FAIM I Prog Test Jan 2004

① (i) $\frac{690}{600} = 1.15$ (i.e. 15% increase over 7 months)

Hence, simple interest per annum $= 15 * \frac{12}{7}$

[2] $= 25.71\%$ pa

(ii) Under compound interest:

$(1.15)^{12/7} = 1.2707$

[2] $\Rightarrow$ Compound interest rate $= 27.07\%$ pa

(iii) Under simple interest, interest earned does not itself earn interest, whereas it does under compound interest

[2] $\Rightarrow$ Expect to earn more interest over the whole year under compound interest.

② (i) $1.074 = \left(1 + \frac{i^{(4)}}{4}\right)^4$

$\Rightarrow i^{(4)} = \left((1.074)^{1/4} - 1\right) * 4$

[2½] $= 7.2031\%$

[1½] (ii) A nominal rate of interest of 7.2031% pa convertible quarterly is equivalent to an effective rate of interest of 7.4% p.a.

[2] ③ (i) $100\,(1.10)^6\,(1.04)^{16} = £331.81$

(ii) $100\,(1+i)^{20} = 331.81$

[1] $\Rightarrow i = 6.18043\%$ pa

④ (i) $PV = 250\, a_{\overline{15|}}$ at 7% pa

$= 250\left(\frac{1-V^{15}}{i}\right)$ $\quad i = 0.07$

[2] $= 2{,}276.98$

(ii) $\delta = 0.07 \Rightarrow e^{\delta} = e^{0.07} = 1+i$

[1] $\Rightarrow i = 0.07251$

So $PV = 250\, a_{\overline{15|}}$ @ 7.251% pa

$= 250\left(\frac{1-V^{15}}{i}\right)$ $\quad i = 0.072$

[2] $= 2{,}241.32$

⑤ $PV = \int_0^8 V(t)\,\rho(t)\,dt$

Here $V(t) = e^{-\int_0^t \delta(s)ds} = e^{-\int_0^t 0.09\,ds} = e^{-0.09t}$

and $\rho(t) = e^{0.06t}$

$$\Rightarrow PV = \int_0^8 e^{-0.09t} \cdot e^{0.06t} \, dt$$

$$= \int_0^8 e^{-0.03t} \, dt$$

$$= \left[-\frac{1}{0.03} e^{-0.03t} \right]_0^8$$

[4] $$= \frac{1 - e^{-0.24}}{0.03} = 7.1124$$

(6) Equation of value:

[2] $$110 v^4 + 1600 v^{10} - 1300 - 10 v^2 = 0$$

Since income is dominated by payment at $t = 10$ and expenditure is dominated by payment at $t = 0$, we'll approximate with:

$$(110 + 1600) v^{10} = 1310$$

$$\Rightarrow v^{10} = \frac{1310}{1710} = 0.76608$$

$$\Rightarrow i = 2.7\%$$

So, let's try 2% in original eqn of value:

Try $i = 0.02 \Rightarrow LHS = 104.57$

$i = 0.03 \Rightarrow LHS = -21.14$

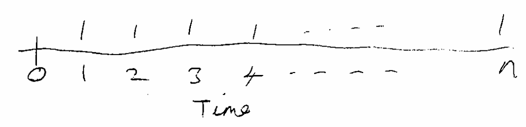
[3] $\Rightarrow i = 3\%$ pa to nearest 1% pa

(as value it produces is closer to 0)

(7) (i) $a_{\overline{n}|}$ is the present value of an annuity of 1 unit per time unit which is paid at the end of each time unit for a total of n time units

[2]

(ii)



PV of above cash flow, $a_{\overline{n}|}$

$$= a_{\overline{n}|} = v + v^2 + v^3 + \cdots + v^n$$

$$= v \left(\frac{1 - v^n}{1 - v} \right)$$

$$= \frac{(1 - v^n)}{(1+i)(1-v)} \quad \text{as } v = \frac{1}{1+i}$$

[3] $$= \frac{1 - v^n}{1 + i - 1} = \frac{1 - v^n}{i}$$

(8) Work in years:–

Accumulation after 10 years =

[2] $200\, s^{(4)}_{\overline{5}|\,8\%}\,(1.015)^{20} + 50\, s_{\overline{20}|\,1\frac{1}{2}\%}$

where $s^{(4)}_{\overline{5}|\,8\%} = \dfrac{(1.08)^5 - 1}{i^{(4)}}$

and $\left(1 + \dfrac{i^{(4)}}{4}\right)^4 = 1.08 \Rightarrow i^{(4)} = 0.0777062$

[$2\frac{1}{2}$] $\Rightarrow s^{(4)}_{\overline{5}|} = 6.039776$

and $s_{\overline{20}|\,1\frac{1}{2}\%} = \dfrac{(1.015)^{20} - 1}{0.015} = 23.12367$ [$1\frac{1}{2}$]

$\Rightarrow$ Accum $= 200 \times 6.039776 \times (1.015)^{20}$

$+ 50 \times 23.12367$

[1] $= 2{,}783.12$

(7) Accumulation at $t = 15$

$= 100 \exp\left\{\int_0^{15} \delta(t)\,dt\right\}$

$= 100\left[\exp\int_0^{10}(0.05 + 0.002t)\,dt * \exp\int_{10}^{15} 0.04\right.$

$= 100\, e^{\left[0.05t + 0.001t^2\right]_0^{10}} \cdot e^{\left[0.04t\right]_{10}^{15}}$

$= 100\, e^{[0.5 + 0.1]} \cdot e^{[0.04(15-10)]}$

[4] $= 100 e^{0.8} = £222.55$

(10) $t < 5$ $\quad V(t) = \exp\left(-\int_0^t \delta(r)\,dr\right) \times 100$

$= 100 \exp\left(-\int_0^t (0.02 + 0.007r)\,dr\right.$

$= 100 \exp\left(-\left[0.02r + \dfrac{0.007r^2}{2}\right]_0^t\right.$

[$2\frac{1}{2}$] $= 100\, e^{-(0.02t + 0.0035t^2)}$

$t \geq 5$ $\quad V(t) = \exp\left(-\int_0^t \delta(r)\,dr\right) * 100$

$= 100 \exp\left(-\int_0^5 \delta(r) - \int_5^t \delta(r)\,dr\right.$

$$= 100\, e^{-0.1875} \cdot \exp\left(-\int_5^t (0.12 - 0.01r)\, dr\right)$$

$$= 100\, e^{-0.1875} \cdot e^{-\left[0.12r - \frac{0.01r^2}{2}\right]_5^t}$$

$$= 100\, e^{-0.1875} \cdot e^{-(0.12t - 0.005t^2 - 0.475)}$$

[3] $= 100\, e^{0.2875} \cdot e^{(-0.12t + 0.005t^2)}$

[1½] (ii) $PV = 500\, e^{0.2875} \cdot e^{(-0.12t + 0.005t^2)}$

with $t = 8$

$$\Rightarrow PV = 500\, e^{0.2875} \cdot e^{-0.64}$$

$$= 351.46$$

(ii) $PV = 600\left(a_{\overline{12}|\,8\%} + v^{12}_{8\%}\, a_{\overline{5}|\,6\%}\right)$

[2]

[1] where $a_{\overline{12}|\,8\%} = \dfrac{1 - v^{12}}{0.08} = 7.5361$

[1] $a_{\overline{5}|\,6\%} = \dfrac{1 - v^5}{0.06} = 4.2124$

$$\Rightarrow PV = 600\left(7.5361 + 0.39711 \times 4.2124\right)$$

[1] $= 5{,}525.33$