Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. Evaluate each of the following limits:

   (i) $\lim_{x \to 0} \left(\dfrac{\sin 2x}{\sin 3x} \right);$

   (ii) $\lim_{x \to 0} \left(\dfrac{e^x - \cos x}{\sin \frac{1}{2} x} \right).$

2. Solve for x the equation

$$4 \sinh x - \cosh x = 1$$

   giving your solution in logarithmic form.

3. Find the general solution of the differential equation

$$y \left(1 + 4x^2\right) \frac{dy}{dx} = 1 + y^2.$$

4. By using the substitution $x = \sinh u$ show that

$$\int_0^{\pi} \sqrt{1 + x^2} dx = \frac{1}{2} \left(ar \sinh \pi + \pi \sqrt{1 + \pi^2} \right).$$

5. Show that the length of the curve $y = \frac{1}{2} \ln \sec 2x$ between the points where $x = -\frac{\pi}{12}$ and $x = \frac{\pi}{12}$ is $\frac{1}{2} \ln 3$.

6. (a) Find the n^{th} derivative with respect to x of

$$\ln (1 + 2x).$$

   (b) Use Leibnitz's Theorem to find the 5^{th} derivative with respect to x of

$$x^3 \sin x.$$

Turn over ...

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. The function f of two variables is given by

$$f(x, y) = 3x^2y - 2xy + 2y^2 - y + 2.$$

(i) Find the first order and second order partial derivatives of f.

(ii) Find the Taylor polynomial of second order for the function f at the point where $x = 1$, $y = 3$.

(iii) Find all the stationary points of the function.

(iv) Classify the stationary points.

8. (a) The function f is defined as

$$f(x) = xe^{3x}.$$

Use the Principle of Mathematical Induction to prove that $f^{(n)}(x)$, the n^{th} derivative of f, is given by

$$f^{(n)}x = 3^{n-1}(3x + n)e^{3x}$$

for all positive integers n.

(b) Given that

$$I_n = \int_0^1 x^n e^x \, dx,$$

show that, for $n \geq 1$,

$$I_n = e - nI_{n-1}$$

Hence evaluate I_5, leaving your answer in terms of e.

Turn over ...

9. (a) Find the general solution of the differential equation

$$x\frac{dy}{dx} - 2y = x^3 \ln x, \qquad x > 0.$$

(b) Find the solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 20 - 26\sin 3x$$

for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$.

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