FAIM Coursework

① (i) [$1\frac{1}{2}$] $100\left(1+\frac{0.04}{4}\right)^{40} = 148.886$

[$1\frac{1}{2}$] (ii) $100\left(1+\frac{0.06}{2}\right)^{10} = 134.39$

[2] (iii) $\dfrac{100}{(1-0.03)^8} = 127.59$

[2] (iv) $100\, e^{0.04 \times 3.75} = 116.18$

[$2\frac{1}{2}$] (v) $100\,(1.05)^{10}(1.04)^5(1.025)^3 = 213.42$

[$2\frac{1}{2}$] (vi) $100 \times 1.16183 \times 1.48886 = 172.98$

[(vi) uses answers to (i) and (iv)]

② (a) Rate of interest per quarter

$= \dfrac{0.02}{1-0.02} = 2.0408\%$

$\Rightarrow$ effective rate of interest per annum $=$

[2] $(1.020408)^4 - 1 = 0.084165$ ie. 8.417% pa

[2] (b) $e^{0.04} = 1.0408108$

$\Rightarrow 4.08108\%$ pa

③ (a) $1+i = \exp\left\{\int_9^{10} \delta(t)\,dt\right\}$

$= \exp\left[0.01t + 0.0025t^2\right]_9^{10}$

$= \exp(0.1 + 0.25 - 0.09 - 0.2025)$

ie. $1+i = e^{0.0575} = 1.05919$

[2] ie $i = 5.92\%$ pa

(b) Accumulation $= 100 \exp\left\{\int_2^{12} \delta(t)\,dt\right\}$

$\int_2^{12} \delta(t)\,dt = \int_2^8 0.05\,dt + \int_8^{12} (0.01 + 0.005t)\,dt$

$= \left[0.05t\right]_2^8 + \left[0.01t + 0.0025t^2\right]_8^{12}$

$= 0.4 - 0.1 + 0.12 + 0.36 - 0.08 - 0.16$

$= 0.54$

[4] $\Rightarrow$ Accumulation $= e^{0.54} \times 100 = £171.60$

③ ctd

(c) In lectures, we saw that if $\rho(t)$ is rate of payment per unit time at time t, and $V(t)$ is the present value of a unit at time t then:

The present value of a payment stream between time 0 and time T is:

$$\int_0^T V(t)\,\rho(t)\,dt$$

Here, we are being asked to consider the accumulation rather than present value.

Method 1

Step A We consider the accumulation of the payment stream to time 5.

Step B We then accumulate this amount to time 10

Step A

Accumulation to time 5:

$$\int_0^5 e^{\int_t^5 \delta(s)\,ds}\rho(t)\,dt \quad \text{where } \rho(t) = 1$$

$$= \int_0^5 e^{\int_t^5 0.05\,ds} \cdot 1\,dt$$

$$= \int_0^5 e^{(0.25 - 0.05t)}\,dt$$

③ ctd

$$= e^{0.25}\left[\frac{-e^{-0.05t}}{0.05}\right]_0^5$$

$$= e^{0.25}\left[\frac{1-e^{-0.25}}{0.05}\right] = \frac{e^{0.25}-1}{0.05}$$

$$= 5.6805$$

Step B

Accumulate this amount to time 10 (remember the payments have stopped).

$$= 5.6805 \times e^{\int_5^{10}\delta(s)\,ds}$$

$$= 5.6805 \times e^{\int_5^8 0.05\,ds + \int_8^{10} 0.01+0.005s\,ds}$$

$$= 5.6805 \times e^{0.15} \times e^{0.11}$$

[4] $= £7.37$ ← answer

Alternative approach

Step A Find the present value of the payment stream paid between time 0 and time 5

Step B Accumulate this present value to time 10

③ ctd

Step A

Present value of payment stream

$$= \int_0^5 e^{-\int_0^t \delta(s)\,ds} \cdot 1 \, dt$$

$$= \int_0^5 e^{-\int_0^t 0.05\,ds} \, dt$$

$$= \int_0^5 e^{-0.05t} \, dt = \left[-\frac{e^{-0.05t}}{0.05} \right]_0^5$$

$$= 4.42398$$

Now accumulate this amount of money from time 0 to time 10:

Step B

$$4.42398 \times e^{\int_0^{10} \delta(s)\,ds}$$

$$= 4.42398 \times e^{\left(\int_0^8 0.05\,ds + \int_8^{10} 0.01 + 0.005s\,ds\right)}$$

$$= 4.42398 \times e^{0.40} \times e^{0.11}$$

$= £7.37$ ← answer

④ (i) $1000\, e^{\int_0^5 (a+bt)\,dt} = 1300$

$$\therefore \exp\left[\int_0^5 (a+bt)\,dt\right] = 1.3$$

$$\Rightarrow \int_0^5 (a+bt)\,dt = \ln 1.3$$

$$\Rightarrow \left[at + \tfrac{1}{2}bt^2\right]_0^5 = 0.262364$$

[$2\frac{1}{2}$] $\Rightarrow 5a + 12.5b = 0.262364 \quad \text{--- ①}$

Similarly $1000\, e^{\int_0^8 (a+bt)\,dt} = 1500$

[2] leads to $8a + 32b = \ln 1.5 = 0.405465 \quad \text{--- ②}$

Hence $b = \dfrac{0.405465 - 8a}{32}$

Using ① $\Rightarrow 5a + 12.5\left(\dfrac{0.405465 - 8a}{32}\right) = 0.26236$

$\therefore a = 0.0554556$

[$1\frac{1}{2}$] and $b = -0.0011931$

(ii) $V(5) = \dfrac{1000}{1300}$ and $V(8) = \dfrac{1000}{1500}$

$$\Rightarrow \frac{V(8)}{V(5)} = \frac{1.5}{1.3}$$

$\therefore$ accumulation of 100 between $t = 5$ and $t = 8$ is

[2] $$100 \times \frac{1.5}{1.3} = £115.38$$

[Students who work from 1st principles lose time — but still get the marks

eg. $$100 \exp\left\{\int_5^8 (0.0554556 - 0.0011931t)\,dt\right\}$$]

5 (i)(a) $100(1 - 0.06 \times 5) = 70$

[2]

[2] (b) $$100\left(1 - \frac{0.06}{12}\right)^{12 \times 5} = 74.03$$

[1] (ii)(a) $70e^{5\delta} = 100 \Rightarrow \delta = 0.07133$

[1] (b) $74.03e^{5\delta} = 100 \Rightarrow \delta = 0.06015$

6 (a) Require $$\int_0^5 500e^{-0.06t}\,dt$$

[Since $V(t) = e^{-\int_0^t 0.06\,ds} = e^{-0.06t}$]

[3] which is $$500\left[\frac{-e^{-0.06t}}{0.06}\right]_0^5 = 2159.85$$

(b) Again $V(t) = e^{-0.06t}$

[3] Require $$\int_0^5 500e^{0.03t}e^{-0.06t}\,dt = 2321.53$$