MAY 2008 ANSWERS

1. (a) —

(b) $a = \sigma^2 + \alpha, \quad b = \frac{\beta}{2}, \quad c = \frac{\gamma}{3}$

$$S = \frac{A}{2B} \exp\left\{ (\sigma^2+\alpha)t + \tfrac{1}{2}\beta t^2 + \tfrac{1}{3}\gamma t^3 \right\}$$

2. $\alpha = \frac{1}{2}(1-k), \quad \beta = -\frac{1}{4}(1+k)^2$

3. (i) $U = \tau^{-\frac{1}{2}} \frac{1}{2\sqrt{\pi}} e^{-\frac{x^2}{4\tau}}$

4. $\left\{ \Delta\tau \frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} - (\Delta x)^2 \left(\frac{1}{12}\frac{\partial^4 U}{\partial x^4} + \frac{5}{3}\frac{\partial^3 U}{\partial x^3} \right) \right\}_{i,j}$

0.2924, 0.5208, 0.7673, 1.005

5. 0.2574, 0.4164, 0.4164, 0.2574

$F = e^{-(\pi^2+10)\tau}$, 0.2655, 0.4296.

MAY 2009 ANSWERS

1. $F' + (\sigma^2 + r)F = 0, \quad G' = 0, \quad V = S - S^2 e^{(b+r)(T-t) + a(\cos t - \cos T)}$

$\Delta = 1 - 2S e^{(b+r)(T-t) + a(\cos t - \cos T)}$

$\frac{1}{4} e^{(b+r)(t-T) + a(\cos T - \cos t)}$

2. —

3. $U = \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{x^2}{4\tau}}, \quad U = \operatorname{erf}\left(\frac{x}{2\sqrt{\tau}}\right) - 1$

4. $\left(0, \frac{35}{64}, \frac{11}{16}, \frac{35}{64}, 0\right), \quad \left(0, \frac{7}{12}, \frac{2}{3}, \frac{7}{12}, 0\right)$

5. $(1-2\theta)\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial x^2}$

$B_n = \frac{4(1-(-1)^n)}{n^3\pi^3}$

0.2395, 0.24