

## PART II ACTUARIAL SCIENCE

## TABLES OF LAPLACE TRANSFORMS

## GENERAL FORMULÆ

$f(t)$	$F(p)$	$f(t)$	$F(p)$
$f'(t)$	$pF(p) - f(0)$	$f''(t)$	$p^2 F(p) - sf(0) - f'(0)$
$f^{(n)}(t)$	$p^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$		
$tf(t)$	$-F'(p)$	$\frac{f(t)}{t}$	$\int_1^\infty F(p') dp'$
$\int_0^t f(t') dt'$	$\frac{1}{p} F(p)$	$\int_0^t \int_0^\tau f(\nu) d\nu d\tau$	$\frac{1}{p^2} F(p)$
$t$ -shifting		$p$ -shifting	
$f(t-a)u(t-a)$	$e^{-ap} F(p)$	$e^{at} f(t)$	$F(p-a)$
$f(t)$ periodic with period $s$		convolution	
$f(t)$	$\frac{1}{1-e^{-sp}} \int_0^s e^{-pt} f(t) dt$	$\int_0^t f_1(t-\tau) f_2(\tau) d\tau$	$F_1(p) F_2(p)$

## SPECIFIC FORMULÆ

$f(t)$	$F(p)$	$f(t)$	$F(p)$
1	$1/p$	$t$	$1/p^2$
$t^n$	$n!/p^{n+1}$	$1/\sqrt{\pi t}$	$1/\sqrt{p}$
$2\sqrt{t/\pi}$	$1/\sqrt{p^3}$	$\frac{2^n t^{n-1/2}}{1 \cdot 3 \cdot 5 \dots (2n-1)\sqrt{\pi}}$	$p^{-(n+1/2)}$
$t^{k-1}$ ( $k > 0$ )	$\Gamma(k)/p^k$	$e^{-at}$	$1/(p+a)$
$te^{-at}$	$1/(p+a)^2$	$t^{n-1}e^{-at}/(n-1)!$	$1/(p+a)^n$
$\frac{e^{-at}-e^{-bt}}{b-a}$ $a \neq b$	$\frac{1}{(p+a)(p+b)}$	$\frac{ae^{-at}-be^{-bt}}{a-b}$ $a \neq b$	$\frac{p}{(p+a)(p+b)}$
$\frac{1}{a} \sin at$	$1/(p^2 + a^2)$	$\cos at$	$p/(p^2 + a^2)$
$\frac{e^{ct}}{a} \sin at$	$\frac{1}{(p-c)^2 + a^2}$	$e^{ct} \cos at$	$\frac{p-c}{(p-c)^2 + a^2}$
$\frac{1}{a} \sinh at$	$1/(p^2 - a^2)$	$\cosh at$	$p/(p^2 - a^2)$
$a^{-2}(1 - \cos at)$	$\frac{1}{p(p^2 + a^2)}$	$a^{-3}(at - \sin at)$	$\frac{1}{p^2(p^2 + a^2)}$
$\delta(t-k)$	$e^{-pk}$	$u(t-k)$	$\frac{1}{p} e^{-kp}$
$u(t) - u(t-k)$	$\frac{1-e^{-kp}}{p}$	$(t-k)u(t-k)$	$\frac{1}{p^2} e^{-kp}$
$\frac{(t-k)^{\mu-1}}{\Gamma(\mu)} u(t-k)$	$\frac{1}{p^\mu} e^{-kp}$ , $(\mu > 0)$	$\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^2}{4t}\right)$	$\frac{1}{\sqrt{p}} e^{-k\sqrt{p}}$
$\frac{k}{2\sqrt{\pi t^3}} \exp\left(-\frac{k^2}{4t}\right)$	$e^{-k\sqrt{p}}$ , $(k > 0)$	$\text{erfc}\frac{k}{2\sqrt{t}}$	$\frac{1}{p} e^{-k\sqrt{p}}$

Here the unit step function (or Heaviside function) is

$$u(x) = 0 \quad \text{if } x < 0, \quad 1 \quad \text{if } x > 0,$$

and the gamma function by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

Note:

$$\Gamma(n+1) = n!, \quad \text{erfc}(x) = 1 - \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-\zeta^2} d\zeta.$$