Homological Methods in Algebra, Geometry & Physics

City University London

Dennis Borisov (Oxford) *Virtual fundamental classes for moduli spaces of sheaves on Calabi-Yau four-folds*

Chris Braun (Münster) *Noncommutative localisation of algebras and modules*

The localisation of a dg algebra (or more generally, a dg category) can be seen to be, in a certain precise sense, equivalent to the Bousfield localisation of its category of dg modules. In this talk I will explain what this means and show how this general abstract result has a range of concrete applications including the group completion theorem, characteristic classes of A-infinity algebras and a derived Riemann-Hilbert correspondence.

Benoit Fresse (Lille) *Koszul duality and homotopy automorphisms of operads*

Homotopy automorphism spaces of operads encode the internal symmetries of homotopy categories of algebras governed by operads. I will explain the definition of these objects in a first introductory part of my talk. In a second step, I will explain the general definition of spectral sequences to compute these homotopy automorphism spaces, and I will show how the method applies in the E_n-operad case.

I will also explain how the equations of the graded Grothendieck-Teichmller group show up in the spectral sequence computation in the E_2-operad case. The main outcome of this computation is that the Grothendieck-Teichmller group represents the group of rational homotopy automorphisms of E_2-operads.

Alastair Hamilton (Texas Tech) *Noncommutative BV-formalism, functional integrals and pairing classes in the moduli space of Riemann surfaces*

A well-known theorem of Kontsevich links noncommutative geometry with moduli spaces of Riemann surfaces. This theorem states that the (stable relative) Chevalley-Eilenberg homology of a certain Lie algebra – a noncommutative analogue of the Poisson algebra of formal Hamiltonian vector fields – captures the cohomology of moduli spaces of curves with marked points. Kontsevich also described constructions producing classes in this moduli space.

In this talk we will describe how to build a noncommutative version of the BV-formalism upon this initial framework of Kontsevich, and use this to construct homology and cohomology classes in certain compactifications of the moduli space. We will construct a differential graded Lie algebra from Kontsevich's original Lie algebra that plays a role similar to the usual BV algebra of polynomial superfunctions on the space of fields. Our constructions will be formulated as functional integrals over finite-dimensional spaces of fields and we will provide examples that use such functional integrals to calculate the pairing between homology and cohomology classes.

Richard Hepworth (Aberdeen) *String topology of classifying spaces*

Let G be a Lie group. Form the classifying space BG, then the space of free loops LBG, then the homology $H^*(LBG)$. What structure does this homology possess? Chataur and Menichi gave one answer: it is a homological conformal field theory, meaning that it admits operations parameterised by the homology of mapping class groups of surfaces with boundary. This talk will explain that there is far more structure, governed not by surfaces and their diffeomorphisms, but by certain singular objects and their homotopy equivalences. This is joint work with Anssi Lahtinen. (arxiv.org/abs/1308.6169)

Vladimir Hinich (Haifa) *Rectification of operad algebras*

Given a topological operad O, we describe the infinity-category of O-algebras (in the sense of Lurie) with values in complexes of k-modules, via the model category of "classical" O-algebras.

Isamu Iwanari (Tohoku) *Tannaka duality theory in derived algebraic geometry and applications*

Tannaka duality theory gives a correspondence between pro-algebraic groups and symmetric monoidal abelian categories which satisfy some categorical condition. We discuss a tannakian theory for symmetric monoidal stable infinity categories and describe also backgrounds, applications to motivic Galois theory of mixed motives, and prospects.

Peter Jørgensen (Newcastle) *The cluster categories of Igusa and Todorov*

Igusa and Todorov have introduced several new cluster categories which are "continuous" or "infinite". The talk will explain some aspects of these categories. We will also examine conditions for subcategories to be functorially finite, cluster tilting, and, more generally, for a subcategory to be half of a torsion pair.

Hovhannes Khudaverdian (Manchester) *Geometrical foundations of the Batalin-Vilkovisky formalism*

The aim of this talk is to explain how the Batalin-Vilkvosiky (BV) formalism follows from the basic principles of field theory and geometry.

To obtain the partition function of the theory one needs to integrate the exponent of action functional over all fields. If a Lagrangian is degenerate (like for gauge theories), then by integrating exponent of action first over symmetries one arrives to the integral of a non-local measure functional over the 'surface' defined in the space of fields by gauge conditions. In order to make this functional local one needs to expand the space of fields by ghosts. One comes finally to gauge independent local action in the space of fields and ghosts. This is the famous 'Fadeev-Popov trick' which in particular works for Yang-Mills gauge theory.

