Sheet 2: phase diagrams of 1-dimensional systems and linearisation

1. For each of the following equations

$$\begin{aligned} \text{(i)} \quad & \frac{dy}{dx} = y\cos(y), \\ \text{(ii)} \quad & \frac{dy}{dx} = y^2(1-y), \\ \text{(iii)} \quad & \frac{dy}{dx} = \tan(y), \\ \text{(iv)} \quad & \frac{dy}{dx} = y^2 + 2y - 3. \end{aligned}$$

   - Obtain all fixed points.
   - Hence draw the corresponding phase space diagram and classify all the fixed points as either repellors, attractors or shunts. In cases when there is an infinite number of fixed points (like the first equation), classify them all and draw your phase diagrams just in some suitable interval.

   Notice that you do not need to solve the equations to do this exercise!

2. Consider now equation (iv) from the previous exercise with the initial condition $y(0) = -1$:

   - Solve the equation for y as a function of x (you may have already solved it in exercise sheet 1, question 1). Determine the interval I of values of x for which your solution is valid. Sketch the solution in I.
   - Write down the general expression for the linearisation of this equation about a generic fixed point $y = a$.
   - From exercise 1 (iv) you should have found that the equation has fixed points at $y = 1$ and $y = -3$. Write down the linearisation about each of these fixed points.
   - Solve both linearised equations with the initial condition $y(0) = -1$ and plot the solutions in the same graph where you plotted the exact solution to the equation before.