END OF CHAPTER EXERCISES

Chapter 10 : Portfolio Theory And Asset Returns

Investments : Spot and Derivatives Markets

(Keith Cuthbertson, Dirk Nitzsche)

1. If you have one risky asset and one riskless (safe) asset, what is meant by the opportunity set? (This particular opportunity set is often referred to as "the transformation line").

2. If you have two risky assets, what is meant by the opportunity set?

3. What is the Sharpe Ratio and intuitively why would an investor choose asset proportions in such a way as to maximise the Sharpe ratio ?

4. The returns on a Treasury Bill (i.e. safe asset) and on equity (i.e. risky asset) are given below (for 4 'states' of the economy) :

State of the Economy	Prob. of State of Economy occurring	Return (% pa) T-Bill	Equity
1	0.25	4	-10
2	0.25	4	0
3	0.25	4	15
4	0.25	4	50

The maturity of the T-bill equals the holding period of 1 year and hence is the risk-free asset.

(a.) Calculate the expected return ER and the standard deviation SD of the returns on (i.) the Treasury Bill and (ii.) the equity share. (Use n=4 rather than n-1 = 3 when calculating SD).

(b.) Calculate the expected return ER and the standard deviation SD of **a portfolio** consisting of the risk free asset and the risky asset (equity), corresponding to the following proportions "x" held in each asset.

Share of T-Bill (x)	Equity (1-x)
1	0
0.5	0.5
0	1
-0.5	1.5

(c.) Plot a graph of the expected return against the standard deviation for each value of "x" and indicate on the graph the proportions of each asset held.

(d.) What is meant by 'leverage' in the context of this graph ? Why do you obtain leverage when x = -0.5 and (1-x)=1.5, if you have "own funds" of $100?

5. The return from Equity-1 (risky asset) and Equity-2 (risky asset) depends on the state of the economy:

State of the economy	Probability P_i	Rate of return (%) Equity-1	Equity-2
1	0.25	0	35
2	0.25	0	15
3	0.25	7.5	10
4	0.25	15	10

(a.) Calculate the expected return ER and the standard deviation σ_i of the returns on equity-1 and equity-2, and the correlation coefficient between the two returns.

(Note: ER = $\sum_{j=1}^{4} p_j R_{1j}$, for each asset and σ_1 = $\sum_{j=1}^{4} p_j (R_j - ER_1)^2$, σ_{12} = and $\rho = \sigma_{12} / \sigma_1 \ \sigma_2$.

(b.) Calculate the expected return ER_p and the standard deviation σ_p **on a portfolio** of equity-1 and equity-2 (i.e. 2 risky assets) corresponding to the following proportions w_i held in each asset.

(Note: $ER_p = w_1 ER_1 + w_2 ER_2$ and $\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \ \sigma_2^2 + 2 w_1 w_2 \rho \sigma_1 \sigma_2$)

Share of Equity-1 W_1	Equity-2: $w_{2=} (1-w_1)$
1	0
0.75	0.25
0.5	0.5
0	1

(c.) For each portfolio (i.e. combination of w_1, w_2) plot a graph of the expected return (ER_p) against the standard deviation (SD = σ_p) and indicate on the graph the proportions of each asset held. What is the opportunity set and the efficient frontier ?

(d.) What is the proportion of each risky asset held in the minimum variance (SD) portfolio ?

(e.) Suppose there is a safe asset (e.g. T-Bill) which has a (safe) return of 4%. What are the optimal proportions of the 2 risky assets (Equity-1 and Equity-2), any investor would hold ? What is meant by "optimal" here ?

(f.) How would your answer to (e) change if the rate of return on the risk-free asset is 10% rather than 4% ?

6. Assume investors like high (expected) returns but do not like high levels of risk (where we measure risk by the standard deviation the portfolio of risky assets held). Mathematically, this can be expressed as :

$$U = U(ER_p, \sigma_p)$$

where U = level of satisfaction (or "utility") and when ER increases so does U, but when σ_p increases there is a fall in U.

(a.) Briefly explain what we mean by this investors "indifference curve" and what general shape does it have (in an ER_p, against σ_p graph) ?

(b.) Draw two indifference curves one for a very risk averse investor and the other for a person for whom risk is not a major concern.

(c.) According to mean-variance portfolio theory how would the portfolio held by the extremely risk averse investor and the not so risk averse investor differ. For example, would one of them hold 70% of a 'high risk' share and 30% of a 'low risk' share, while the other held exactly the opposite *proportions* of these two risky assets ?

7. The CAPM predicts that the equilibrium rate of return on any risky asset-i is given by :

$$ER_i = r + \beta_i (ER_m - r)$$

where ER_m = expected market return, r = risk free rate.

(a.) Explain the intuitive logic behind the CAPM relationship. How would you measure beta ?

(b.) Intuitively, when a risky asset earns less than the risk free return r, why would you continue to hold this risky asset?