One can consider the surface of gauge conditions as a Lagrangian surface in the symplectic space of fields and anti-fields provided with the canonical odd symplectic structure. In this case the measure functional over the surface of gauge conditions becomes half-density, the master-half-density, in this symplectic space. The gauge-independence can be formulated as a condition of vanishing of this master-half-density under the action of the canonical odd Laplacian. This is the complete description of the BV quantum master equation. The initial action and symmetries of the theory are boundary conditions which define this master half-density. Such a formulation is equivalent to the Fadeev-Popov trick in the case of so called 'closed algebra of symmetries' (e.g. for Yang-Mills theory). On the other hand the formulation in terms of half-densities is invariant with respect to wider algebra of transformations, it works for an arbitrary degenerate Lagrangian, and it becomes necessary if we have so called 'open algebra of symmetries'. In the classical limit the quantum BV equation on master half-density becomes well-known BV equation on the master action.

Finally we explain the Severa interpretation of the BV quantum master equation in terms of specially constructed spectral sequence.

Wajid Mannan (Lancaster) *Closed model category structures for coalgebras and curved Lie algebras*

I will describe a closed model category structure on cocommutative dg coalgebras extending Hinich's construction for the conilpotent case. Working with the linear duals of the coalgebras, this involves extending Hinich's category to include acyclic algebras and formal products. I will also discuss the Koszul dual notion of adding acyclic algebras to the Hinich category; namely extending the category of Lie algebras to include curved Lie algebras. This is joint work with Joe Chuang and Andrey Lazarev.

Raphaël Rouquier (UCLA) *Perverse equivalences and higher representations*

Perverse equivalences are equivalences of derived categories that amount to a shift of the t-structure on each slice of some filtration. It is hoped that they will give a geometric group theory approach to spaces of stability conditions.

I will discuss the example of certain triangulated categories that can be viewed as 0-weight spaces of 2-representations of simple Lie algebras (for example coming from 2d quotient singularities). Braid group actions arise from such 2-representations, and are particular instances of perverse equivalences. (Joint work with Joseph Chuang)

Christoph Schweigert (Hamburg) *Boundary conditions and defects in three-dimensional topological field theory*

Codimension-one defects in quantum field theories have proven to be natural objects that carry much interesting structure. In the talk, we concentrate on topological quantum field theories in three dimensions, with a special focus on Dijkgraaf-Witten theories with abelian gauge group. We explain that symmetries in these topological field theories are naturally defined in terms of invertible topological surface defects and are thus Brauer-Picard groups.

Alexander Voronov (Minnesota) *r_∞-matrices and quantum ∞-groups*

The goal of the talk is to propose an ∞-version of Drinfeld's scheme of quantization. We introduce r_∞-matrices, which are solutions to the Maurer-Cartan equation in the exterior algebra of a dg Lie algebra or, more generally, an L_∞-algebra. This equation generalizes the classical Yang-Baxter equation $[r, r] = 0$ in the exterior algebra of a Lie algebra. We show have such r_∞-matrices give rise to triangular L_∞-bialgebras and quantum ∞-groups. This is a joint work with my graduate student Denis Bashkirov.

Theodore Voronov (Manchester) *Homological vector fields, higher Koszul brackets and related structures*

Homological vector fields are a powerful tool for encoding geometric and algebraic structures, especially various brackets. We shall recall the necessary language and discuss known and less-known examples related with L-infinity algebras, Lie algebroids and their generalizations.

In particular, we shall consider the 'higher Koszul brackets arising from a homotopy Poisson or Schouten structure on a supermanifold and show how they are connected with L-infinity bialgebroids.

(Based on joint work with Hovhannes Khudaverdian.)

Benjamin Ward (Stony Brook) *Universal operations and cyclic operads*

First, I will briefly recall joint work with Ralph Kaufmann which associates an operad of universal operations to a Feynman category. This operad typically contains the Lie operad and we can then ask for a description of the universal operations after twisting by a Maurer-Cartan element. When the Feynman category encodes non symmetric operads, this line of investigation leads to brace operations and homotopy Gerstenhaber algebras, as is well known. In analogy, I will discuss the case of cyclic operads, introducing cyclic brace operations and a class of homotopy gravity algebras.