Dynamical Systems: Coursework 1	T1

Student's Name:……………………………………………………………………………………

Instructions: For question 1, each wrong answer will contribute -5 points. For all other questions tick only one box. For questions 2,3,4 and 5, ticking more than one box or the wrong box will result in zero marks.

Q1 [20 points] Consider the following 1-dimensional dynamical system

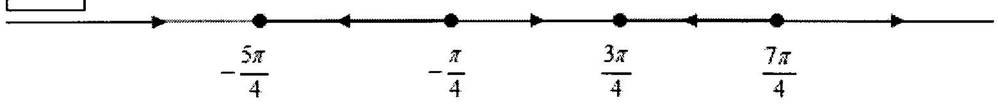
$$\frac{dy}{dx} = \sin(y) + \cos(y)$$

Tick all the boxes which provide correct statements about the equation above

- [ ] The system has three fixed points in the region $-\pi \le y \le \pi$
- [x] The system is autonomous
- [ ] The fixed points of the system are of the form $y = -\frac{\pi}{2} + n\pi$ with $n = 0, \pm 1, \pm 2 \ldots.$
- [x] The system has a fixed point at $y = \frac{11\pi}{4}$
- [x] The system has infinitely many fixed points
- [x] There is a fixed point at $y = -\frac{\pi}{4}$ and it is a repellor

Q2 [20 points] Identify the phase diagram of the system of question 1 in the region $-2\pi \le y \le 2\pi$

- [x]



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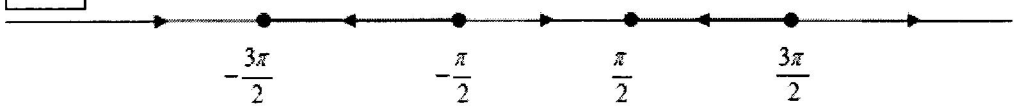

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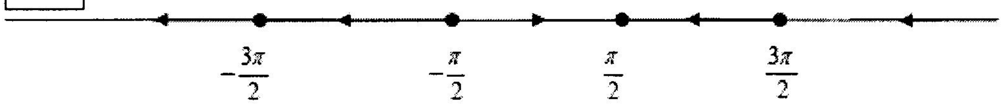

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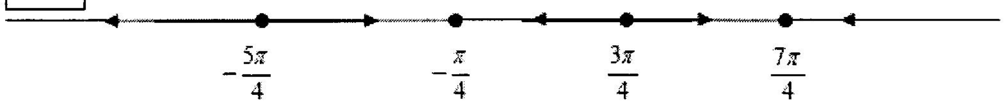


Turn over…

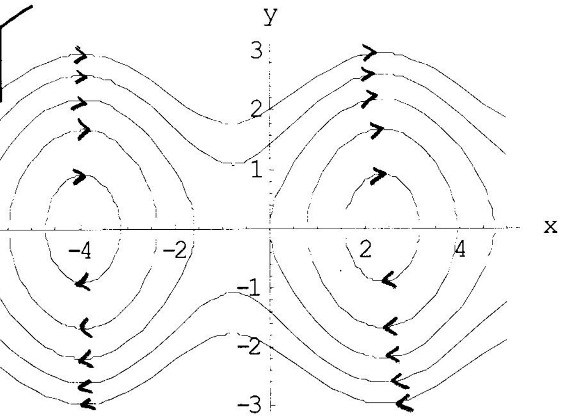
Q3 [20 points] Consider again the equation of question 1. The equation has a fixed point at y=a, with $-\frac{\pi}{2} \le a < 0$. Find the value of a. Thus select the option which gives the solution to the linearized version of the equation of question 1 about the fixed point a, with initial condition y=0 for x=1.

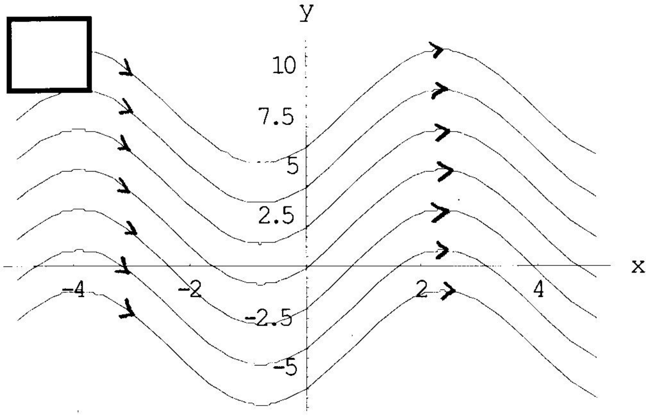
☐ $y = -\frac{\pi}{2} + \frac{\pi}{2} e^{x-1}$ ☑ $y = -\frac{\pi}{4} + \frac{\pi}{4} e^{\sqrt{2}(x-1)}$ ☐ $y = -\frac{\pi}{4} + \frac{\pi}{4} e^{x-1}$ ☐ $y = -\frac{\pi}{2} + \frac{\pi}{2} e^{\sqrt{2}(x-1)}$

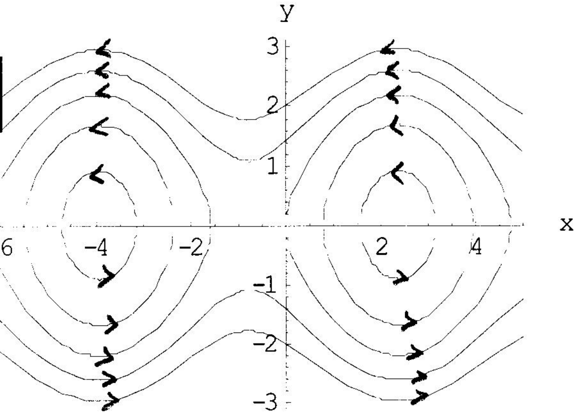
Q4 [20 points] Consider the 2-dimensional dynamical system

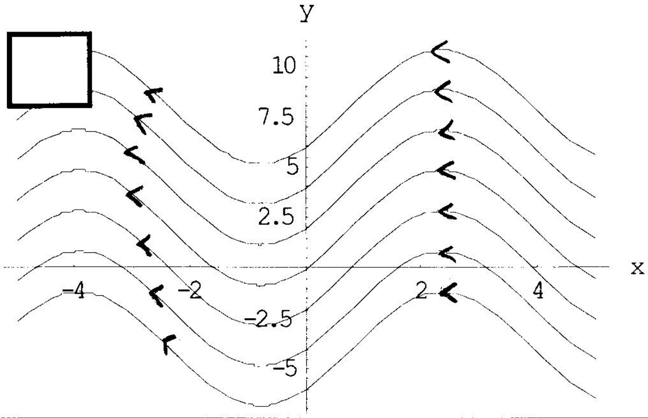
$$\frac{dy}{dt} = \sin(x) - \cos(x) \qquad \text{and} \qquad \frac{dx}{dt} = y$$

Identify the phase diagram of this system of equations

☑ 

☐ 

☐ 

☐ 

Q5 [20 points] Consider the first order differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin(x)$. Identify which of the functions below is a particular solution to this equation

☑ $y = \frac{\sin(x) - x\cos(x) + 1}{x}$ ☐ $y = \frac{\sin(x) + x\cos(x)}{x}$

☐ $y = \frac{1 + \sin(x) + x\cos(x)}{x}$ ☐ $y = \frac{-\sin(x) - x\cos(x)}{x}$