Diploma – Subject 104

Assignment 2

Q1 A life insurer assumes that the force of mortality of smokers at all ages is twice the force of mortality of non-smokers, which is taken from the A1967–70 ultimate life table.

Calculate the difference between the median future lifetimes of a non-smoker and a smoker, both aged exactly 50. [5]

Q2 The random variable K represents the curtate future lifetime of a life now aged x.

Describe the insurance benefit which has present value equal to the random variable $g(K)$ where

$$g(K) = \begin{cases} v^{K+1}.\ddot{a}_{\overline{n-K}|} & \text{if } K < n \\ 0 & \text{if } K \geq n \end{cases}$$ [3]

Q3 A mortality table is defined such that ${}_tp_x = \left(1 - \frac{t}{100-x}\right)^{0.5}$ for $x < 100$, $t < 100 - x$, and ${}_tp_x = 0$ for $t \geq 100 - x$.

Calculate the complete expectation of life at exact age 40. [3]

Q4 Let T_x and K_x be respectively the complete and curtate future lifetime of a life aged x. Express the present value of the following assurance and annuity benefits as functions of T_x or K_x

(a) A whole life assurance with sum assured S payable immediately on death.

(b) An annuity of X per annum payable annually in advance for a maximum of n years.

(c) A pure endowment with term n years and a sum assured of S.

(d) An endowment assurance with term n years and a sum assured of S, payable at the end of the year of death or on survival to the end of the term.

(e) A deferred annuity of X per annum, payable annually in arrears after a deferred period of n years.

(f) An annuity of X per annum, payable continuously, under which payment is guaranteed for 5 years (that is, for 5 years the payments are certain, and not contingent on the survival of the life assured. If the life survives the 5 year guarantee period, payment continues as long as the annuitant survives.) [12]

Q5 What are the symbols, according to standard actuarial notation corresponding to the following expected present values?

(i) $1 + v.p_x + v^2.{}_2p_x + v^3.{}_3p_x + \ldots + v^n.{}_np_x$

(ii) ${}_m|q_x.v^{m+1} + {}_{m+1}|q_x.v^{m+2} + \ldots + {}_{m+n-1}|q_x.v^{m+n}$

(iii) $\int_s^r v^t.{}_tp_x.\mu_{x+t}\,dt \qquad r > s$ [10]

Question 6

Write down an expression in terms of commutation functions for the expected present value of each of the following products for a life aged 65, and evaluate your expression using the basis indicated.

(a) A 12 year temporary assurance where the sum assured is £2,000 and is payable at the end of the year of death.

[Basis: AM92 ultimate at 4% per annum]

(b) A 16 year temporary annuity-due paying £1,000 pa for the first 9 years and £1,500 for the remaining 7 years. Payments are made annually in advance.

[Basis: PMA92C20 ultimate at 4% pa]

[9 marks]

Question 7

An annuity of £2,500 pa is payable annually in arrears to a life aged 67 for a maximum of 3 years, ceasing on earlier death.

Calculate the standard deviation of the present value of the annuity, using PFA92C20 ultimate mortality, and interest at 9.5% pa

[7 marks]

Question 8

Evaluate the following expected present values, using AM92 mortality at 4% pa interest:

(i) $\ddot{a}_{35:\overline{15}|}$

(ii) $\overline{A}_{35:\overline{15}|}$

(iii) ${}_{5|}a_{35:\overline{15}|}$

[10 marks]