

Sheet 3 Answers

1. $0.3113, 0.5036, 0.5036, 0.3113$
 $0.4853, 0.7853, 0.7853, 0.4853$
 $e^{-\pi^2 z} \sin \pi x, 0.7807$

2. $\left\{ \frac{\Delta z}{2} \frac{\partial^2 U}{\partial z^2} - \frac{(\Delta x)^2}{12} \frac{\partial^4 U}{\partial x^4} - \frac{(\Delta x)^2}{6} U \frac{\partial^3 U}{\partial x^3} \right\}_{i,j}$
 $-1, -\frac{73}{128}, 0, \frac{73}{128}, 1$

3. $\left\{ \frac{1}{2} \Delta z \frac{\partial^2 U}{\partial z^2} - \frac{1}{12} (\Delta x)^2 \frac{\partial^4 U}{\partial x^4} \right\}_{i,j}$
 $0.105, 0.11051, 0.11105$

4. $\frac{\sqrt{3}}{27}, \frac{\sqrt{3}}{27}, U = \frac{1}{\pi^2} (1 - e^{-\pi^2 z}) \sin \pi x$
 Error 0.0058

5. $(1-2\theta) \frac{\partial U}{\partial z} = \frac{\partial^2 U}{\partial x^2}, \quad \alpha = \frac{1}{6} (1-2\theta)^2$

6. $\alpha \leq \frac{27}{8}, \quad 0.2475$

7. $\left\{ \frac{\Delta z}{2} \frac{\partial^2 U}{\partial z^2} - (\Delta x)^2 \left(\frac{1}{12} \frac{\partial^4 U}{\partial x^4} + \frac{5}{3} \frac{\partial^3 U}{\partial x^3} \right) \right\}_{i,j}$
 $0.2924, 0.5208, 0.7673, 1.005$

8. $0.2574, 0.4164, 0.4164, 0.2574$
 $F = e^{-(\pi^2 + 10)z}, \quad 0.2655, 0.4296$