FAIM 1 Prog Test Dec 2002 Solutions

1 (i) Let i' = effective rate of interest per $\frac{1}{2}$-yr

[2] $(1+i')^2 = 1.06 \Rightarrow i' = 2.956\%$ per $\frac{1}{2}$-yr

(ii) $2 \times 2.956 = 5.912\%$ per annum convertible $\frac{1}{2}$-yrly

[1]

[3]

2 (i) Accum $= 300\,(1.015)^{20}\,(1.0125)^{40}\,(1.08)^{3}$

[3] $= £836.59$

(ii) Yield is $i\%$ pa such that:

$$(1+i)^{26} = \frac{836.59}{300}$$

[1] $\Rightarrow i = 4.0233\%$ pa

[4]

3 (ii) $(1.12)^{12/11} = 1.13160$

[2] $\Rightarrow$ Compound interest $= 13.160\%$ pa

(iii) Let d = Simple discount rate pa

$$560\left(1 - d \times \frac{11}{12}\right) = 500$$

[2] $\Rightarrow d = 11.688\%$ pa

[4]

4 (i) The most valuable annuity would be (b) when we receive £200 at the start of each year (and can start to earn interest the soonest).

The next most valuable annuity is (c) when we receive £50 at the end of each quarter.

The least valuable annuity is (a) when we receive £200 at the end of each year, (when we start earning interest relatively late)

[3]

(ii) (a) 8% pa convertible quarterly

$\Rightarrow$ 2% per quarter effective rate

$(1.02)^4 = 1 + i = 1.08243$

[1] $\Rightarrow$ effective rate is 8.243% pa

$$PV = 200\, a_{\overline{15}|} \quad @\ 8.243\%$$

$$= 200\, \frac{1 - v^{15}}{i}$$

$$= 200 \times \left(\frac{1 - \frac{1}{(1.08243)^{15}}}{0.08243} \right)$$

[2] $= 200 \times 8.43393 = £1,686.79$

(b) $PV = 200\, \ddot{a}_{\overline{15}|}$ @ 8.243%

where $\ddot{a}_{\overline{15}|} = (1+i)\, a_{\overline{15}|}$

$= 1.08243 \times 8.43393 = 9.12914$

[2] $\Rightarrow PV = 200 \times 9.12914 = £1,825.83$

(c) Work in quarters:

$PV = 50\, a_{\overline{60}|}$ at 2%

$= 50 \times \dfrac{1 - \frac{1}{(1.02)^{60}}}{0.02}$

[3] $= £1,738.04$

[11]

(5) (i) $PV = \int_0^{20} v(t)\rho(t)\, dt$

Here $v(t) = e^{-\int_0^t \delta(s)\, ds} = e^{-\int_0^t 0.08\, ds} = e^{-0.08t}$

and $\rho(t) = 20e^{0.03t}$

$\Rightarrow PV = \int_0^{20} 20e^{0.03t} \cdot e^{-0.08t}\, dt$

$= 20\int_0^{20} e^{-0.05t}\, dt$

$= \frac{20}{0.05}\left[-e^{-0.05t}\right]_0^{20}$

$= 400(1 - e^{-1})$

[3½] $= 252.85$

(ii) Accumulation after 20 yrs is:

$252.85 \times e^{\int_0^{20} \delta(s)\, ds}$

$= 252.85 \times e^{\int_0^{20} 0.08\, ds}$

$= 252.85 \times e^{20 \times 0.08}$

[1½] $= 1,252.37$

(iii) If $\delta(t)$ had been a constant less than 0.08 (i.e. 0.06 for example), then the rate of interest used to calculate the present value decreases.

We know that for constant $\delta(t)_0 = \delta$,

$V^t = e^{-\delta t}$ = P.V. of £1 paid at time t

$\Rightarrow$ the smaller δ is, the greater $e^{-\delta t}$ is

$\Rightarrow$ the greater the present value is

[2] Conclusion, PV in (i) would have been greater than 252.85

[7]

6 (i) Eqn of value:

$$12{,}500\,V^9 + 1{,}000\,V^7 - 200 - 10{,}000\,V^5 = 0$$

Since income is dominated by payment at $t=9$ and outgo is dominated by payment at $t=5$, we try

$$(12{,}500 + 1{,}000)\,V^9 = (200 + 10{,}000)\,V^5$$

$$\Rightarrow \quad V^4 = \frac{10200}{13500} = 0.7556$$

$$\Rightarrow \quad i \simeq 7.3\%$$

Try 7% in original eqn of value:

