of A. For what values of A do oscillations occur? Give the phase plane diagram for this case. Find the complete solution and give the phase plane diagram for $A = 0$. Discuss the period of the motion for A in the range $-\frac{4}{27} \leq A \leq 0$.

6. If $\dot{x}^2 = x^3 - 3x^2 - 6x + 8$, for what range of x do oscillations occur? Give the phase plane diagram.

7. If $\ddot{x} = f(x)$ where (i) $f = 1 - x$ (ii) $f = 1 - x^3$ (iii) $f = \cos x$ (iv) $f = x^3 - x$ find the equilibrium positions and determine the stability properties in each case.

8. A rocket is launched vertically from the Earth's surface $r = R$ with speed $\dot{r} = V$, so that the angular momentum $h = 0$. Use Newton's law and the inverse square law of gravitational attraction to determine the relationship

$$\dot{r}^2 = \frac{2\mu}{r} + V^2 - \frac{2\mu}{R}.$$

Use the phase plane to discuss the motion for different values of V and find the complete solution for r as a function of t in the case when $V^2 = 2\mu/R$.

9. An infinite string $(-\infty < x < \infty)$ is given a displacement $\eta = \operatorname{sech} x$ at $t = 0$ and is released from rest. Find and sketch its subsequent displacement $\eta(x,t)$.

10. A semi-infinite string $x \geq 0$ is held fixed with zero displacement η at $x = 0$ for all $t \geq 0$. If it is released from rest at $t = 0$ with displacement $\eta = xe^{-3x}$, find and sketch its subsequent displacement $\eta(x,t)$. Find the slope of the string at $x = 0$.

11. A semi-infinite string $x \geq 0$ is forced to oscillate by a vertical displacement $\eta = a \sin \omega t$ of its end $x = 0$. If the string is at rest with zero displacement at $t = 0$, find and sketch its displacement $\eta(x,t)$ for $t \geq 0$.

12. The displacement x of a damped spring system is governed by the equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 0.$$

If the system is released with zero displacement and initial speed $\frac{dx}{dt} = 1$ at $t = 0$, find x as a function of t. Find the maximum displacement and show that it occurs at time $t = \frac{1}{3}\ln 4$. Sketch x as a function of t.

The system is now forced so that x satisfies the equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 34\cos t$$

and the system is now released from rest with zero displacement at $t = 0$. Find x as a function of t and show that at large times there is a periodic motion with amplitude $\sqrt{34}$.