CITY UNIVERSITY
LONDON

BSc Honours Degree in Actuarial Science
BSc Honours Degree in Mathematics and Finance
Bsc Honours Degree in Mathematical Science,
Finance and Economics.

FINANCIAL AND INVESTMENT MATHEMATICS

Progress Test

9th December 2002

Answer all of the questions.

The number of marks allocated is shown at the end of each part of the question.

The total number of marks is 55.

The time allowed is 90 minutes.

Financial and Investment Maths

Progress Test - Formulae Sheet

$$a_{\overline{n}|} = \frac{1-v^n}{i} \qquad s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

$$\left(1+\frac{i^{(p)}}{p}\right)^p = 1+i = e^{\delta}$$

$$\left(1-\frac{d^{(p)}}{p}\right)^p = v = 1-d = e^{-\delta}$$

Financial and Investment Maths

Progress Test

Question 1

The effective rate of interest is 6% per annum. Calculate the equivalent:

(a) effective rate of interest per half-year

(b) nominal rate of interest per annum convertible half-yearly.

[3]

Question 2

A single lump sum of £300 is invested for a period of 26 years at the following nominal rates of interest:

Years 1-10: 3% per annum convertible half-yearly
Years 11-20: 5% per annum convertible quarterly
Years 21-26: 4% per annum convertible every two years.

(i) Calculate the total accumulation after 26 years

[3]

(ii) Calculate the yield expressed as an effective rate per annum

[1]

[Total 4 marks]

Question 3

£500 accumulates to £560 in eleven months.

(i) If the interest rate is expressed as a compound rate, calculate the compound interest rate per annum.

[2]

(ii) Calculate the simple discount rate per annum.

[2]

[Total 4 marks]

Question 4

You are informed that the rate of interest is a nominal rate of 8% pa convertible quarterly. You are to receive an annuity of £200 pa for 15 years which might be payable:

(a) annually in arrears, or

(b) annually in advance, or

(c) quarterly in arrears

(i) Using general reasoning, which of (a), (b) or (c) would you expect to have the largest present value, and which the smallest?

(ii) Calculate the present value for each of (a), (b) and (c).

[11]

Question 5

A continuous payment is received for 20 years and the rate of payment at time t is $20e^{0.03t}$. The force of interest, $\delta(t)$, is constant at 0.08.

(i) Calculate the present value of the payment stream.

(ii) Calculate the accumulation of the payment stream after 20 years.

(iii) Without doing any further calculations, explain how your answer to (i) would change if $\delta(t)$ had been a constant which was less then 0.08.

[7]

Question 6

An investor invests:

£200 at $t = 0$, and
£10,000 at $t = 5$

and receives

£1,000 at $t = 7$, and
£12,500 at $t = 9$

(i) Calculate the yield obtained on the investment to the nearest 0.1% per annum.

(ii) Check your answer to (i)

[7]

TURN OVER

PTO

Question 7

The force of interest $\delta(t)$ is given by

$$\delta(t) = \begin{cases} 0.06 + 0.001t & 0 \le t < 5 \\ 0.07 & 5 \le t \end{cases}$$

(i) Calculate the amount to which an investment of £100 made at $t = 0$ will have accumulated :

(a) by time $t = 3$

(b) by time $t = 7$

(ii) Hence, or otherwise, calculate the amount to which an investment of £100 made at time $t = 3$ will have accumulated to by time $t = 7$.

[7]

Question 8

The force of interest $\delta(t)$ is given by

$$\delta(t) = \begin{cases} 0.02 + 0.003t & 0 \le t < 7 \\ 0.08 - 0.002t & 7 \le t \end{cases}$$

(i) Derive expressions in terms of t for the present value of a payment of £100 due at time t. You should find separate expressions for $t < 7$ and $t \ge 7$.

(ii) Calculate the present value of £1,500 at $t = 10$.

[7]

Question 9

Calculate the present value of the following payments:

£400 made at the end of each of the years 1 to 15

£700 made at the end of each of the years 16 to 30

if the interest rate is:

6% p.a. for the first twenty years
8% p.a. thereafter

[5]