$i = 0.07$, LHS = 92.06

$i = 0.075$ LHS = -43.04

So yield is between 7 and 7.5%. Now use linear interpolation:

$$i = 0.07 + \frac{92.06}{92.06 - (-43.04)} \times (0.075 - 0.07)$$

[6]

$= 0.0734$ i.e. 7.3% to nearest 0.1%

(ii) Check: Put $i = 0.073$ in original eqn of value

[1] LHS = 9.9 $\rightarrow$ close enough to zero

[illegible]

7 (i)(a) Accumulation $= 100e^{\int_0^3 \delta(s)\,ds}$

$= 100e^{\int_0^3 (0.06+0.001s)\,ds}$

$= 100e^{\left[0.06s + \frac{0.001s^2}{2}\right]_0^3}$

[2] $= 100e^{0.1845} = £120.26$

(b) Accumulation $= 100e^{\int_0^7 \delta(s)\,ds}$

$= 100e^{\left(\int_0^5 0.06+0.001s)\,ds + \int_5^7 0.07\,ds\right)}$

$= 100e^{\left(\left[0.06s+\frac{0.001s^2}{2}\right]_0^5 + \left[0.07s\right]_5^7\right)}$

$= 100e^{0.3125+0.14}$

[3] $= 100e^{0.4525} = £157.22$

(ii) From (i), we know that £120.26 at $t=3$ accumulates to £157.22 at $t=7$

[Since in (i)(a) and (b), the starting point was the same — i.e. £100 was invested at $t=0$]

[2] Hence, £100 at $t=3$ accumulates to

$\frac{100}{120.26} \times 157.22 = £130.73$ at $t=7$

[7] Alternatively, consider $100e^{\int_3^7 \delta(s)\,ds}$ (which $= 130.73$)

8 (i) $t<7$ $V(t) = \exp\left(-\int_0^t \delta(s)\,ds\right) \times 100$

$\Rightarrow V(t) = 100\exp\left(-\int_0^t (0.02+0.003s)\,ds\right)$

$= 100\exp\left(-\left[0.02s + \frac{0.003s^2}{2}\right]_0^t\right)$

$= 100\exp\left(-0.02t - 0.0015t^2\right)$

[2½] $t \geq 7$ $V(t) = \exp\left(-\int_0^t \delta(s)\,ds\right) \times 100$

$\Rightarrow V(t) = 100\exp\left(-\int_0^7 \delta(s)\,ds - \int_7^t \delta(s)\,ds\right)$

$= 100\,V(7)\cdot\exp\left(-\int_7^t \delta(s)\,ds\right)$

$= 100\,V(7)\cdot\exp-\left[0.08s - 0.001s^2\right]_7^t$

$= 100\,V(7)\cdot\exp-\left(0.08t - 0.001t^2 - 0.56 + 0.049\right)$

[3½] $= 100\,e^{-0.2135}\cdot e^{(0.001t^2 - 0.08t + 0.511)}$

$= 100\,e^{(0.001t^2 - 0.08t + 0.2975)}$

[1] (ii) $PV = 1500\,e^{(0.001\times10^2 - 0.08\times10 + 0.2975)}$

[7] $= 1500\,e^{-0.4025} = £1,002.97$

9 $PV = 700\, a_{\overline{20|}}^{6\%} - 300\, a_{\overline{15|}}^{6\%} + 700\, v_{6\%}^{20}\, a_{\overline{10|}}^{8\%}$

$$a_{\overline{20|}}^{6\%} = \frac{1 - v^{20}}{i} = \frac{1 - \frac{1}{(1.06)^{20}}}{0.06} = 11.4699$$

$$a_{\overline{15|}}^{6\%} = \frac{1 - v^{15}}{0.06} = 9.7122$$

$$a_{\overline{10|}}^{8\%} = \frac{1 - v^{10}}{0.08} = 6.7101$$

$$v_{6\%}^{20} = 0.311805$$

Hence $PV = 700 \times 11.4699 - 300 \times 9.7122$

$+ 0.311805 \times 6.7101 \times 700$

[5] $= £6,579.84$