Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. Solve for x the equation
$$10\cosh^2 x + \sinh x = 31$$
giving your answer in logarithmic form.

2. (a) Write down an expression for the Taylor series of a function f about a value c.
   (b) Determine the Taylor series of the function $f(x) = \sin^2 x$ about $\frac{\pi}{2}$ up to the term in $(x - \frac{\pi}{2})^4$.

3. Evaluate each of the following integrals:
   (a) $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2+9}}dx$;
   (b) $\displaystyle\int x\cos x\, dx$.

4. Evaluate each of the following limits:
   (a) $\displaystyle\lim_{x\to 0}\left(\frac{\cosh x - e^x\cos x}{\sin 2x}\right)$;
   (b) $\displaystyle\lim_{x\to\infty}\left(\frac{3x^2+2x-1}{1-2x^2}\right)$.

5. Find the general solution to the differential equation
$$\frac{dy}{dx} + 3x^2 y = x^2 e^{-x^3}.$$

6. Find the length of the curve given by
$$x^{2/3} + y^{2/3} = 1$$
between $x = 0$ and $x = 1$.

Turn over ...

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. A function of two variables, $f(x, y)$, has stationary points where both $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ simultaneously. What test should be used for identifying whether a stationary point is a maximum, minimum or a saddle point?

   Show that two stationary points of the function

   $$f(x, y) = x^2y + xy^2 - x^2 - y^2 - 3xy + 2x + 2y,$$

   are to be found at $x = y = 1$ and at $x = 1$, $y = 0$. Find the other two stationary points.

   Identify the types of *all* of the stationary points of this function.

8. (a) Using a suitable double angle identity show that

   $$\cos(x) = \frac{1 - t^2}{1 + t^2}$$

   where $t = \tan(\frac{x}{2})$.

   (b) If $t = \tan(\frac{x}{2})$ determine $\frac{\mathrm{d}x}{\mathrm{d}t}$ as a function of t.

   (c) Hence (or otherwise) calculate

   $$\int \frac{dx}{3\cos x + 5}.$$

   (d) Write down a version of the identity in part (a) involving cosh and tanh, and verify it directly using the definitions of these functions.

9. (a) Find the solutions of the differential equation

   $$\left(2y + x^2 \cosh y\right) \frac{dy}{dx} + 2x(1 + \sinh y) = 0.$$

   (b) Find the solution of the differential equation

   $$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 1 + \cos x,$$

   with $y = 1$, $\frac{dy}{dx} = 0$ at $x = 0$.

Internal Examiners:	Dr A. G. Cox
	Dr O. S. Kerr
External Examiners:	Professor D. J. Needham
	Professor M. E. O'Neill