CITY City University
London

The University for business and the professions

Faculty of Actuarial Science and Statistics

BSc Honours Degree in Actuarial Science

Financial and Investment Mathematics I

Progress Test

January 2004

Answer all of the questions

The number of marks allocated is shown at the end of each question

The total number of marks is 55

The time allowed is 90 minutes.

Financial and Investment Maths I
Progress Test – Formula Sheet

$$a_{\overline{n}|} = \frac{1-v^n}{i} \qquad\qquad s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

$$\ddot{a}_{\overline{n}|}^{(p)} = \frac{1-v^n}{d^{(p)}} \qquad\qquad s_{\overline{n}|}^{(p)} = \frac{(1+i)^n - 1}{i^{(p)}}$$

$$\left(1 + \frac{i^{(p)}}{p}\right)^p = 1 + i = e^{\delta}$$

$$\left(1 - \frac{d^{(p)}}{p}\right)^p = v = 1 - d = e^{-\delta}$$

Question 1

£600 accumulates to £690 in seven months.

   (i)   If the interest rate is expressed as a simple rate, calculate the simple interest rate **per annum**.

   (ii)   If the interest rate is expressed as a compound rate, calculate the compound interest rate **per annum**.

   (iii)   Why is your answer to (ii) more than your answer to (i)?

                                     [6]

Question 2

You are informed that $i = 7.4\%$

   (i)   Calculate $i^{(4)}$

   (ii)   From your answer to (i), complete the following statement:

A nominal rate of interest of _____% per annum convertible _____ is equivalent to an effective rate of interest of _____% per annum.

                                     [4]

Question 3

£100 is invested at a rate of interest of 5% per annum convertible every two years for 12 years followed by a rate of 8% per annum convertible half yearly for a further eight years.

   (i)   Calculate the total accumulation after 20 years.

   (ii)   Calculate the yield obtained by the investor.

                                     [3]

Question 4

An annuity of £250 is payable annually in arrears for 15 years.

   (i)   Calculate the present value of the annuity at a **rate** of interest of 7% per annum

   (ii)   Calculate the present value of the annuity at a **force** of interest of 7% per annum.

                                     [5]

Question 5

A continuous payment stream is received for 8 years and the rate of payment at time t is $e^{0.06t}$. The force of interest $\delta(t)$ is constant at 0.09.

Calculate the present value of the payment stream.

[4]

Question 6

An investor invests:

$$£1,300 \text{ at } t = 0 \text{ and}$$

$$£ \quad 10 \text{ at } t = 2$$

and receives

$$£ \quad 110 \text{ at } t = 4 \text{ and}$$

$$£1,600 \text{ at } t = 10.$$

Calculate the yield obtained on the investment to the nearest 1% per annum.

[5]

Question 7

(i)   Describe in detail the annuity which has a present value equal to $a_{\overline{n}|}$.

(ii)   Show, from first principles, that

$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

[5]

Question 8

An individual invests in a 10 year savings plan by making contributions of £50 at the end of every quarter.

Calculate the accumulated fund after 10 years if the interest rate was:

8% per annum effective for the first 5 years.

6% per annum convertible quarterly for the last 5 years.

[7]

Question 9

The force of interest $\delta(t)$ is given by the formula:

$$\delta(t) = 0.05 + 0.002t \qquad 0 \le t < 10$$
$$= 0.04 \qquad\qquad 10 \le t$$

Calculate the value to which an investment of £100, made at time t=0, will have accumulated by time t=15.

[4]

Question 10

The force of interest $\delta(t)$ is given by the formula:

$$\delta(t) = 0.02 + 0.007t \qquad 0 \le t < 5$$
$$= 0.12 - 0.01t \qquad\quad 5 \le t$$

(i)   Derive expressions in terms of t for the present value of a payment of £100 due at time t. You should find separate expressions for $t < 5$ and $t \ge 5$.

(ii)   Calculate the present value of a payment of £500 due at time $t = 8$

[7]

Question 11

Calculate the present value of payments of £600 received at the end of each of the next 17 years if the interest rate is:

8% per annum effective for the first 12 years.
6% per annum effective for the last 5 years.

[5]