_{61} \int_0^1 {}_tp_{62}\,dt $$

$$ = \int_0^1 e^{-0.015t}\,dt + e^{-0.015} \int e^{-0.017t}\,dt $$

$$ = \frac{1 - e^{-0.015}}{0.015} + e^{-0.015} \times \frac{1 - e^{-0.017}}{0.017} $$

$$ = 0.99254 + 0.97679 = 1.96932 $$

$$ \Rightarrow \mathring{e}_{63} = \frac{20 - 1.96932}{0.96851} $$

$$ = 18.617 $$