Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. Calculate

$$\int \frac{1-2x+5x^2}{(1-2x)(1+x^2)}\,\mathrm{d}x.$$

[8]

2. (a) Give the definition of the Taylor series of a function f about c.

[2]

(b) Find the first three terms in the Taylor expansion of $\sqrt{(2-x)}$ and state its region of convergence.

[6]

3. (a) Using integration by parts calculate

$$\int \ln x\,\mathrm{d}x.$$

[4]

(b) By using the definition of $\cot x$ in terms of $\cos x$ and $\sin x$ calculate

$$\int \cot x\,\mathrm{d}x.$$

[4]

4. Show that $y = \dfrac{1}{\sqrt{1+x^2}}$ satisfies

$$(1+x^2)\frac{\mathrm{d}y}{\mathrm{d}x} + xy = 0.$$

Use Leibnitz's theorem to deduce from this that for $n \geq 1$ we have

$$(1+x^2)\frac{\mathrm{d}^{n+1}y}{\mathrm{d}x^{n+1}} + (2n+1)x\frac{\mathrm{d}^n y}{\mathrm{d}x^n} + n^2\frac{\mathrm{d}^{n-1}y}{\mathrm{d}x^{n-1}} = 0.$$

[8]

Turn over ...

5. Find the general solution to the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = x.$$

[**8**]

6. Find the length of the curve given by

$$y = \frac{\cosh 3x}{3}$$

between $x = 0$ and $x = 1$. [**8**]

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. (a) State and prove an identity involving $\mathrm{sech}^2 x$ and $\tanh^2 x$.

[3]

(b) If $\tanh x = \alpha$ find an expression for x.

[2]

(c) Solve for x the equation

$$3\mathrm{sech}^2 x + 4\tanh x + 1 = 0.$$

[5]

(d) Find the area bounded by the curve

$$y = x^2 \sin\left(x - \frac{\pi}{4}\right)$$

the x-axis, and the lines $x = -\frac{5\pi}{4}$ and $x = \frac{3\pi}{4}$. (Remember that area beneath the x-axis should *not* be regarded as negative.)

[16]

8. A function of two variables, $f(x, y)$, has stationary points where both $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ simultaneously. What is the test that can be used to determine the nature of the stationary points? **[2]**

The function

$$f(x, y) = x^4 + 2x^2y^2 + y^4 - 2(x + y)^2$$

has three stationary points. Verify that $(x, y) = (0, 0)$ is one of these points, and find the other two. **[8]**

Show that the usual test for identifying stationary points fails for $(0, 0)$ but works for the other two points. Identify whether each of these other two stationary points of this function is a maximum, minimum or saddle point. **[10]**

By examining $f(x, y)$ along $x = y$ and $x = -y$ show that $(0, 0)$ cannot be either a maximum or minimum. **[6]**

Turn over ...

9. (a) Find the solutions of the differential equation

$$\left(y + xe^{x^2}\right)\frac{dy}{dx} + x + (1 + 2x^2)ye^{x^2} = 0.$$

[8]

(b) Find the solutions of the differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - 8y = 1 + x.$$

[9]

(c) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = e^x + e^{-x}.$$

[9]

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