Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. An ellipse has foci at $(\pm 1, 2)$ and major axis of length 6. Find its Cartesian equation.

2. Prove by induction that

$$\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1).$$

3. Using the method of differences to find the sum the first n terms of the series

$$S_n = 3 + 5x + 7x^2 + 9x^3 + \cdots + (2n+1)x^{n-1} + \cdots.$$

4. Solve the recurrence relations

   (i) $u_{n+1} = 4u_n + 1 \quad n = 0, 1, 2, \ldots,$

   (ii) $u_{n+2} - 3u_{n+1} + 2u_n = 3, \quad n = 0, 1, 2, \ldots \quad u_0 = 0,\ u_1 = 1.$

5. A is the matrix given by

$$\mathsf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 4 & 3 & a \end{pmatrix}.$$

   (i) Find the values of a for which A is singular.

   (ii) Find the inverse matrix of A when $a = 0$.

6. Find all six complex solutions to

$$z^6 = -1.$$

Turn over ...

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. (a) Find the general solution of the equation

$$\cos x + \sqrt{3}\sin x = 1$$

expressing your answer in terms of π.

(b) Express $\cos 3\theta$ in terms of $\cos\theta$.

(c) Write down the definition of $\tan^{-1} x$, giving the domain and range of the function.

(d) Express $\sin(2\tan^{-1} x)$ in terms of x only.

8. (a) A relation $\sim$ is defined on the set $A = \{a, b, c, d\}$. Give the definitions of symmetric, reflexive and transitive relations.

Say whether the following relations are are symmetric, reflexive or transitive, giving reasons:

(i) $\sim = \{(a,a),(a,d),(b,b),(c,c),(d,a),(d,d)\}$,

(ii) $\sim = \{(a,a),(a,c),(b,b),(c,a),(c,c)\}$,

(iii) $\sim = \{(a,a),(a,b),(b,a),(b,c),(c,b),(c,c),(d,d)\}$.

(b) For any set A, the power set $P(A)$ is the set of all subsets of A.

(i) Write down all the elements of $P(A)$ when $A = \{0,1,2\}$.

(ii) Draw a Hasse diagram for the partially ordered set $(P(A), \subseteq)$

(iii) Give the lower bounds for the subset of $P(A)$ given by $\{\{0,1\},\{0,2\}\}$. What is the greatest lower bound?

9. (a) Write the following system of equations in matrix form:

$$\begin{aligned} x + y + z &= 1, \\ 2x + 4y + 3z &= 3, \\ 2x + 6y + (3 + a^2)z &= -4a, \end{aligned}$$

where a is a constant. By considering augmented matrix for this problem, say how many solutions there are for this system of equations for the three cases $a = -1$, 0 and 1. Do *not* calculate the solutions to the equations.

(b) Using row manipulation methods, find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 4 & 0 \\ 2 & 1 & 3 \\ 2 & 1 & 2 \end{pmatrix}.$$

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