

Differential Equations for Finance

MAY 2006 ANSWERS

1. $F = e^{-(\sigma^2 + r)t}$, $G = e^{-(3\sigma^2 + 2r)t}$, $\Delta = 2SF - 3S^2G$
 $S = \frac{2}{3} e^{(2\sigma^2 + r)t}$

2. (i) $U = \frac{1}{2\sqrt{\pi z}} e^{-x^2/4z}$ (ii) $U = \text{erf}\left(\frac{x}{2\sqrt{z}}\right)$

3. $\beta = -1$

4. $\left\{ \frac{1}{2} \Delta z \frac{\partial^2 U}{\partial z^2} - \frac{1}{12} (\Delta x)^2 \frac{\partial^4 U}{\partial x^4} - \frac{1}{3} (\Delta x)^2 U \frac{\partial^3 U}{\partial x^3} \right\}_{i,j}$

0, 0.21665, 0.43331, 0.64996, 0.84328, 1.

5. 0.4551, 0.6436, 0.4551, $U = \left\{ \left(1 - \frac{2}{\pi^2}\right) e^{-\pi^2 z} + \frac{2}{\pi^2} \right\} \sin \pi x$
 0.0107

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1. (i) $X = e^{rt}$, $Y = e^{-(\sigma^2 + r)t}$, $Z = -e^{-(6\sigma^2 + 3r)t}$, $\Delta = 2SY + 4S^3Z$
 $S = \frac{1}{\sqrt{2}} e^{(\frac{5}{2}\sigma^2 + r)t}$ (ii) $a = -r, b = -\alpha$

2. —
 3. $U = 1 + 2 \text{erf}\left(\frac{x}{2\sqrt{z}}\right)$

4. $\left\{ \frac{1}{2} \Delta z \frac{\partial^2 U}{\partial z^2} - \frac{1}{12} (\Delta x)^2 \frac{\partial^4 U}{\partial x^4} \right\}_{i,j}$, $\alpha = \frac{1}{6}$, 0.63807, 0.90237

$F = e^{-\pi^2 z}$, 0.00005, 0.00007

5. 0.0818, 0.1356, 0.1356, 0.0818