(Part II) Lab-session 3

1) Carry out the following integrations numerically:

$$\begin{aligned} I_1 &= \int_1^4 \frac{1}{x} dx = \ln 4 \\ I_2 &= \int_1^2 \exp(x)/x dx \approx 3.059116539, \\ I_3 &= \int_0^\pi \sin(x) x^3 dx = \pi^3 - 6\pi, \\ I_4 &= \int_{-\infty}^{\infty} e^{-(x-5)^2} dx = \sqrt{\pi} \end{aligned}$$

Use the following two methods:

a) Write a subroutine to carry out this task which writes the final answer into a cell on the Excel worksheet. Use the trapezoid rule

$$I = \int_a^b f(x)dx \approx \Delta \left[\frac{1}{2}(f(a) + f(b)) + \sum_{i=2}^{n} f(x_i) \right] \quad \text{with} \quad \Delta = \frac{b-a}{n},$$

as an approximation (just like the 1st example in the lecture). Perform the computations by separating the integration interval $[a, b]$ into $n = 10$, $n = 100$, $n = 1000$ and $n = 10000$ subintervals. You may improve the lecture's subroutine by having your code read the values of a, b and n from some cells in the Excel Worksheet.

b) Write instead a user defined function to carry out the same task by taking a, b and n as input variables and returning the value of the integral as output. Use Simpson's one-third rule

$$I = \int_a^b f(x)dx \approx \frac{\Delta}{3} \left[\sum_{i=1,3,5,\ldots}^{n-2} f(x_i) + 4f(x_{i+1}) + f(x_{i+2}) \right] \quad \text{with} \quad \Delta = \frac{b-a}{n},$$

as an approximation. Test your function for various values of n and find a large n', such that the final answer on your worksheet does not change for any $n > n'$ up to an accuracy of 6 decimal places. The value for n' does not have to be precise, just try to find the correct order of magnitude.

2) Use the Excel built-in function Goal Seek to solve the following equations numerically:

$$\begin{aligned} 110x^2 + 1650x - 40040 &= 0, \\ x^3 - 17x^2 + 71x - 55 &= 0. \end{aligned}$$